

Wikipedia mining of hidden links between political leaders

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Abstract. We describe a new method of reduced Google matrix which allows to establish direct and hidden links between a subset of nodes of a large directed network. This approach uses parallels with quantum scattering theory, developed for processes in nuclear and mesoscopic physics and quantum chaos. The method is applied to the Wikipedia networks in different language editions analyzing several groups of political leaders of USA, UK, Germany, France, Russia and G20. We demonstrate that this approach allows to recover reliably direct and hidden links among political leaders. We argue that the reduced Google matrix method can form the mathematical basis for studies in social and political sciences analyzing Leader-Members eXchange (LMX).

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1 Introduction

At present a free online encyclopaedia Wikipedia [1] becomes the largest open source of knowledge being close to Encyclopaedia Britannica [2] by an accuracy of scientific entries [3] and overcoming the later by an enormous amount of available information. A detailed analysis of strong and weak features of Wikipedia is given at [4,5].

Since Wikipedia articles make citations to each other they generate a larger directed network with a rather clear meaning of nodes defined by article titles. Due to these reasons it is interesting to apply algorithms developed for search engines of World Wide Web (WWW), those like the PageRank algorithm [6] (see also [7]), to analyze the ranking properties and relations between Wikipedia articles. The clear meaning of Wikipedia nodes allows also to use its network as a test bed for machine learning algorithms computing semantic relatedness [8].

It is convenient to describe the network of N Wikipedia articles by the Google matrix G constructed from the adjacency matrix A_{ij} with elements 1 if article (node) j points to article (node) i and zero otherwise. Then the matrix elements of the Google matrix take the standard form [6,7]

$$G_{ij} = \alpha S_{ij} + (1 - \alpha)/N, \quad (1)$$

where S is the matrix of Markov transitions with elements $S_{ij} = A_{ij}/k_{out}(j)$, $k_{out}(j) = \sum_{i=1}^N A_{ij} \neq 0$ being the node j out-degree (number of outgoing links) and with $S_{ij} = 1/N$ if j has no outgoing links (dangling node). Here $0 < \alpha < 1$ is the damping factor which for a random surfer determines the probability $(1 - \alpha)$ to jump to any node. The properties of spectrum and eigenstates of

G have been discussed in detail for Wikipedia and other directed networks (see e.g. [9,10]).

The right eigenvectors $\psi_i(j)$ of G are determined by the equation:

$$\sum_{j'} G_{jj'} \psi_i(j') = \lambda_i \psi_i(j). \quad (2)$$

The PageRank eigenvector $P(j) = \psi_{i=0}(j)$ corresponds to the largest eigenvalue $\lambda_{i=0} = 1$ [6,7]. It has positive elements which give a probability to find a random surfer on a given node in the stationary long time limit of the Markov process. All nodes can be ordered by a monotonically decreasing probability $P(K_i)$ with the highest probability at $K = 1$. The index K is the PageRank index. Left eigenvectors are biorthogonal to right eigenvectors of different eigenvalues. The left eigenvector for $\lambda = 1$ has identical (unit) entries due to the column sum normalization of G . One can show that the damping factor α in (1) only affects the PageRank vector (or other eigenvectors for $\lambda = 1$ of S in case of a degeneracy) while other eigenvectors are independent of α due to their orthogonality to the left unit eigenvector for $\lambda = 1$ [7]. Thus all eigenvalues, except $\lambda = 1$, are multiplied by a factor α when replacing S by G . In the following we use the notations ψ_L^T and ψ_R for left and right eigenvectors respectively (here T means vector or matrix transposition).

In many real networks the number of nonzero elements in a column of S is significantly smaller than the whole matrix size N which allows to find efficiently the PageRank vector by the PageRank algorithm of power iterations [7]. Also a certain number of largest eigenvalues (in modulus)

and related eigenvectors can be efficiently computed by the Arnoldi algorithm (see e.g. [10]).

For various language editions of Wikipedia it was shown that the PageRank vector produces a reliable ranking of historical figures over 35 centuries of human history [11,12,13,14] and Wikipedia ranking of world universities (WRWU) [11,15]. Thus the Wikipedia ranking of historical figures is in a good agreement with the well-known Hart ranking [16], while the WRWU is in a good agreement with the Shanghai Academic ranking of world universities [17]. At the same time Wikipedia ranking produces some new elements additional to these classifications.

In addition to the matrix G it is useful to introduce a Google matrix G^* constructed from the adjacency matrix of the same network but with inverted direction of all links. The statistical properties of the eigenvector P^* of G^* with the largest eigenvalue $\lambda = 1$ have been studied first for the Linux Kernel network [18] showing that there are nontrivial correlations between P and P^* vectors of the network. More detailed studies have been done for Wikipedia and other networks [11,10]. The vector $P^*(K^*)$ is called the CheiRank vector and the index numbering nodes in order of monotonic decrease of probability P^* is noted as CheiRank index K^* . Thus, nodes with many ingoing links have small value of $K^* = 1, 2, 3, \dots$ and nodes with many outgoing links have $K^* = 1, 2, 3, \dots$ [7,10]. Examples of density distributions for Wikipedia editions EN, DE, FR, RU from year 2013 (see network data in [14]) are shown in Fig. 1.

Other eigenvectors of G have $|\lambda| \leq \alpha$ [7,10]. For Wikipedia it was shown that the eigenvectors with a large modulus of λ select some specific communities of Wikipedia network [9,10]. However, a priori it is not possible to know what are the meanings of these communities. Thus other methods are required to determine effective interactions between N_r nodes of a specific subset (group) of the global network of a large size N .

Recently, the method of reduced Google matrix has been proposed for analysis of effective interactions between nodes of a selected subset embedded into a large size network [19]. This approach uses parallels with the quantum scattering theory, developed for processes in nuclear and mesoscopic physics and quantum chaos. In this work we apply this method to subsets (groups) of Wikipedia articles about political leaders (politicians) considering English, French, German and Russian Wikipedia editions and politicians of USA, UK, Germany (DE), France (FR) and Russia. The total number of nodes for these Wikipedia networks is $N = 4212493$ (EN), 1532978 (DE), 1352835 (FR), 966284 (RU) [14]. We also analyze interactions between political leaders of G20 Los Cabos summit in 2012 [20]. The selected subsets have networks of 20 or 40 nodes and are well suited for analysis of direct and hidden links between politicians. In our analysis we use the Wikipedia networks collected in 2013 and described in [14]. The location of selected nodes on the PageRank-CheiRank plane ($\ln K, \ln K^*$) is shown in Fig. 1. The obtained results al-

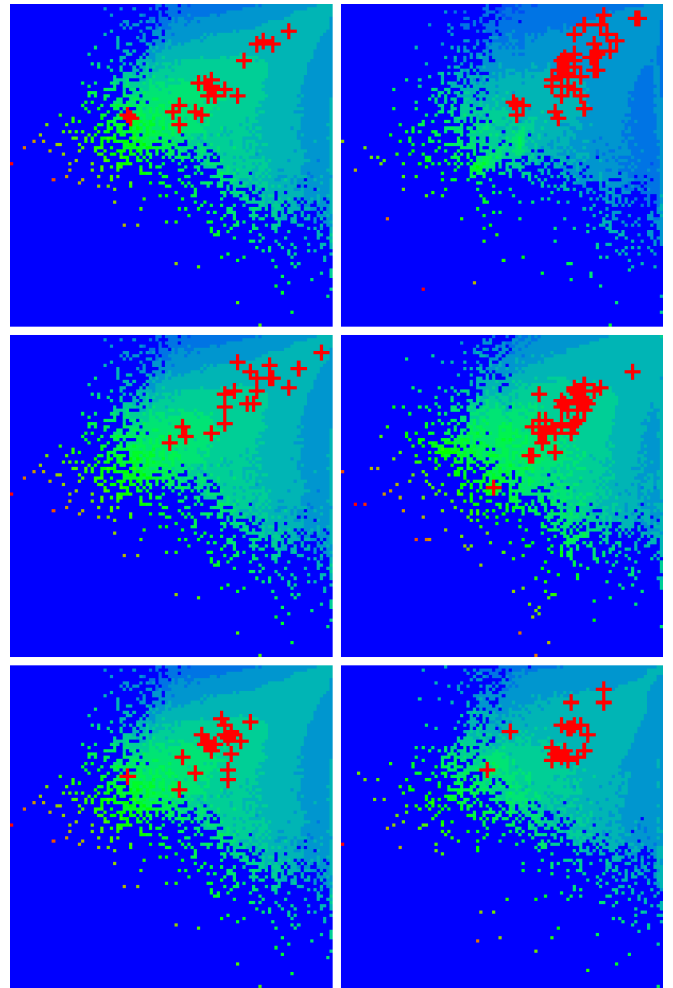


Fig. 1. Density of nodes $W(K, K^*)$ on the PageRank-CheiRank plane averaged over 100×100 logarithmic equidistant grids for $0 \leq \ln K, \ln K^* \leq \ln N$. The density is averaged over all nodes inside each cell of the grid with the x -axis corresponding to $\ln K$ and the y -axis to $\ln K^*$. The left column of panels correspond all to the English (Enwiki) and the right column of panels to the German (Dewiki, top), French (Frwiki, center) and Russian (Ruwiki, bottom) Wikipedia editions of 2013 collected in [14]. The colors represent maximum (red), intermediate (green) and minimum (blue) values. The red crosses show the positions of the selected reduced network nodes for 20 US (top left), 20 UK (center left), 40 German (DE, top right), 40 French (FR, center right), 20 Russian (RU, bottom right) politicians and 20 state leaders of G20 states (bottom left) with the list of names given in Tables 2-7 and in Figs.2-7.

low to determine interesting direct and hidden relations between political leaders of the selected countries.

We should note that the analysis of interactions and relations between political leaders represents a hot topic in social and political sciences (see e.g. [21]). Thus the interactions between leader and group members, known at Leader-Members eXchange (LMX), attracts at present active investigations of researchers in social and political

sciences (see e.g. [22,23,24]). However, only very recently the methods of complex networks (see e.g. [25]) started to be used by in the LMX analysis (see e.g. [26]). In this work we argue that the reduced Google matrix G_R approach represents a useful efficient tool for the LMX analysis of interactions inside a group of people. Thus for a group of politicians (a group of their articles at Wikipedia) we find that those at the top of PageRank index K are the dominant leaders being usually country presidents or prime-ministers. It turns out that the obtained G_R matrix describing the interactions between group members is composed of three matrix components which describe the direct interactions between group members, G_{rr} , a projector part G_{pr} which is mainly imposed by the PageRank of group members given by the global G matrix and a component G_{qr} from hidden interactions between members which appear due to indirect links via the global network. Thus the reduced matrix $G_R = G_{rr} + G_{pr} + G_{qr}$ allows to obtain precise information about the group members taking into account their environment given by the global Wikipedia network. We think that this G_R matrix approach provides mathematical grounds for the LMX studies.

The paper is composed as follows: Section 2 shortly describes the method of reduced Google matrix, Section 3 presents distributions of selected subsets on the PageRank-Cheirank plane of global and reduced networks, Sections 4, 5, 6, 7, 8, and 9 describe the results of the reduced Google matrix analysis for politicians of USA, UK, Germany, France, Russia and G20 respectively. Section 10 provides a particular analysis for the group of French politicians in terms of effective networks of strongest friends or followers using either the matrix G_R or the hidden interactions given by the component G_{qr} . The discussion of the obtained results is given in Section 11. All numerical data of the reduced Google matrix of groups of political leaders considered here are publicly available at the web site [27].

2 Reduced Google matrix

The concept of reduced Google matrix G_R was introduced in [19] on the basis of the following observation. At present directed networks of real systems can be very large (e.g. 4.2 millions for the English Wikipedia edition in 2013 [10] or 3.5 billion web pages for a publicly accessible web crawl that was gathered by the Common Crawl Foundation in 2012 [28]). In certain cases one may be interested in the particular interactions among a small reduced subset of N_r nodes with $N_r \ll N$ instead of the interactions in the entire network. However, the interactions between these N_r nodes should be correctly determined taking into account that there are many indirect links between the N_r nodes via all other $N_s = N - N_r$ nodes of the network. This leads to the problem of the reduced Google matrix G_R with N_r nodes which describes the interactions of a subset of N_r nodes.

In a certain sense we can trace parallels with the problem of quantum scattering appearing in nuclear and mesoscopic physics (see e.g. [29,30,31,32,33]) and quantum chaotic

scattering (see e.g. [34]). Indeed, in the scattering problem there are effective interactions between open channels to localized basis states in a well confined scattering domain where a particle can spend a certain time before its escape to open channels. Having this analogy in mind we construct the reduced Google matrix G_R which describes interactions between selected N_r nodes and satisfies the standard requirements of the Google matrix.

Let G be a typical Google matrix of Perron-Frobenius type for a network with N nodes such that $G_{ij} \geq 0$ and the column sum normalization $\sum_{i=1}^N G_{ij} = 1$ are verified. We consider a sub-network with $N_r < N$ nodes, called “reduced network”. In this case we can write G in a block form :

$$G = \begin{pmatrix} G_{rr} & G_{rs} \\ G_{sr} & G_{ss} \end{pmatrix} \quad (3)$$

where the index “ r ” refers to the nodes of the reduced network and “ s ” to the other $N_s = N - N_r$ nodes which form a complementary network which we will call “scattering network”.

We denote the PageRank vector of the full network as

$$P = \begin{pmatrix} P_r \\ P_s \end{pmatrix} \quad (4)$$

which satisfies the equation $GP = P$ or in other words P is the right eigenvector of G for the unit eigenvalue. This eigenvalue equation reads in block notations:

$$(\mathbf{1} - G_{rr}) P_r - G_{rs} P_s = 0, \quad (5)$$

$$-G_{sr} P_r + (\mathbf{1} - G_{ss}) P_s = 0. \quad (6)$$

Here $\mathbf{1}$ is a unit diagonal matrix of corresponding size N_r or N_s . Assuming that the matrix $\mathbf{1} - G_{ss}$ is not singular, i.e. all eigenvalues G_{ss} are strictly smaller than unity (in modulus), we obtain from (6) that

$$P_s = (\mathbf{1} - G_{ss})^{-1} G_{sr} P_r \quad (7)$$

which gives together with (5):

$$G_R P_r = P_r \quad , \quad G_R = G_{rr} + G_{rs}(\mathbf{1} - G_{ss})^{-1} G_{sr} \quad (8)$$

where the matrix G_R of size $N_r \times N_r$, defined for the reduced network, can be viewed as an effective reduced Google matrix. Here the contribution of G_{rr} accounts for direct links in the reduced network and the second term with the matrix inverse corresponds to all contributions of indirect links of arbitrary order. We note that in mesoscopic scattering problems one typically uses an expression of the scattering matrix which has a similar structure where the scattering channels correspond to the reduced network and the states inside the scattering domain to the scattering network [31].

The matrix elements of G_R are non-negative since the matrix inverse in (8) can be expanded as:

$$(\mathbf{1} - G_{ss})^{-1} = \sum_{l=0}^{\infty} G_{ss}^l. \quad (9)$$

In (9) the integer l represents the order of indirect links, i. e. the number of indirect links which are used to connect indirectly two nodes of the reduced network. The matrix inverse corresponds to an exact resummation of all orders of indirect links. According to (9) the matrix $(\mathbf{1} - G_{ss})^{-1}$ and therefore also G_R have non-negative matrix elements. It remains to show that G_R also fulfills the condition of column sum normalization being unity. For this let us denote by $E^T = (1, \dots, 1)$ the line vector of size N with unit entries and by E_r^T (or E_s^T) the corresponding vectors for the reduced (or scattering) network with N_r (or N_s) unit entries such that $E^T = (E_r^T, E_s^T)$. The column sum normalization for the full Google matrix G implies that $E^T G = E^T$, i. e. E^T is the left eigenvector of G with eigenvalue 1. This equation becomes in block notation:

$$E_r^T (\mathbf{1} - G_{rr}) - E_s^T G_{sr} = 0, \quad (10)$$

$$-E_r^T G_{rs} + E_s^T (\mathbf{1} - G_{ss}) = 0. \quad (11)$$

From (11) we find that $E_s^T = E_r^T G_{rs} (\mathbf{1} - G_{ss})^{-1}$ which implies together with (10) that $E_r^T G_R = E_r^T$ using G_R as in (8). This shows that the column sum normalization condition is indeed verified for G_R justifying that this matrix is indeed an effective Google matrix for the reduced network.

We can question how to evaluate practically the expression (8) of G_R for a particular sparse and quite large network with a typical situation when $N_r \sim 10^2$ - 10^3 is small compared to N and $N_s \approx N \gg N_r$. If N_s is too large (e. g. $N_s \sim 10^5$) a direct naive evaluation of the matrix inverse $(\mathbf{1} - G_{ss})^{-1}$ in (8) by Gauss algorithm is not possible. In this case we can try the expansion (9) provided it converges sufficiently fast with a modest number of terms. However, this is most likely not the case for typical applications.

Therefore, we consider the situation where the full Google matrix has a well defined gap between the leading unit eigenvalue and the second largest eigenvalue (in modulus). For example if G is defined using a damping factor α in the standard way, as in (1), the gap is at least $1 - \alpha$ which is 0.15 for the standard choice $\alpha = 0.85$ [7]. For such a situation we expect that the matrix G_{ss} has a leading real eigenvalue close to unity (but still different from unity so that $\mathbf{1} - G_{ss}$ is not singular) while the other eigenvalues are clearly below this leading eigenvalue with a gap comparable to the gap of the full Google matrix G . In order to evaluate the expansion (9) efficiently, we need to take out analytically the contribution of the leading eigenvalue close to unity which is responsible for the slow convergence.

Below we denote by λ_c this leading eigenvalue and by ψ_R (ψ_L^T) the corresponding right (left) eigenvector such that $G_{ss}\psi_R = \lambda_c\psi_R$ (or $\psi_L^T G_{ss} = \lambda_c\psi_L^T$). Both left and right eigenvectors as well as λ_c can be efficiently computed by the power iteration method in a similar way as the standard PageRank method. We note that one can easily show that λ_c must be real and that both left/right eigenvectors can be chosen with positive elements. Concerning the normalization for ψ_R we choose $E_s^T \psi_R = 1$ and for ψ_L we choose $\psi_L^T \psi_R = 1$. It is well known (and easy to

Table 1. Values of $1 - \lambda_c$ and of Σ_P for the five groups of politicians and four cases of G20 subnetworks with λ_c being the leading eigenvalue of the matrix G_{ss} and $\Sigma_P = \|P_r\|_1$ being the 1-norm of the projected PageRank representing the relative PageRank weight of the subset N_r in the full network.

Network	Group	$1 - \lambda_c$	Σ_P
Enwiki	Politicians US	0.0003680	0.0003787
Enwiki	Politicians UK	0.0001041	0.0001068
Dewiki	Politicians DE	0.0002705	0.0002802
Frwiki	Politicians FR	0.0003810	0.0004005
Ruwiki	Politicians RU	0.0003136	0.0003247
Enwiki	G20 EN	0.0002465	0.0002508
Dewiki	G20 DE	0.0001492	0.0001513
Frwiki	G20 FR	0.0002131	0.0002159
Ruwiki	G20 RU	0.0002707	0.0002734

show) that ψ_L^T is orthogonal to all other right eigenvectors (and ψ_R is orthogonal to all other left eigenvectors) of G_{ss} with eigenvalues different from λ_c . We introduce the operator $\mathcal{P}_c = \psi_R \psi_L^T$ which is the projector onto the eigenspace of λ_c and we denote by $\mathcal{Q}_c = \mathbf{1} - \mathcal{P}_c$ the complementary projector. One verifies directly that both projectors commute with the matrix G_{ss} and in particular $\mathcal{P}_c G_{ss} = G_{ss} \mathcal{P}_c = \lambda_c \mathcal{P}_c$. Therefore we can write:

$$(\mathbf{1} - G_{ss})^{-1} = (\mathcal{P}_c + \mathcal{Q}_c)(\mathbf{1} - G_{ss})^{-1}(\mathcal{P}_c + \mathcal{Q}_c) \quad (12)$$

$$= \mathcal{P}_c \frac{1}{1 - \lambda_c} + \mathcal{Q}_c (\mathbf{1} - G_{ss})^{-1} \mathcal{Q}_c \quad (13)$$

$$= \mathcal{P}_c \frac{1}{1 - \lambda_c} + (\mathbf{1} - \bar{G}_{ss})^{-1} \mathcal{Q}_c \quad (14)$$

$$= \mathcal{P}_c \frac{1}{1 - \lambda_c} + \mathcal{Q}_c \sum_{l=0}^{\infty} \bar{G}_{ss}^l \quad (15)$$

with $\bar{G}_{ss} = \mathcal{Q}_c G_{ss} \mathcal{Q}_c$ and using the standard identity $\mathcal{P}_c \mathcal{Q}_c = 0$ for complementary projectors. The expansion in (15) has the advantage that it converges rapidly since $\bar{G}_{ss}^l \sim |\lambda_{c,2}|^l$ with $\lambda_{c,2}$ being the second largest eigenvalue which is significantly lower than unity (e. g. $|\lambda_{c,2}| \approx \alpha = 0.85$ for the case with a damping factor). The first contribution due to the leading eigenvalue λ_c close to unity is taken out analytically once the left and right eigenvectors, and therefore also the projector \mathcal{P}_c , are known.

The combination of (8) and (15) provides an explicit algorithm feasible for a numerical implementation for the case of modest values of N_r , large values of N_s and of course for sparse matrices G , G_{ss} etc. The described method can also be modified to take out analytically the contributions of several leading eigenvalues close to unity as described in [19]. We note that the numerical methods described in [35] allow to determine the eigenvalues λ_c (and corresponding eigenvectors) which are exponentially close to unity (e.g. $1 - \lambda_c \sim 10^{-16}$) so that the expression (12) can be efficiently computed numerically.

On the basis of above equations (8)-(12) the reduced Google matrix can be presented as a sum of three compo-

nents

$$G_R = G_{rr} + G_{pr} + G_{qr}, \quad (16)$$

with the first component G_{rr} given by direct matrix elements of G among the selected N_r nodes, the second projector component G_{pr} is given by

$$G_{pr} = G_{rs} \mathcal{P}_c G_{sr} / (1 - \lambda_c), \quad \mathcal{P}_c = \psi_R \psi_L^T. \quad (17)$$

We mention that this contribution is of the form $G_{pr} = \tilde{\psi}_R \tilde{\psi}_L^T / (1 - \lambda_c)$ with $\tilde{\psi}_R = G_{rs} \psi_R$ and $\tilde{\psi}_L^T = \psi_L^T G_{sr}$ being two small vectors defined on the reduced space of dimension N_r . Therefore G_{pr} is indeed a (small) matrix of rank one which is also confirmed by a numerical diagonalization of this matrix. The third component G_{qr} of indirect or hidden links is given by

$$G_{qr} = G_{rs} [\mathcal{Q}_c \sum_{l=0}^{\infty} \bar{G}_{ss}^l] G_{sr}, \quad \mathcal{Q}_c = \mathbf{1} - \mathcal{P}_c, \quad \bar{G}_{ss} = \mathcal{Q}_c G_{ss} \mathcal{Q}_c. \quad (18)$$

Even though the decomposition (16) is at first motivated by the numerical efficiency to evaluate the matrix inverse it is equally important concerning the interpretation of the different terms and especially the last contribution (18) which is typically rather small as compared to (17) plays in an important role as we will see below.

Concerning the numerical algorithm to evaluate all contributions in (16), we mention that we first determine by the power iteration method the leading left ψ_L and right eigenvector ψ_R of the matrix G_{ss} which also provides an accurate value of the corresponding eigenvalue λ_c or better of $1 - \lambda_c$ (by taking the norm of the projection of $G\psi_R$ on the reduced space which is highly accurate even for λ_c close to 1). These two vectors provide directly G_{pr} by (17) and allow to numerically apply the projector \mathcal{Q}_c to an arbitrary vector (with $\sim N$ operations). The most expensive part is the evaluation of the last contribution according to (18). For this we apply successively G_{ss} and \mathcal{Q}_c to an arbitrary column of G_{sr} which can be done by a sparse matrix vector multiplication or the efficient application of the projector. We compute simultaneously the series in (18) which converges rather quickly after about $\sim 200 - 250$ terms since the contribution of the leading eigenvalue (of G_{ss}) has been taken out and the eigenvalues of \bar{G}_{ss} are roughly below the damping factor $\alpha = 0.85$. In the end the resulting vector is multiplied with the matrix G_{rs} which provides one column of G_{qr} . This procedure has to be repeated for each of the N_r columns but the number N_r is typically very modest (20 or 40 in this work) and the computation of the different columns can actually be done in parallel on typical multicore machines.

Concerning the choice of the reduced space we used 5 groups of 20 or 40 political leaders of 5 countries (US, UK, DE, FR, RU) for 4 Wikipedia editions (EN, DE, FR, RU with EN-Wikipedia for both US and UK politicians). We also considered the group of G20 state leaders for which we used all 4 of these Wikipedia editions even though here we concentrate on the G20 data obtained for EN-Wikipedia. A detailed description of these subsets is

given in Section 3. For these data sets of politicians considered in this work we find that typically $1 - \lambda_c \sim 10^{-4}$ and the right eigenvector ψ_R of G_{ss} is rather close to the full PageRank of G (for the leading nodes in the full PageRank not belonging to the reduced space). We mention that for other data sets with more ranking PageRank members, e.g. 40 countries, one may find larger values of $1 - \lambda_c \sim 10^{-2}$. Furthermore, we find that an approximate relation holds: $1 - \lambda_c \approx \Sigma_P = \|P_r\|_1$ where Σ_P is the PageRank probability of the global network concentrated on the subset of N_r selected nodes. The data of Table 1 show that this relation works with an accuracy of a couple of percent. To understand this result mathematically, we replace in (7) the matrix inverse by the first term of (15) which gives $P_s \approx \psi_R \psi_L^T G_{sr} P_r / (1 - \lambda_c)$ (for $N_s \gg N_r$ and $1 - \lambda_c \ll 1$). Furthermore we note that $\psi_L \approx E_s$ (for most nodes) and $E_s^T G_{sr} P_r \approx E_r^T P_r = \|P_r\|_1$ such $P_s \approx \psi_R \|P_r\|_1 / (1 - \lambda_c)$. Since $\|P_s\|_1 \approx \|P_r\|_1 = 1$ we find indeed that $\|P_r\|_1 \approx 1 - \lambda_c$.

The numerical computations show that the vectors $\tilde{\psi}_R$ and $\tilde{\psi}_L^T$ introduced above below equation (17) are approximately given by $\tilde{\psi}_R = P_r$ and $\tilde{\psi}_L^T = E_r^T$ such that $G_{pr} \approx P_r E_r^T / (1 - \lambda_c)$ is rather close to a rank one Google matrix (since $\|P_r\|_1 \approx 1 - \lambda_c$) and with identical columns given by the normalized vector $P_r / (1 - \lambda_c)$. More precisely, we will indeed see in Sections 4-9, that the overall column sums of G_{pr} account for $\sim 95-97\%$ of the total column sum of G_R . In other words, in terms of probability the contribution of G_{pr} is dominant in G_R but it is also kind of trivial with nearly identical columns. Therefore the two small contributions of G_{rr} and G_{qr} are indeed very important for the interpretation even though they only contribute weakly to the overall column sum normalization.

The meaning of G_{rr} is rather clear since it gives direct links between the selected nodes. In contrast, the meaning of G_{qr} is significantly more interesting since it generates indirect links between the N_r nodes due to their interactions with the global network environment. We note that G_{qr} is composed of two parts $G_{qr} = G_{qrd} + G_{qrnd}$ where the first diagonal term G_{qrd} represents a probability to stay on the same node during multiple iterations of \bar{G}_{ss} in (18) while the second nondiagonal term G_{qrnd} represents indirect (hidden) links between the N_r nodes appearing due via the global network. We note that in principle certain matrix elements of G_{qr} can be negative, which is possible due to negative terms in $\mathcal{Q}_c = \mathbf{1} - \mathcal{P}_c$ appearing in (18). However, for all subsets considered in this work the total weight of negative elements was negligibly small (about 10^{-10} for the data of UK politicians, 0 for data of politicians of other countries, and 10^{-5} for the G20 state leader data, of the total weight 1 for G_R).

It is convenient to characterize the strength of 3 components in (16) by their respective weights W_{rr} , W_{pr} , W_{qr} given respectively by the sum of all matrix elements of G_{rr} , G_{pr} , G_{qr} divided by N_r . By definition we have $W_{rr} + W_{pr} + W_{qr} = 1$.

In the following sections we will see that all three components of (16) play important roles.

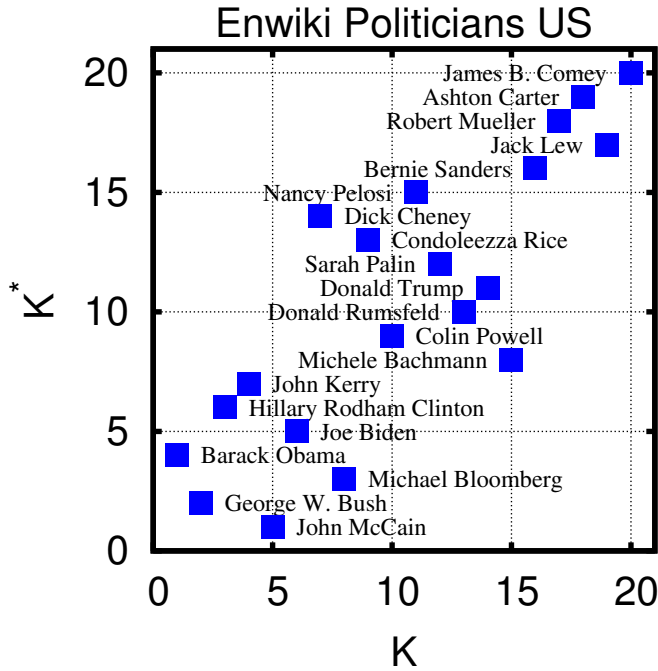


Fig. 2. Position of nodes in the local (K, K^*) plane of the reduced network for 20 US politicians in the Enwiki network. The names are shown on the same lines of the corresponding data points.

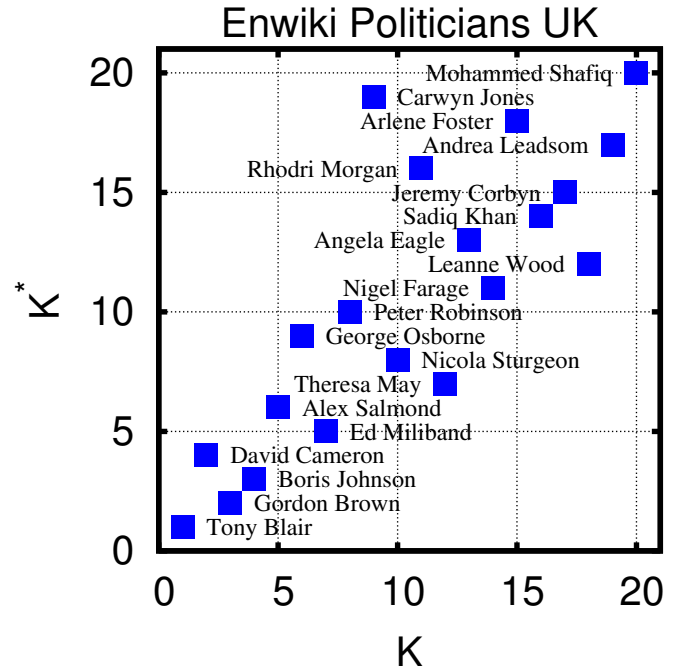


Fig. 3. Position of nodes in the local (K, K^*) plane of the reduced network for 20 UK politicians in the Enwiki network. The names are shown on the same lines of the corresponding data points.

Names (US)	K	K^*	K_G
Barack Obama	1	4	2
George W. Bush	2	2	1
Hillary Rodham Clinton	3	6	6
John Kerry	4	7	4
John McCain	5	1	3
Joe Biden	6	5	7
Dick Cheney	7	14	5
Michael Bloomberg	8	3	14
Condoleezza Rice	9	13	10
Colin Powell	10	9	12
Nancy Pelosi	11	15	8
Sarah Palin	12	12	9
Donald Rumsfeld	13	10	11
Donald Trump	14	11	20
Michele Bachmann	15	8	16
Bernie Sanders	16	16	15
Robert Mueller	17	18	13
Ashton Carter	18	19	19
Jack Lew	19	17	18
James B. Comey	20	20	17

Table 2. List of names of 20 selected US politicians and the corresponding PageRank index values K , K^* and K_G of the reduced network matrices G_R , G_R^* and G_{rr+qr} given as the sum of G_{rr} and G_{qr} (without diagonal elements). All matrices were computed for English Wikipedia edition of 2013.

3 PageRank-CheiRank plane of Wikipedia editions

For our studies we choose 4 independent groups of articles of 20 US and 20 UK politicians from Eniki, 40 German and 40 French politicians from Dewiki and Frwiki respectively and 20 Russian politicians from Ruwiki. The information about number of nodes and links for each Wikipedia is available at [14]. In the selection of names of political leaders of each country we used the names appearing at the top of Google search on e.g. “politicians of Russia”, in addition we take only those politician who were active in the period not more than 10 - 20 years before the collection date of our Wikipedia editions of 2013. A few names are used to have a group of 20 or 40. We do not pretend that we selected all important politicians of a given country but we suppose that the main part of them is present in our selection.

For each group (or subset of N_r nodes) we order politicians by their PageRank probability in the corresponding global Wikipedia network. After such ordering we obtain local rank PageRank index K changing from 1 to 20 (or 40). The best known politicians are found to be at the top values $K = 1, 2, \dots$. In addition we determine the local CheiRank index K^* of the selected names using the CheiRank vector of the global network. At the top of K^* we have most communicative articles of politicians. Then we present the distribution of politicians on the PageRank-CheiRank plane of local indexes (K, K^*) in Figs. 2,3,4,5,6 for US, UK, DE, FR, RU respectively. The full names of political leaders are given respectively in

Names (UK)	K	K^*	K_G
Tony Blair	1	1	2
David Cameron	2	4	3
Gordon Brown	3	2	1
Boris Johnson	4	3	7
Alex Salmond	5	6	5
George Osborne	6	9	6
Ed Miliband	7	5	4
Peter Robinson	8	10	12
Carwyn Jones	9	19	13
Nicola Sturgeon	10	8	9
Rhodri Morgan	11	16	16
Theresa May	12	7	8
Angela Eagle	13	13	15
Nigel Farage	14	11	10
Arlene Foster	15	18	14
Sadiq Khan	16	14	11
Jeremy Corbyn	17	15	17
Leanne Wood	18	12	18
Andrea Leadsom	19	17	19
Mohammed Shafiq	20	20	20

Table 3. Same as Table 2 for 20 selected UK politicians and the English Wikipedia edition of 2013.

Tables 2,3,4,5,6 with corresponding values of local K, K^* indexes (we discuss the meaning of additional index K_G later).

For US case shown in Fig. 2 and Table 2 we find that Obama, Bush and Clinton takes the top three K positions that looks rather natural. However, the most communicative politician being at the top of CheiRank with $K^* = 1, 2, 3$ are McCain, Bush, Bloomberg.

In Fig. 3 and Table 3 the names and distribution of 20 UK politicians are shown. The top 3 positions of PageRank are taken by UK prime ministers Blair, Cameron, Brown with $K = 1, 2, 3$. The distribution in (K, K^*) plane is more centered in a diagonal vicinity comparing to US case and other countries discussed below. For $K^* = 1, 2, 3$ we have Blair, Brown, Johnson. The present prime minister May is rather far in rank indexes.

The PageRank-CheiRank distribution of German politicians is shown in Fig. 4 with the full names and ranks given in Table 4. The top PageRank values $K = 1, 2, 3$ are taken by the chancellors Kohl, Schmidt, Merkel with Schröder at $K = 4$. However, the most communicative politicians are Gauck (DE president, $K^* = 1$), Merkel ($K^* = 2$), Lafontaine ($K^* = 3$).

The French politicians are presented in Fig. 5 and Table 5. Here we choose names of those who are really active in the period 2007 - 2013, thus e.g. Jacques Chirac is not included in the list. The top 3 positions in K and K^* are taken by two presidents Sarkozy, Hollande with J.-M. Le Pen at the third position. We note a large dispersion of positions on (K, K^*) plane for the main part of politicians. Thus Royal ($K = 4$) and Raffarin ($K = 5$) have rather high values $K^* = 13, 32$ respectively. For French

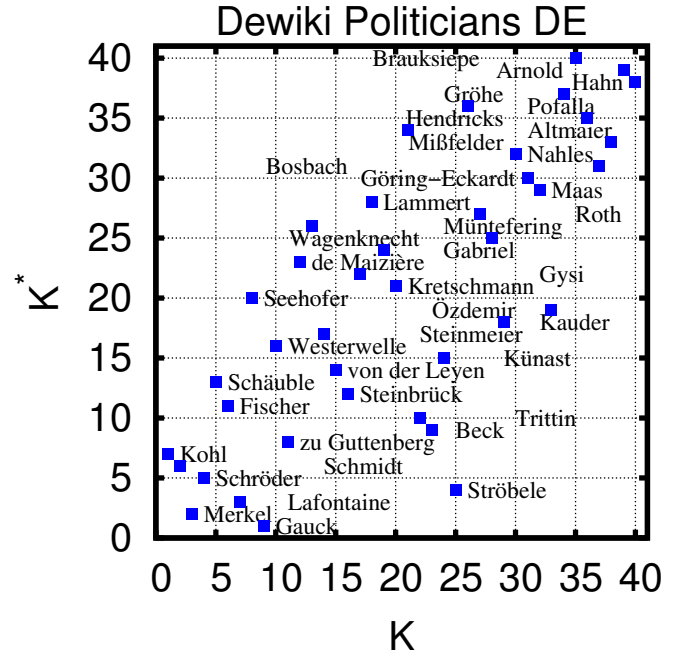


Fig. 4. Position of nodes in the local (K, K^*) plane of the reduced network for 40 DE politicians in the Dewiki network. The names are shown on the same lines of the corresponding data points. Some of the names are horizontally shifted for a better visibility.

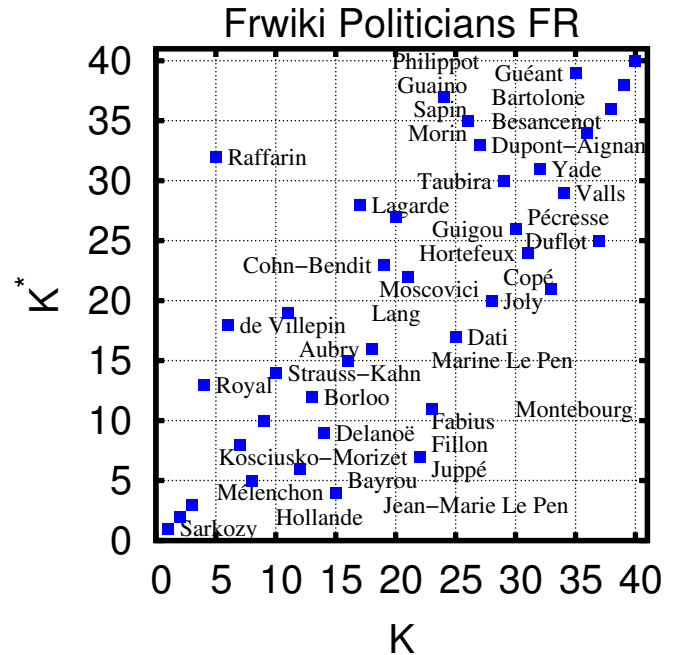


Fig. 5. Position of nodes in the local (K, K^*) plane of the reduced network for 40 FR politicians in the Frwiki network. The names are shown on the same lines of the corresponding data points. Some of the names are horizontally shifted for a better visibility.

Names (DE)	K	K^*	K_G
Helmut Kohl	1	7	3
Helmut Schmidt	2	6	6
Angela Merkel	3	2	1
Gerhard Schröder	4	5	2
Wolfgang Schäuble	5	13	9
Joschka Fischer	6	11	7
Oskar Lafontaine	7	3	4
Horst Seehofer	8	20	20
Joachim Gauck	9	1	24
Guido Westerwelle	10	16	14
Karl-Theodor zu Guttenberg	11	8	13
Thomas de Maizière	12	23	16
Franz Müntefering	13	26	5
Frank-Walter Steinmeier	14	17	10
Ursula von der Leyen	15	14	25
Peer Steinbrück	16	12	8
Gregor Gysi	17	22	17
Norbert Lammert	18	28	22
Sigmar Gabriel	19	24	12
Winfried Kretschmann	20	21	35
Peter Altmaier	21	34	29
Jürgen Trittin	22	10	15
Volker Beck	23	9	32
Renate Künast	24	15	23
Hans-Christian Ströbele	25	4	11
Ronald Pofalla	26	36	19
Claudia Roth	27	27	21
Sahra Wagenknecht	28	25	28
Volker Kauder	29	18	34
Andrea Nahles	30	32	18
Katrin Göring-Eckardt	31	30	26
Heiko Maas	32	29	30
Cem Özdemir	33	19	33
Hermann Gröhe	34	37	31
Ralf Brauksiepe	35	40	38
Barbara Hendricks	36	35	36
Wolfgang Bosbach	37	31	27
Philipp Mißfelder	38	33	37
Rainer Arnold	39	39	39
Florian Hahn	40	38	40

Table 4. Same as Table 2 for 40 selected DE politicians and the German Wikipedia edition of 2013.

politicians in Table 5 we mark by color their membership in political parties. The effect of interactions and links between different parties is discussed in the last Section.

The names of 20 Russian politicians and their distribution on (K, K^*) plane are presented in Fig. 6 and Table 6. Similar to the case of France the two top PageRank positions are taken by presidents Putin, Medvedev with Zhirinovsky at the third position $K = 3$. For $K^* = 1, 2, 3$ we have Putin, Matvienko, Yavlinsky while Medvedev is at far $K^* = 12$ showing low communicative properties of his Wikipedia article.

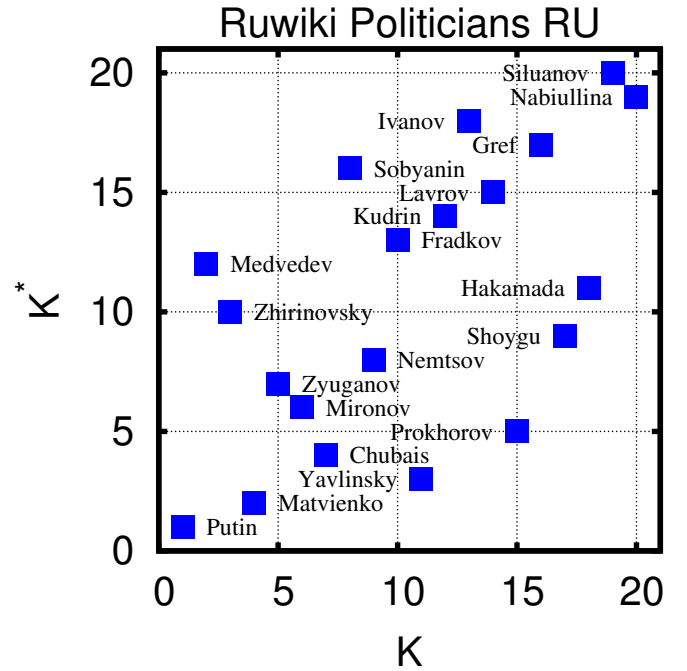


Fig. 6. Position of nodes in the local (K, K^*) plane of the reduced network for 20 RU politicians in the Ruwiki network. The names are shown on the same lines of the corresponding data points.

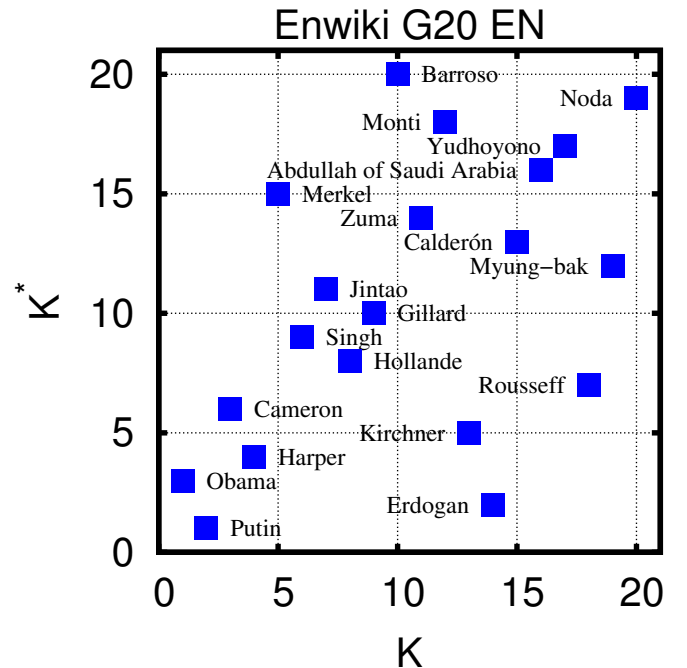


Fig. 7. Position of nodes in the local (K, K^*) plane of the reduced network for 20 state leaders of G20 states in the Enwiki network. The names are shown on the same lines of the corresponding data points.

Names (FR)	K	K^*	K_G
Nicolas Sarkozy	1	1	1
François Hollande	2	2	2
Jean-Marie Le Pen	3	3	11
Ségolène Royal	4	13	3
Jean-Pierre Raffarin	5	32	10
Dominique de Villepin	6	18	12
François Fillon	7	8	4
François Bayrou	8	5	6
Laurent Fabius	9	10	8
Dominique Strauss-Kahn	10	14	7
Jack Lang	11	19	13
Alain Juppé	12	6	5
Jean-Louis Borloo	13	12	19
Bertrand Delanoë	14	9	18
Jean-Luc Mélenchon	15	4	14
Marine Le Pen	16	15	17
Christine Lagarde	17	28	25
Martine Aubry	18	16	9
Daniel Cohn-Bendit	19	23	32
Valérie Pécresse	20	27	29
Jean-François Copé	21	22	15
Nathalie Kosciusko-Morizet	22	7	26
Arnaud Montebourg	23	11	16
Claude Bartolone	24	37	28
Rachida Dati	25	17	35
Olivier Besancenot	26	35	37
Nicolas Dupont-Aignan	27	33	30
Eva Joly	28	20	21
Christiane Taubira	29	30	38
Élisabeth Guigou	30	26	23
Brice Hortefeux	31	24	27
Rama Yade	32	31	39
Pierre Moscovici	33	21	20
Manuel Valls	34	29	22
Claude Guéant	35	39	34
Hervé Morin	36	34	24
Cécile Duflot	37	25	36
Michel Sapin	38	36	31
Henri Guaino	39	38	33
Florian Philippot	40	40	40

Table 5. Same as Table 2 for 40 selected FR politicians and the French Wikipedia edition of 2013. Color marks membership in political parties: red (ultra-left parties), magenta (social party PS), green (green parties), blue (right parties UMP, UDI), violet (ultra-right FN).

All the groups of politicians have been considered above in the frame of Wikipedia editions corresponding to their native language. Indeed, we find that in other language editions the articles about many politicians are rather short or sometimes they are even absent (e.g. for 40 French politicians in Enwiki or Ruwiki). However, the political leaders of countries are usually well present in the editions discussed here. Therefore, we take for our analysis 20 world political leaders participated in the G20 meeting at Los Cabos summit in 2012 [20]. Their names and local

Names (RU)	K	K^*	K_G
Putin, Vladimir Vladimirovich	1	1	1
Medvedev, Dmitry Anatolyevich	2	12	2
Zhirinovsky, Vladimir Wolfovich	3	10	15
Matvienko, Valentina Ivanovna	4	2	11
Zyuganov, Gennady Andreyevich	5	7	13
Mironov, Sergey Mikhailovich	6	6	8
Chubais, Anatoly Borisovich	7	4	6
Sobyanin, Sergey Semenovich	8	16	9
Nemtsov, Boris Yefimovich	9	8	10
Fradkov, Mikhail Yefimovich	10	13	3
Yavlinsky, Grigory	11	3	14
Kudrin, Alexei Leonidovich	12	14	7
Ivanov, Sergey Borisovich	13	18	4
Lavrov, Sergey Viktorovich	14	15	20
Prokhorov, Mikhail Dmitrievich	15	5	18
Gref, Herman Oskarovich	16	17	5
Shoygu, Sergei	17	9	19
Hakamada, Irina Mutsuovna	18	11	17
Siluanov, Anton Germanovich	19	20	16
Nabiullina, Elvira	20	19	12

Table 6. Same as Table 2 for 20 selected RU politicians and the Russian Wikipedia edition of 2013.

G20 / Wikipedia edition	EN	DE	FR	RU
Names	K	K	K	K
Barack Obama	1	1	1	2
Vladimir Putin	2	3	2	1
David Cameron	3	4	6	4
Stephen Harper	4	10	7	16
Angela Merkel	5	2	4	3
Manmohan Singh	6	13	13	10
Hu Jintao	7	9	5	5
François Hollande	8	5	3	6
Julia Gillard	9	14	16	11
José Manuel Barroso	10	6	8	14
Jacob Zuma	11	18	18	18
Mario Monti	12	8	9	9
Cristina Fernández de Kirchner	13	12	10	8
Recep Tayyip Erdoğan	14	7	14	12
Felipe Calderón	15	11	12	7
Abdullah of Saudi Arabia	16	15	15	13
Susilo Bambang Yudhoyono	17	19	20	19
Dilma Rousseff	18	17	11	15
Lee Myung-bak	19	16	17	17
Yoshihiko Noda	20	20	19	20

Table 7. List of the 20 state leaders of the G20 states and the corresponding PageRank index values K of the reduced network matrices G_R for the English (EN), German (DE), French (FR) and Russian (RU) Wikipedia editions of 2013. For each state leader the country name is given in ISO 3166-1 alpha-2.

PageRank indexes according to Enwiki, Dewiki, Frwiki, Ruwiki are given in Table 7. The distribution of politicians on PageRank-Cheirank plane is shown in Fig. 7 for Enwiki. We take the name of country leader Abdullah of Saudi Arabia (Saudi Arabia) since the name of minister of finance, who was presenting Saudi Arabia is not listed in Dewiki, Frwiki, Ruwiki; José Manuel Barroso is taken as EU representative.

Among G20 leaders the top 2 PageRank positions are taken by Obama and Putin (see Table 7) in Enwiki, Frwiki; Putin and Obama in Ruwiki and Obama and Merkel in Dewiki. So there is definite trend that leaders are promoted in their native editions. The language preference is probably the reason to have Singh (India) ahead of Jintao (China) in Enwiki while in other editions Jintao is well ahead of Singh. At the top Cheirank positions K^* of Enwiki we have Putin, Erdogan, Obama (see Fig. 7) showing very different communicative strengths of political leaders.

In the next Sections we consider interactions between selected political leaders using the reduced Google matrix approach.

4 Direct and hidden links of US politicians

The reduced Google matrix G_R of 20 US politicians, listed in Fig. 2 and Table 2, is shown in Fig. 8 with its three matrix components G_{pr} , G_{rr} , G_{qr} from (16). The amplitudes of matrix elements are shown by color with maximum for red and minimum for blue. We mention that for the data of US Politicians in this section there are no negative matrix elements of G_{qr} . The same holds for the cases of DE and FR politicians (see sections below). However for UK politicians (G20 data, also see sections below for both cases) there are few very (rather) small negative elements of G_{qr} of order $\sim 10^{-8}$ (or $\sim 10^{-5}$) with no (or very small) effects on colors. The data of US politicians in Fig. 8 clearly show that the main contribution to G_R is given by the projector component G_{pr} with a weight of approximately 96%. The remaining weight is distributed between component of direct links G_{rr} (1.9%) and the one of indirect links G_{qr} (1.6%) (see Fig. 8). Of course, the total weight of three components is equal to unity by construction.

Thus the main component G_{pr} imposes to G_R the PageRank probability $P(K)$ mainly produced by the environment of a huge remaining part G_{ss} of the global network with $N - N_r \gg N_r$ nodes. Due to this the structure of G_R is close to a solution (discussed above) when each column of G_R is given by the same PageRank vector of N_r nodes which up to a constant factor coincides with the PageRank probabilities of the selected N_r in the global network of N nodes. Due to that the contribution of two other components G_{rr} of direct and G_{qr} of indirect links (friends) play an important role even if their weight is significantly smaller compared to the weight of G_{pr} .

The global structure of G_{rr} of 20 US politicians is shown in Fig. 8 (bottom left panel) and in more detail in Fig. 9 where the lines and columns are marked by short names of politicians (up to 7 letters). We can say that

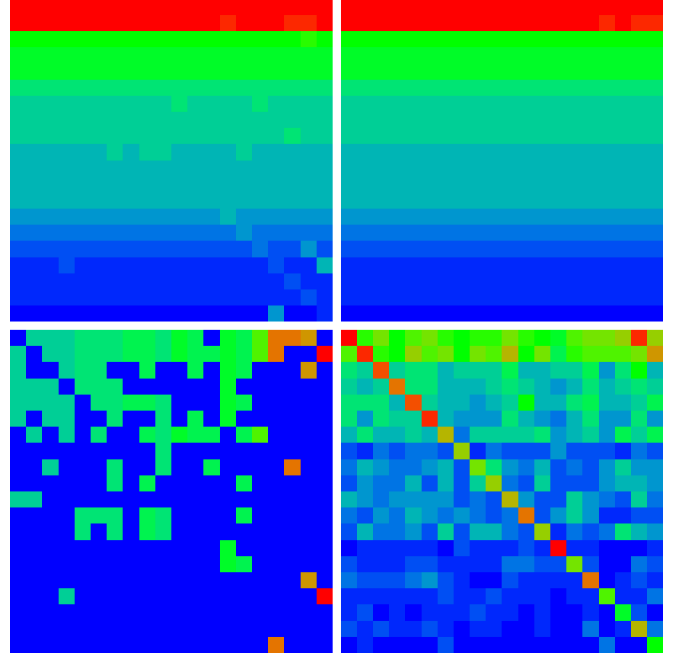


Fig. 8. Density plots of the matrices G_R (top left), G_{pr} (top right), G_{rr} (bottom left) and G_{qr} (bottom right) for the reduced network of 20 US politicians in the Enwiki network. The nodes N_r are ordered in lines by PageRank index changing from $K = 1$ to N_r (left to right) and in columns from $K' = 1$ to N_r from top to bottom. The weights of three matrix components of G_R are $W_{pr} = 0.9640$ for G_{pr} , $W_{rr} = 0.01970$ for G_{rr} , $W_{qr} = 0.01630$ for G_{qr} . The colors represent maximum (red), intermediate (green) and minimum (blue) values.

Politicians	US	Enwiki
Name	Friends	Followers
Obama	Bush Clinton Biden	Lew Comey Carter
Bush	Obama Cheney McCain	Comey Pelosi McCain
Clinton	Obama Bush McCain	Lew Sanders Pelosi
Kerry	Obama Bush Clinton	Pelosi McCain Biden
McCain	Bush Obama Clinton	Palin Comey Sanders

Table 8. List of leading politicians (in PageRank order) of the group of US politicians together with the leading “Friends” (“Followers”) in this group defined by maximal values of the elements of G_{qr} corresponding to links “to” (“from”) the Friends (Followers) for the English Wikipedia edition. Links to/from the same politician, corresponding to diagonal elements of G_{qr} , are omitted.

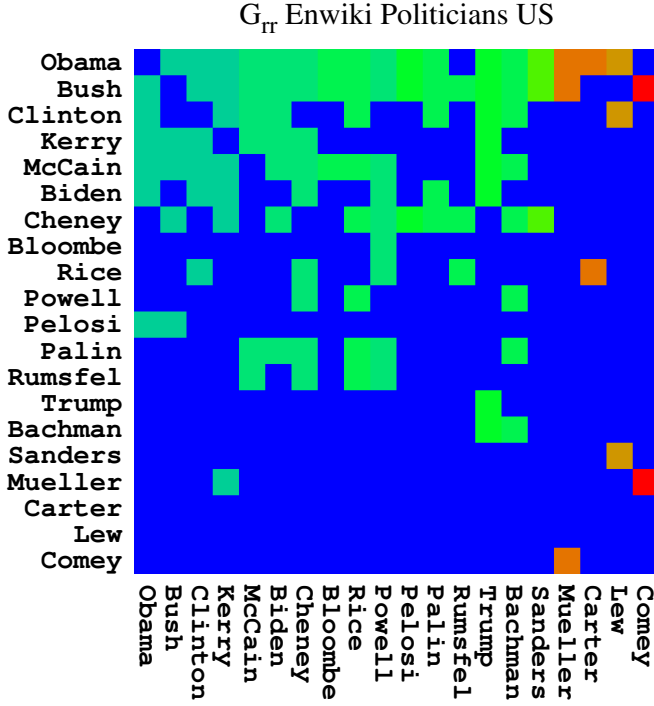


Fig. 9. Density plot of the matrix G_{rr} for the reduced network of 20 US politicians in the Enwiki network with short names at both axes (up to 7 letters, full names are in Table 2).

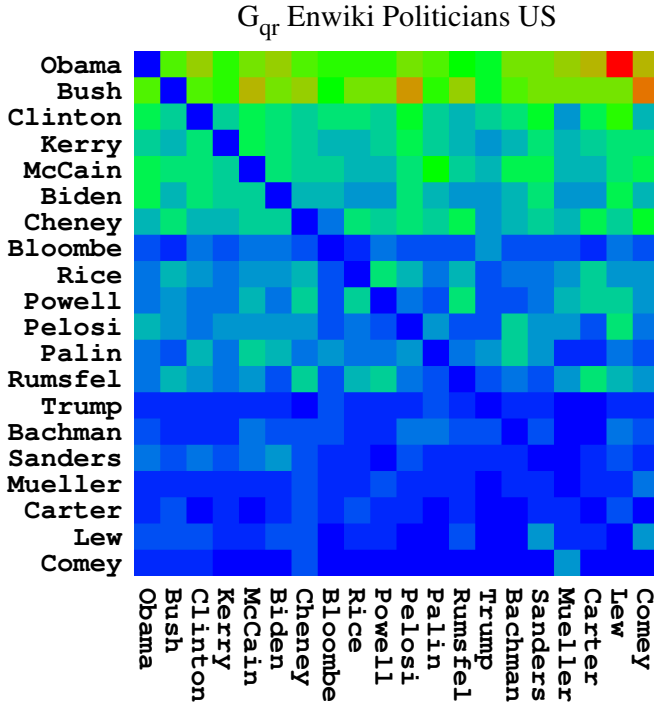


Fig. 10. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 US politicians in the Enwiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrd} = 0.01143$ (see Fig. 8 for W_{qr} weight with diagonal).

large matrix elements in a column of a given politician can be considered as direct friends to whom he/she points in his/her Wikipedia article. However, we should note that by construction of G all elements inside a given column have the same amplitudes given by a fraction of total number of outgoing links of a given article which point inside N_r nodes of selected subset. Thus on the basis of G_{rr} component it is not possible to say that some direct friend (link) of a given politicians is more preferable then another one: all of them have the same amplitude. Of course, when we are speaking about a friend we simply mean that one politician points to another but at present we cannot say if this link have a positive or negative content. Such a classification would required further extension of our G_R analysis. However, since the PageRank probability is on average proportional to a number of ingoing links we can assume that the content is mainly positive,

The large amplitudes inside a given line, attributed to a given politician, mark the followers of this politician (similar to a situation in Twitter). Thus from Fig. 9 we can say the the first (strongest) direct follow of Bush is Comey who is the present FBI director and is closely linked with Bush. At the same time he has no direct link with Obama. Of course the direct links are important but only on their own basis it is not possible to obtain a correct relations between selected persons. Indeed, an attempt [36] to make ranking of historical figures of Wikipedia considering only links between their biographical articles gave a rather strange unrealistic results clearly showing that this approach is not working (see also discussion in [14]).

Due to above reasons the most interesting matrix component G_{qp} is the one of indirect links shown in Fig. 8 (bottom right panel) and without diagonal elements in Fig. 10. The weight $W_{qrd} = 0.00487$ of diagonal component of G_{qr} is approximately twice smaller than the weight of nondiagonal part with $W_{qrd} = 0.01143$ so that the nondiagonal elements play a more significant role. Also as for the projector part G_{pr} the rank of diagonal is unity so that the matrix contributions with high rank are generated by nondiagonal part of $G_{pr} + G_{qr}$.

As discussed above the largest matrix elements of G_{qr} in a column of a given politician give his/her strongest indirect or hidden friends while those in a line give his/her strongest hidden followers. The names of top friends and top followers of top 5 PageRank US politicians are given in Table 8. Surprisingly we find that the strongest hidden followers of Obama are not his colleagues from democratic party but Lew (actual secretary of treasury), Comey (actual FBI director), Carter (actual secretary of defense). Interestingly, Clinton has as main followers Lew, Sanders, Pelosi so that already in 2013 there existed the strong hidden links between Sanders and Clinton. At the same time the top 3 hidden friends (followers) of Trump are Obama, Bush, Clinton (Palin, Bloomberg, Clinton) highlighting hidden links between these two political leaders who are now fighting for the US presidency (see data at [27]).

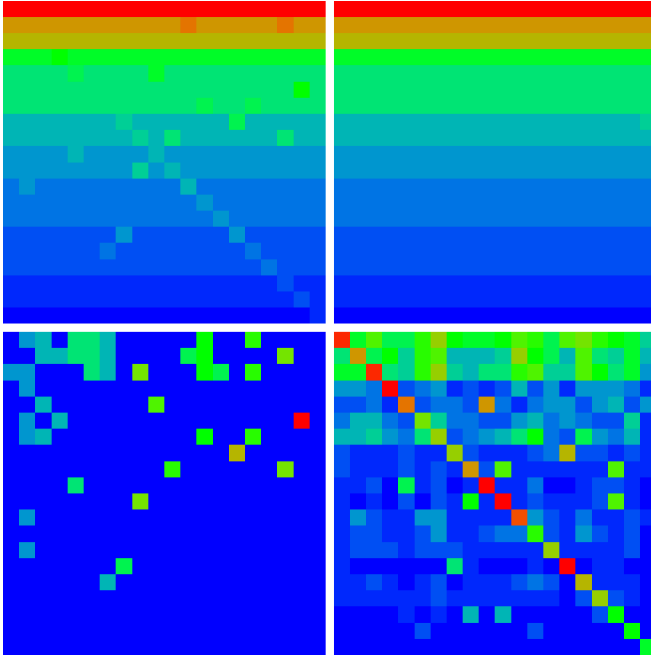


Fig. 11. Same as Fig. 8 for 20 UK politicians in the Enwiki network. The weights of three matrix components of G_R are $W_{pr} = 0.9668$ for G_{pr} , $W_{rr} = 0.01729$ for G_{rr} , $W_{qr} = 0.01588$ for G_{qr} .

It is possible to try not to take into account the projector component and construct modified reduced Google matrix \tilde{G}_R obtained from $G_{rr} + G_{qrrnd}$ by renormalization of each column to unity. Then the PageRank vector of \tilde{G}_R gives the new ranking of the selected group with index K_G given in Table 2. We see then that the top 3 positions are taken by Bush, Obama, McCain. This gives rearrangement of ranking which stresses in a stronger way previous presidential team. We will see that for other countries the effects can work in a different direction (e.g. for DE).

5 Direct and hidden links of UK politicians

The reduced Google matrix analysis of 20 UK politicians from Fig. 3 and Table 3 is presented in Figs. 11, 12, 13. As for the US case we find that G_R has the dominant contribution from the projector component G_{pr} which has a similar weight of 96%. The weights of other two components of direct G_{rr} (1.7%) and hidden G_{qr} (1.5%) links also is similar to those of US. However, on average the direct links are distributed in a more homogeneous manner for UK then for US (see Figs. 12, 9). The strongest direct link is from Leadsom to Osborne.

The distribution of hidden links in Fig. 13 is much more broad comparing to direct ones in Fig. 12. The top 3 hidden friends and follows of top 5 PageRank politicians are given in Table 9. There strong links between top 3 leaders Blair, Brown, Cameron. More surprisingly we find that already in 2013 May was a strongest follower of Cameron (there is only a moderate direct link between them) and

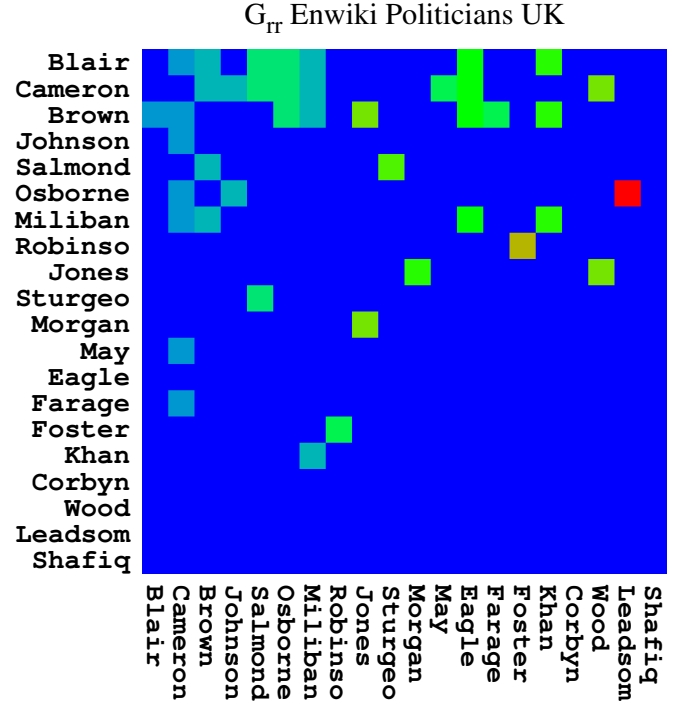


Fig. 12. Density plot of the matrix G_{rr} for the reduced network of 20 UK politicians in the Enwiki network with short names at both axes.

Jonhson (there is no direct link). The strongest amplitudes of links are from Sturgeon (actual first minister of Scotland) to Salmond (previous first minister of Scotland) and from Foster to Robinson (first ministers of Northern Ireland). Even if the direct links are present in G_{rr} they are definitely not so pronounced as in the indirect part G_{qr} . Also we find that Khan (actual mayor of London) is a second by strength indirect follower of Cameron even if there are no direct links between them.

The ranking index K_G of UK politicians from $G_{rr} + G_{qrrnd}$ is given in the last column of Table 3. It places on top positions Brow, Blair, Cameron followed by Milibard and Salmond. Such a ranking looks to be less natural comparing to the global rank index K showing that the projector component G_{pr} plays an important role.

6 Direct and hidden links of DE politicians

For 40 German politicians the matrix G_R and its three components are shown in Figs. 14, 15, 16. The component weights are similar to the cases of US, UK but the percent of G_{rr} is now slightly higher.

The strongest direct links are from Maas to Lafontaine (Maas was supported by Lafontaine, chairman of the Social Democratic Party - SPD) and Arnold to Ströbele.

For the indirect links component G_{qr} the strongest links remains the one of Maas to Lafontaine influenced by their direct link. However, in global the number of indirect links is significantly larger compared to direct links. The

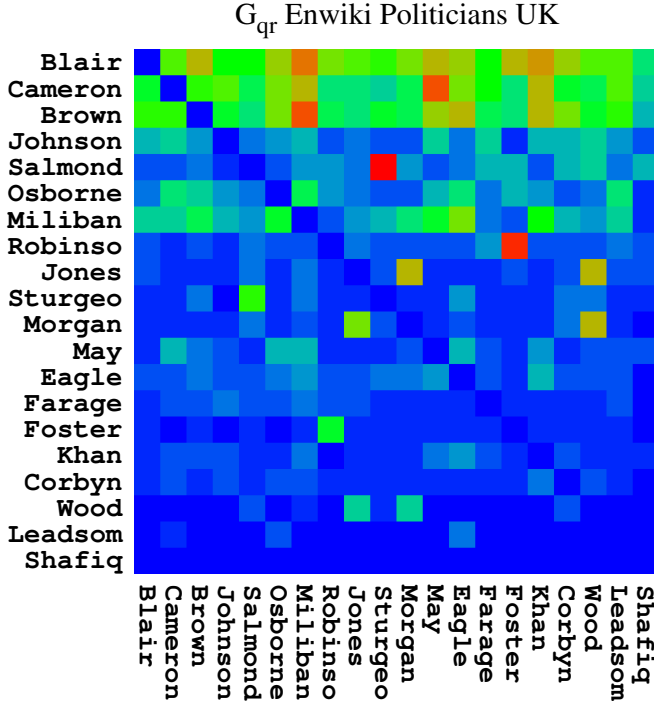


Fig. 13. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 UK politicians in the Enwiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrd} = 0.009875$ (see Fig. 10 for W_{qr} weight with diagonal).

Politicians	UK	Enwiki
Name	Friends	Followers
Blair	Brown Cameron Miliband	Miliband Khan Foster
Cameron	Blair Brown Osborne	May Khan Miliband
Brown	Blair Cameron Miliband	Miliband Khan Eagle
Johnson	Cameron Blair Brown	May Wood Cameron
Salmond	Sturgeon Blair Cameron	Sturgeon Wood Foster

Table 9. Same as Table 8 for leading UK politicians and the English Wikipedia edition of 2013.

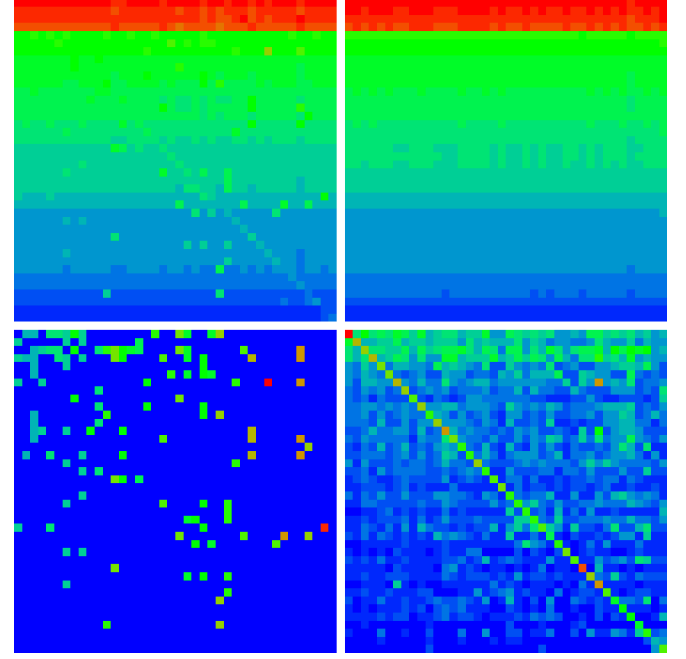


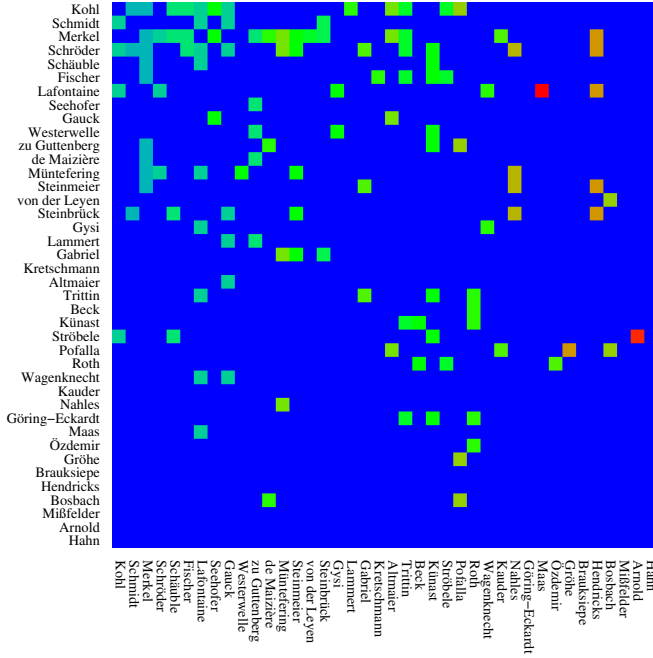
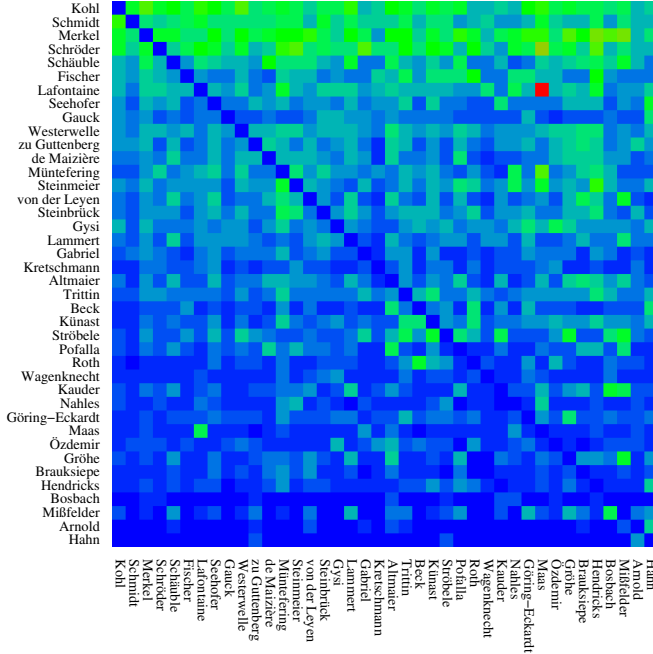
Fig. 14. Same as Fig. 8 for 40 DE politicians in the Dewiki network. The weights of three matrix components of G_R are $W_{pr} = 0.9590$ for G_{pr} , $W_{rr} = 0.02520$ for G_{rr} , $W_{qr} = 0.01580$ for G_{qr} .

top 5 PageRank politicians have strongest direct friend links mainly between their own group as it is seen in Table 10. However, the list of followers is rather different, Thus Merkel is the first follower of Kohl who strongly supported her. Kohl is the strongest follower of Schmidt, probably because many Wikipedia artikels refer to the change of power between them in 1982 but also because Kohl was the opposition leader during the Schmidt government of 1974-1982. Furthermore, Hendricks is the first follower of Merkel. She is a recent member of Merkel's government (since end 2013 and at least up to 2016) despite being member of the socialist party. The strongest follower of Schöder is Maas. Despite the fact that both belong to the socialist party it is difficult to establish a direct political link due to a considerable time difference of more than 10 years between their time periods in government. However, their names appear both (and together with other names) in certain news articles discussing general marital problems of certain german politicians which may indeed produce indirect links on a different than professional level.

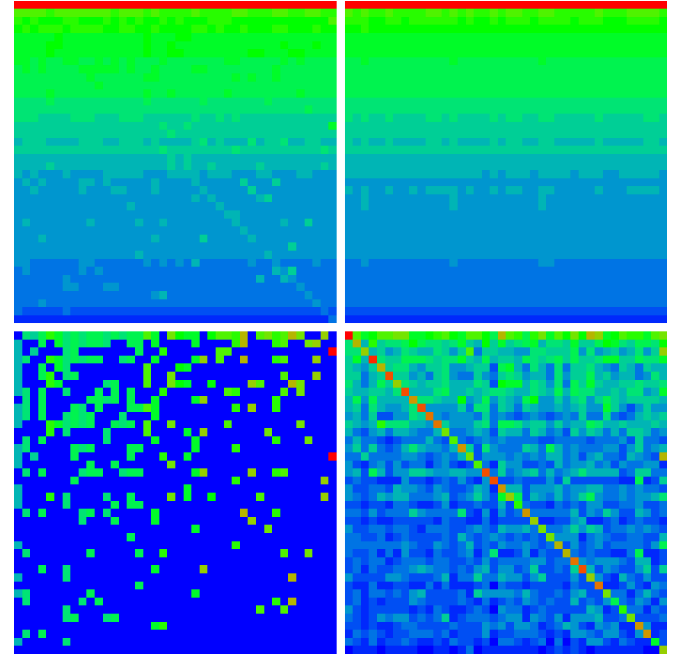
The ranking K_G from the matrix $G_{rr} + G_{qrd}$ is given for 40 German politicians in the last column of Table 4 placing on the top positions Merkel, Schröder, Kohl.

7 Direct and hidden links of FR politicians

The reduced Google matrix G_R and its three components for 40 French politicians (see Fig. 5 and Table 5) are shown in Figs. 17, 18, 19. Here the weight of W_{rr} is the largest among all groups of politicians considered in this

G_{rr} Dewiki Politicians DE**Fig. 15.** Density plot of the matrix G_{rr} for the reduced network of 40 DE politicians in the Dewiki network with short names at both axes. G_{qr} Dewiki Politicians DE**Fig. 16.** Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 40 DE politicians in the Dewiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrnd} = 0.01202$ (see Fig. 14 for W_{qr} weight with diagonal).

Politicians	DE	Dewiki
Name	Friends	Followers
Kohl	Schmidt Merkel Schröder	Merkel Westerwelle Lafontaine
Schmidt	Kohl Merkel Schröder	Kohl Lafontaine Steinbrück
Merkel	Kohl Schröder Schäuble	Hendricks Mißfelder Gröhe
Schröder	Kohl Merkel Schmidt	Maas Steinmeier Hendricks
Schäuble	Kohl Merkel Schmidt	Maizière Mißfelder Lammert

Table 10. Same as Table 8 for leading DE politicians and the German Wikipedia edition of 2013.**Fig. 17.** Same as Fig. 8 for 40 FR politicians in the Frwiki network. The weights of three matrix components of G_R are $W_{pr} = 0.9481$ for G_{pr} , $W_{rr} = 0.03248$ for G_{rr} , $W_{qr} = 0.01941$ for G_{qr} .

work. Also the distribution of direct links (see Fig. 18) is very broad comparing to the case of 40 DE politicians in Fig. 15. The strongest direct links are from Philippot to Jean-Marie Le Pen and Marine Le Pen.

The hidden links between 40 FR politicians without diagonal term are shown in Fig. 19. The strongest 3 friends and followers are given in Table 11.

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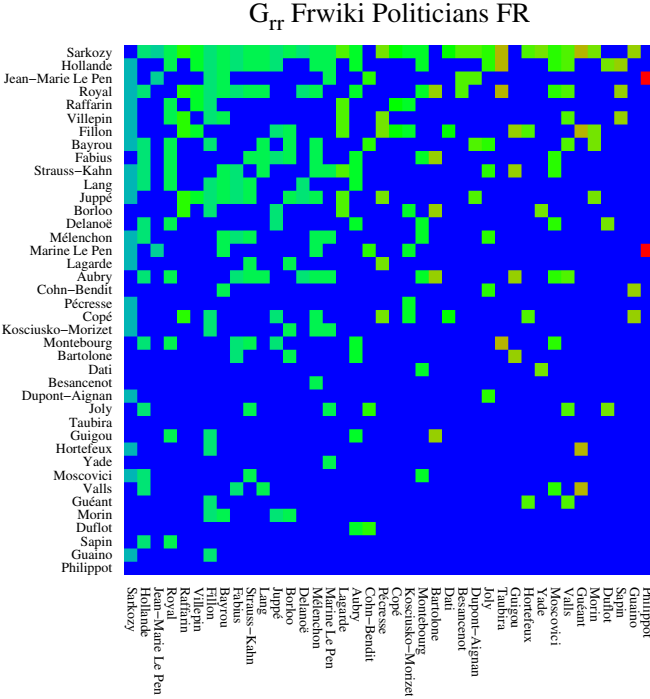


Fig. 18. Density plot of the matrix G_{rr} for the reduced network of 40 FR politicians in the Frwiki network with short names at both axes.

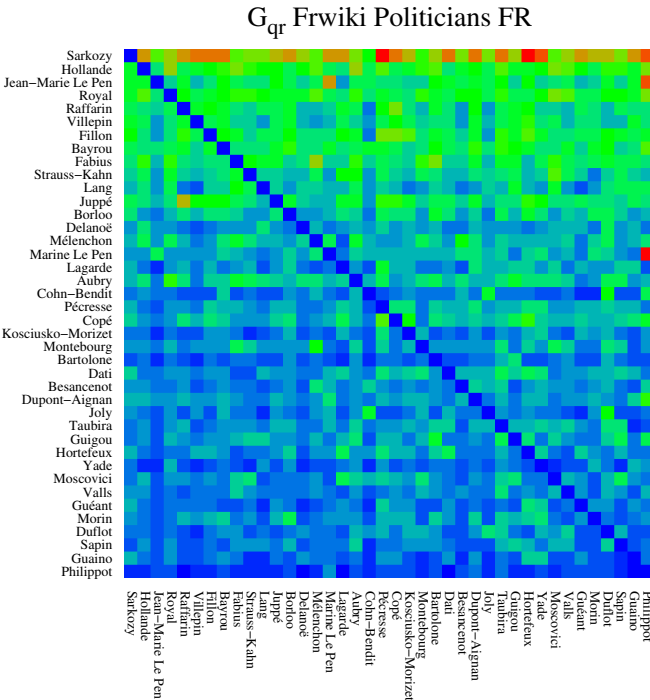


Fig. 19. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 40 FR politicians in the Frwiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrd} = 0.01543$ (see Fig. 17 for W_{qr} weight with diagonal).

Politicians	FR	Frwiki
Name	Friends	Followers
Sarkozy	Fillon J.-M. Le Pen Hollande	Hortefeux Pécresse Yade
Hollande	Sarkozy Royal Fabius	Royal Aubry Fabius
J.-M. Le Pen	Sarkozy M. Le Pen Bayrou	Philippot M. Le Pen Taubira
Royal	Hollande Sarkozy Fabius	Moscovici Philippot Hollande
Raffarin	Sarkozy Juppé Fillon	Copé Pécresse Hortefeux

Table 11. Same as Table 8 for leading FR politicians and the French Wikipedia edition of 2013.

Politicians	RU	Ruwiki
Name	Friends	Followers
Putin	Medvedev Chubais Ivanov	Hakamada Fradkov Nabiullina
Medvedev	Putin Chubais Ivanov	Putin Mironov Gref
Zhirinovskiy	Putin Medvedev Zyuganov	Zyuganov Hakamada Yavlinsky
Matvienko	Putin Medvedev Mironov	Mironov Kudrin Nemtsov
Zyuganov	Putin Medvedev Zhirinovskiy	Hakamada Yavlinsky Zhirinovskiy

Table 12. Same as Table 8 for leading RU politicians and the Russian Wikipedia edition of 2013.

8 Direct and hidden links of RU politicians

The reduced Google matrix G_R for 20 Russian politicians from Fig. 6 and Table 6 are shown in Figs. 20, 21, 22. The weight of the component of direct links as a bit larger then in the case of US and UK but is comparable with DE. The strongest direct links are from Siluanov (minister of finance) to Putin and Medvedev and Kudrin (previous minister of finance).

The matrix of hidden links G_{qr} without diagonal is shown in Fig. 22 with the list of 2 top friends and followers for top 5 RU politicians in Table 12. Among hidden friends of Putin we find naturally Medvedev, Chubais, Ivanov who are closely linked with him during his political

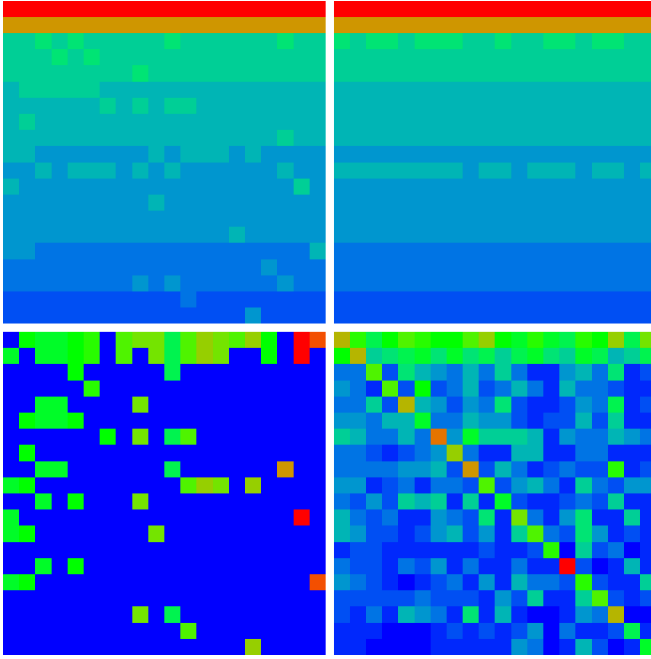


Fig. 20. Same as Fig. 8 for 20 RU politicians in the Ruwiki network. The weights of three matrix components of G_R are $W_{pr} = 0.9644$ for G_{pr} , $W_{rr} = 0.02408$ for G_{rr} , $W_{qr} = 0.011148$ for G_{qr} .

G_{rr} Ruwiki Politicians RU

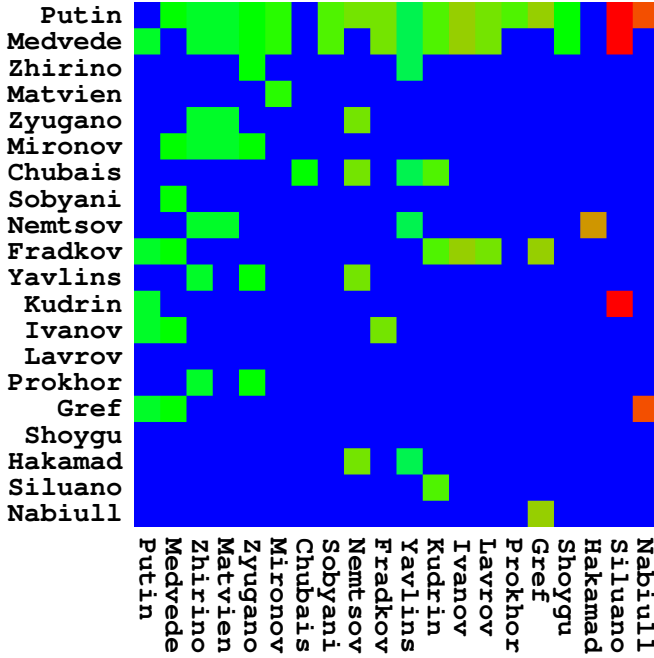


Fig. 21. Density plot of the matrix G_{rr} for the reduced network of 20 RU politicians in the Ruwiki network with short names at both axes.

G_{qr} Ruwiki Politicians RU

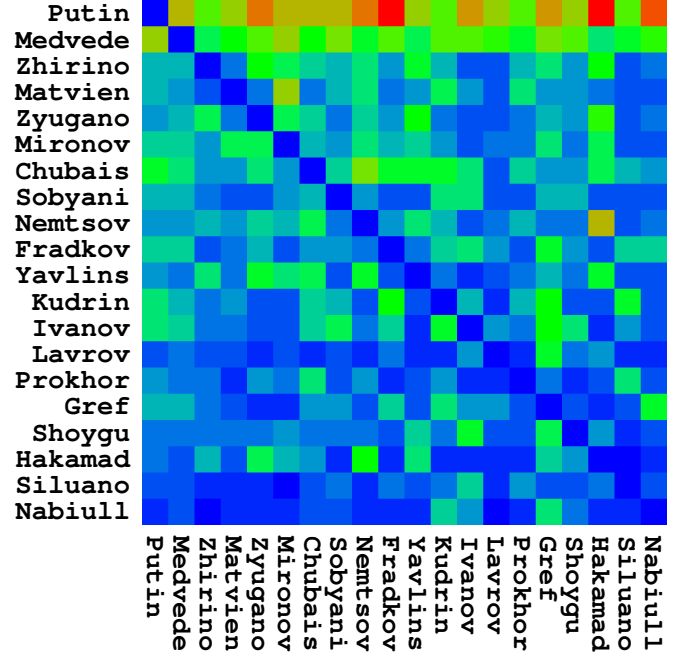


Fig. 22. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 RU politicians in the Ruwiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrd} = 0.007912$ (see Fig. 20 for W_{qr} weight with diagonal).

career. More surprising is that his strongest followers are Hakamada, Fradkov, Nabiullina. Among friends relations somewhat unexpected is that Zhirinovskiy and Zyuganov are friends of each other in a third position. It is also interesting that Russian democratically oriented politicians (Hakamada, Yavlinsky) are among top followers of Zhirinovskiy and Zyuganov. More naturally, top 2 to 5 PageRank politicians (Medvedev, Zhirikovskiy, Matvienko, Zyuganov) have Putin as the first friend (see data at [27]). In fact for all 19 politicians the first friend from matrix of indirect links G_{qr} is Putin confirming well the vertical power of politics in Russia. On a level of direct friends from G_{rr} only Chubais and Hakamada do not point directly to Putin but on a level of indirect links of G_{qr} all 19 politicians point first to Putin. For RU democrats like we have among top 3 friends (follows)

- for Yavlinky: Putin, Zyuganov, Chubais (Hakamada, Nemtsov, Zyuganov);
- for Nemtsov: Putin, Chubais, Hakamada (Hakamada, Chubais, Yavlinsky);
- for Hakamada: Putin, Nemtsov, Zyuganov (Nemtsov, Zyuganov, Yavlinsky).

This shows relatively strong links between democrats even contacts with Zyuganov (leader of communist party) are surprisingly well present.

The rank index J_G from the matrix $G_{rr} + G_{qrd}$ is given in last column of Table 6 with the top leaders being Putin, Megvedev, Fradkov that seems to overestimate im-

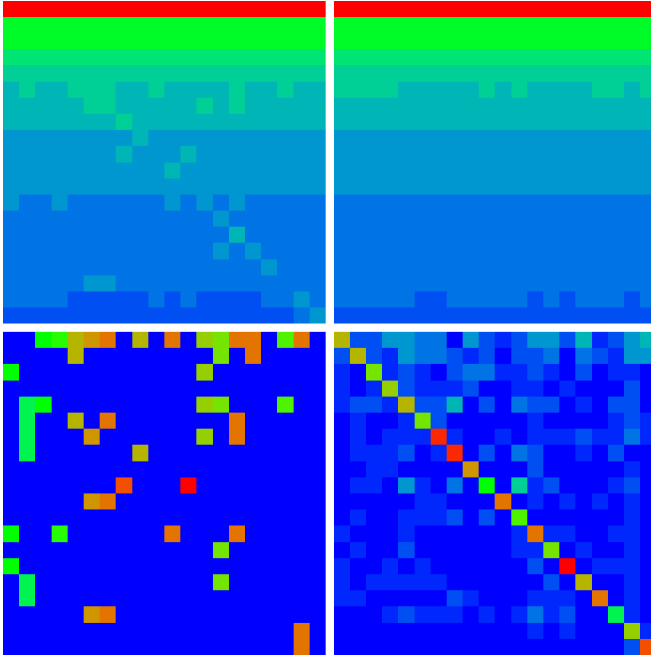


Fig. 23. Same as Fig. 8 for 20 state leaders of G20 states in the Enwiki network. The weights of three matrix components of G_R are $W_{pr} = 0.9793$ for G_{pr} , $W_{rr} = 0.009976$ for G_{rr} , $W_{qr} = 0.01070$ for G_{qr} .

portance of Fradkov, even if he is followed by Ivanov and Gref that looks to be more reasonable. So the above analysis of politicians of US, UK, DE, FR, RU suggests that the projector component should be taken into account even if we want to analyze the relations inside the selected group of politicians since still the environment links of global matrix G play an important role.

9 Direct and hidden links of G20 state leaders

Above we considered interactions between political leaders of the same country from the view point of Wikipedia edition language native for the selected country. It is interesting to see the results of reduced Google matrix analysis for interactions of state leaders of G20 summit of 2012 [20]. We analyze these interactions from the view point of 4 Wikipedia editions EN, DE, FR, RU.

The list of names of politicians of G20 and their distribution over PageRank-CheRank plane are given in Fig. 7 and Table 7. In our presentation below we keep the names in the PageRank order of Enwiki of Table 7. For EN Wikipedia the reduced Google matrix G_R and its three components are shown in Figs. 23, 24, 25. We see that, comparing to previous cases inside one country, the weights W_{rr} and W_{qr} are reduced approximately by a factor 2. Indeed, there are significantly less direct links between leaders of different states (see Fig. 24). There are even less direct links for DE, FR, RU editions (see data at [27]).

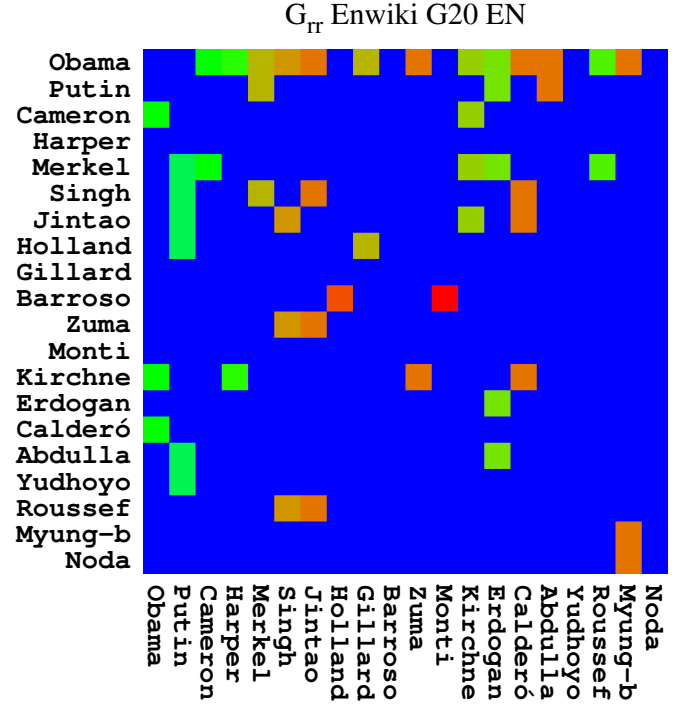


Fig. 24. Density plot of the matrix G_{rr} for the reduced network of 20 state leaders of G20 states in the Enwiki network with short names at both axes.

For example, the only direct links for RU Wikipedia are between Merkel and Putin, Erdoğan, Putin and Jintao.

Thus the importance of indirect links from G_{qr} becomes more significant even if the weight of nondiagonal matrix elements is reduced by a factor 4-5 comparing to the case of politicians in the same country. In Fig. 25 for Enwiki we find the strongest indirect links between Monti and Barroso (EU), Abdullah (SA) and Obama (even if direct links also exist), Holland and Merkel, Noda (JP) and Obama (without direct links). The list of top 3 friends (followers) of top 5 PageRank leaders of Enwiki is given in Table 13. Among friends the first positions are taken by Putin, Merkel, Obama. For Merkel the top friends are Barroso, Putin, Obama; for Cameron we have Putin, Obama, Merkel. Among top followers of Obama we do not find EU leaders but Noda (JP), Abdullah (SA), Myung-bak (KR). The followers of Putin are Noda, Myung-bak, Merkel. The first follower of Merkel is Holland while Merkel is the top follower of Harper (CA).

The reduced Google matrices G_R from 4 Wikipedia editions are shown in Fig. 26. It is clear that each culture (which, in first approximation, can be associate with language) has its own view on relations between state leaders of G20. In deed, even top 3 PageRank leaders are different for these cultures that creates different structure of matrix elements.

The indirect links of G_{qr} are shown in Fig. 27 for Dewiki, Fig. 28 for Frwiki and Fig. 29 for Ruwiki. For Dewiki the strongest indirect links are from Gillard (AU) and Barroso to Merkel, Abdullah to Erdoğan. For Fr-

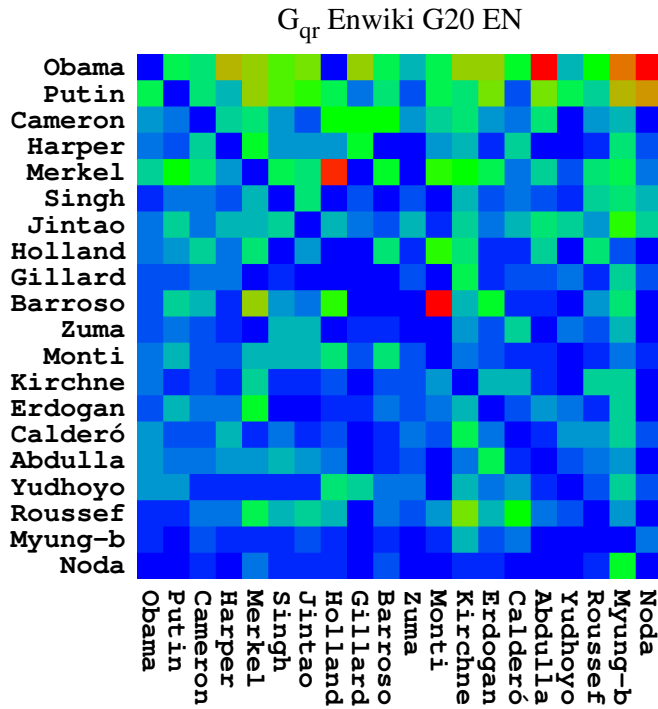


Fig. 25. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 state leaders of G20 states in the Enwiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrd} = 0.001878$ (see Fig. 23 for W_{qr} weight with diagonal).

G20	EN	Enwiki
Name	Friends	Followers
Obama	Putin Merkel Calderón	Noda Abdullah Myung-bak
Putin	Merkel Obama Barroso	Noda Myung-bak Merkel
Cameron	Putin Obama Merkel	Gillard Barroso Hollande
Harper	Obama Cameron Putin	Merkel Gillard Myung-bak
Merkel	Barroso Putin Obama	Hollande Monti Kirchner

Table 13. Same as Table 8 for leading G20 state leader and the English Wikipedia edition of 2013.

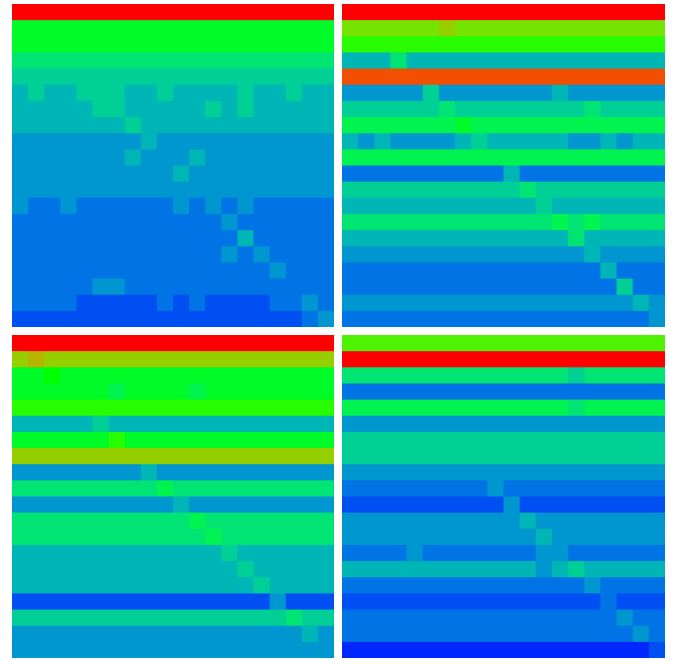


Fig. 26. Density plots of the matrix G_R for the reduced networks of 20 state leaders of G20 states in the Enwiki (top left), Dewiki (top right), Frwiki (bottom left) and Ruwiki (bottom right) networks. The order of the 20 state leaders is in all cases given by the PageRank order for the Enwiki network and corresponds to the same order of Figs. 23-25.

wiki the strongest link is from Yudhoyono (ID) to Putin, Monti and Merkel to Barroso. In contrast for Ruwiki the strongest links are from Barosso to Obama, Myung-bak (KR) to Putin. Being at the top of PageRank in Rywiki Putin accumulates the largest number of followers. This demonstrates a large variety of cultural view on interactions between state leaders.

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10 Network of political leaders of France

To have a more direct pictorial representation of interactions in the framework of the reduced matrix G_R we choose the case of 40 politicians of France discussed above (see Fig. 5 and Table 5). Here all politicians are attributed to the main political parties marked by corresponding 5 colors in Table 5. For each color we take the top PageRank politicians: Sarkozy (blue, right parties, UMP-UDI), Holland (magenta, left, PS), Jean-Marie Le Pen (violet, ultra right, FN), Mélanchon (red, ultra left), Cohn-Bendit (green, green party). These 5 “leaders”¹ are positioned on a circle of fixed radius forming the first level. Then for the directed network of G_R of these 5 leaders we show for each

¹ This definition of leader does not necessarily correspond to the official “leader” a political party or group since in the Wikipedia PageRank index also older, more historical aspects, e. g. “former leadership”, are taken into account.

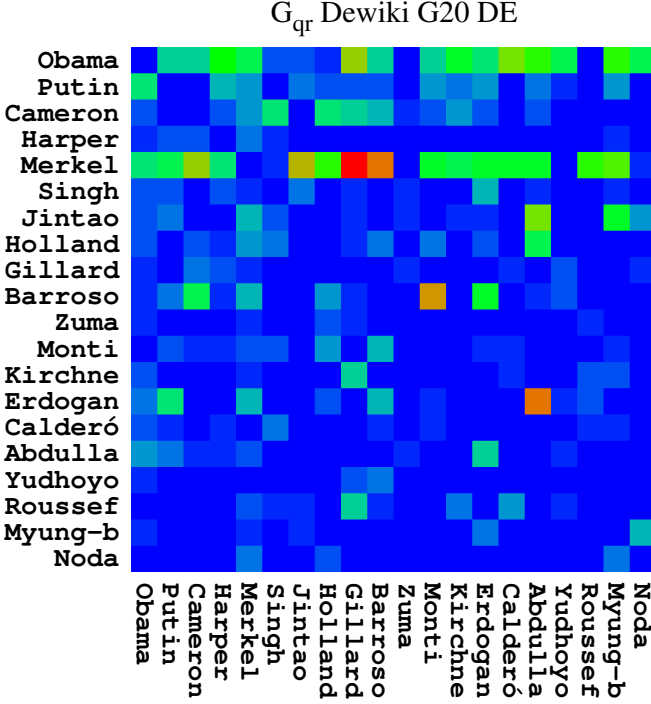


Fig. 27. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 state leaders of G20 states in the Dewiki network with short names (in English) at both axes using the same order of Fig. 25. The weight of this matrix component without diagonal is $W_{qrd} = 0.001125$.

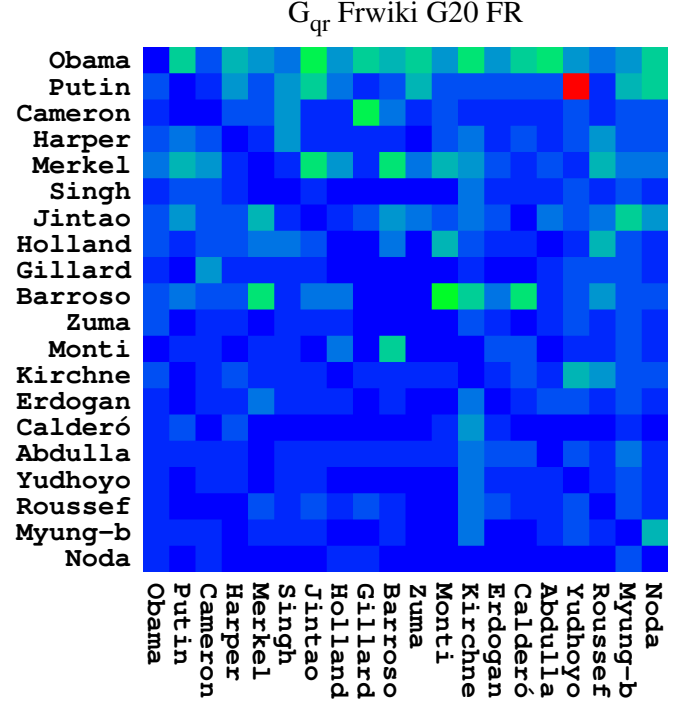


Fig. 28. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 state leaders of G20 states in the Frwiki network with short names (in English) at both axes using the same order of Fig. 25. The weight of this matrix component without diagonal is $W_{qrd} = 0.001554$.

of them 4 strongest links from them to other politicians (considered as their direct friends). These links are shown by directed bold black lines with arrows in the top panel of Fig. 30. The new (secondary) politicians appeared from these links are placed on second level circles around the (primary) politicians of the first level to which they correspond (the preference is given to the primary politicians with the same color if there are several corresponding primary politicians). For the politicians of the second level new red links with arrows are drawn for each of them with top 4 strongest links forming the third level circles if any. After these two iteration of 4 strongest friend we obtain the network of friends of G_R with only 7 politicians. The politicians are marked by their PageRank numbers K from Table 5. At blue color we find Sarkozy with Raffarin, at magenta we have Holland with Royal, and for other colors we have only one politician of the first circle. Thus we find that the circle of close friends is very narrow. We also mention that for this case the subnetwork actually saturates completely at level 3 (i.e. including the group of tertiary politicians visible in Fig. 30) with only these 7 politicians and does not increase even if we try to include higher level circles/politicians. The reason of the saturation on a small sub network is that G_R is dominated by the rank one contribution G_{pr} which selects essentially top PageRank nodes. Therefore we simply find the top 5 PageRank nodes and the two other nodes (Mélanchon and Cohn-Bendit), who occupy rather late PageRank po-

sitions, belong to the subnetwork by construction since they belong to the chosen primary nodes.

We now preform the same procedure for G_R for followers, using the strongest 4 incoming links on each level, instead of friends, as it is shown in the bottom panel Fig. 30. Again after two iterations (black and red links) we obtain 32 politicians showing that the number of followers is significantly larger then the number of friends. The network of followers has an interesting structure: violet, blue and green parties form mainly compact groups while the magenta party is clearly divided on two groups one centered around Holland and another centered around Mélanchon. This clearly indicate deep structural problems in French PS. We note that for this case of followers (for G_R) the subnetwork would saturate at all 40 politicians after 5 iterations (the corresponding higher level links are not shown in Fig. 30 in order to keep a simple presentation).

We also used the same approach to construct the network of hidden friends or hidden followers using the matrix G_{qr} (instead of G_R) with the result shown in both panels of Fig. 31. Now, the network of hidden friends contain a significantly larger number of politicians after two iterations (15 instead of 7 for G_R but also with saturation at level 3). At the same time the network of followers has a similar number of nodes (34 instead of 32 for G_{rmR} with saturation at 35 after level 4). For the hidden follower network the separation of the magenta PS party in two groups is less pronounced with a smaller number of followers for

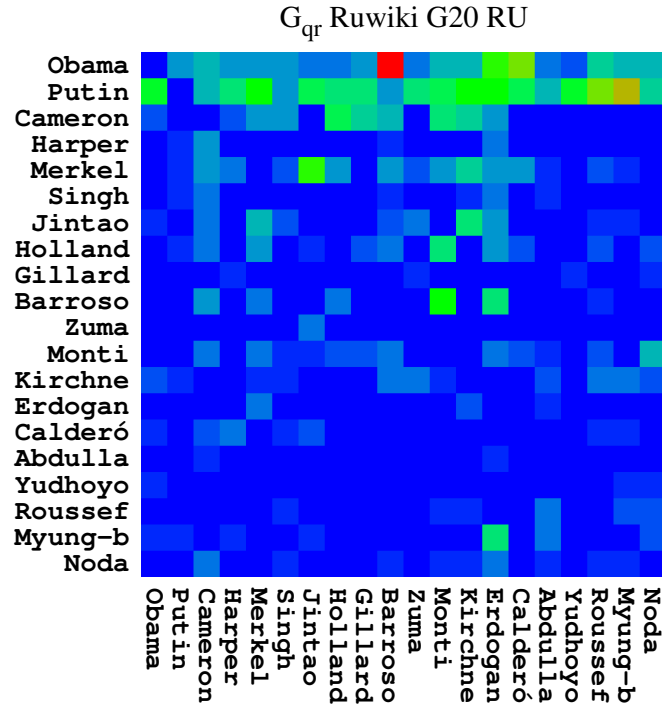


Fig. 29. Density plot of the matrix G_{qr} without diagonal elements for the reduced network of 20 state leaders of G20 states in the Ruwiki network with short names (in English) at both axes using the same order of Fig. 25. The weight of this matrix component without diagonal is $W_{qrd} = 0.001089$.

Mélanchon. The group of the violet party becomes more compact having all its 4 members begin selected. There is also Taubira ($K = 29$) who is closely linked to this violet group apparently due to her strong criticism on J.-M. Le Pen.

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11 Discussion

In conclusion, we presented the new mathematical method which establish effective directed network for a selected subset of nodes of significantly larger network. This approach was tested on examples of several groups of political leaders of 5 countries and world state leaders of G20 analyzed in the frame of Wikipedia network. Our results show that the proposed method works in a reliable way determining direct and hidden links between political leaders. We think that this approach can provide firm mathematical grounds for the LMX studies [22,23,24] in social and political sciences. Our results show that the Wikipedia network can be used in an efficient way to determine direct and hidden relations between different subjects appearing in Wikipedia. We also point that the reduced Google matrix approach can be applied to a variety of directed networks where the relations between selected subgroup of nodes are not evident.

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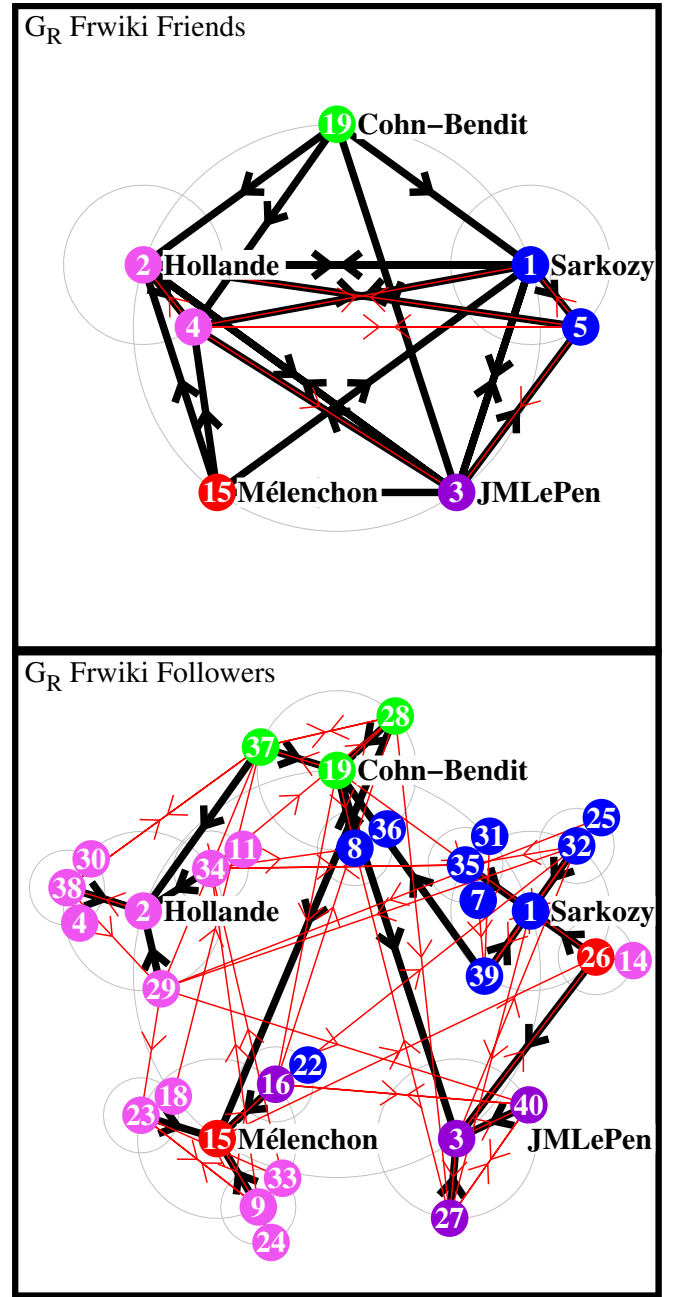


Fig. 30. Network structure of friends (followers) in top (bottom) panel induced by the largest four non-diagonal elements per column (row) of the matrix G_R obtained for the group of 40 French politicians in the French Wikipedia edition of 2013. The five colors of the nodes represent the five main political movements and the numbers of the nodes represent the local PageRank index in Table 5. For each movement the leading node (in PageRank ordering) is drawn on the largest primary circle (with name). The thick black arrows represent links from each primary node to its four most relevant nodes and the thin red arrows represent links from each secondary node to its four most relevant nodes. New secondary (tertiary) nodes are drawn on the secondary (tertiary) circles. Most of these circles contain less than four nodes due to possible multiple links with center nodes on other circles of the same level. When possible, a secondary (tertiary) node was associated to a primary (secondary) node of the same color.

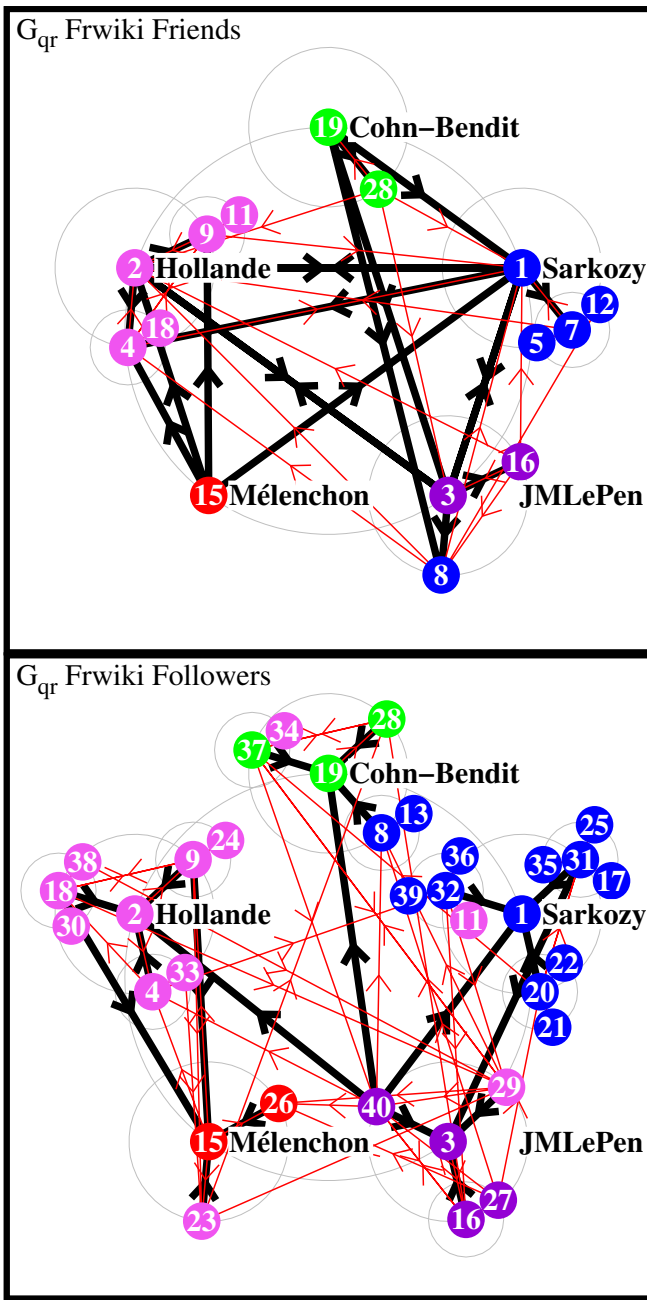


Fig. 31. Same as Fig. 30 for the matrix G_{qr} which recovers hidden links between politicians.

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