ACADEMY OF SCIENCES OF THE USSR

SIBERIAN DIVISION

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Preprint

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SOME ESTIMATES OF A SINGLE PARTICLE LIFE TIME IN A GEOMAGNETIC TRAP DUE TO A WEAK NONADIABATICITY

Communication
at the International
Geophysical Congress,
Madrid, 1969

Novosibirsk
1969

In this brief communication I would like to attract your attention to a rather little known weak instability of a many-dimensional Hamiltonian system, so-called Arnold' diffusion, and to its influence on a single particle motion in a geomagnetic trap. The first example of such an instability and its mechanism were exposed by Arnold /I/. Theory of instability is given in /2/.

The instability in question is connected with so-called resonant surfaces in particle's phase space which, in the problem under consideration, are defined by the equation:

$$p\Omega_{\mathcal{D}} + q\Omega + \ell\bar{\omega} = 0 \qquad \text{(I)}$$

Here drift ($\Omega_{\mathcal{D}}$), longitudinal (Ω) and mean Larmor ($\bar{\omega}$) frequencies depend upon canonical momenta of a particle ($\Omega_{\mathcal{D}}/\Omega \sim \Omega/\bar{\omega} \sim \varepsilon \ll 1$); ε is a small perturbation (adiabaticity) parameter (see below (4)) and p, q, ℓ are integer (harmonic numbers). Under very general conditions /2-4/ there exists so-called stochastic layer in a vicinity of every nonlinear resonance separatriss which runs at both sides of resonant surface (I) at the distance $\sim\sqrt{\varepsilon_n} \sim \sqrt{\varepsilon}\cdot e^{-n/n_0}$ where usually n = |q|, and $n_0 \sim$ I is a parameter related to the magnetic field configuration /2/.

The instability is a result of interference of a given resonance with other resonances or, better to say, an influence of nonresonant perturbation terms which are usually neglected. The trajectories running along resonant surface and that with a random-like motion were proved to exist in the stochastic layer by Arnold /I/ and Alexeev /5/. This result have been confirmed to some extend in numerical model experiments summarized in /2/.

A qualitative theory developed in /2/ leads to the following estimate for the coefficient of Arnold' diffusion:

$$\mathcal{D}_A \sim \bar{\omega} \mu^2 \varepsilon \beta^3 \cdot \exp\left(-\frac{2e^{1/6\varepsilon}}{(\beta\varepsilon)^{1/3}}\right) \qquad (2)$$

Here μ is particle magnetic moment and $\beta^2 = (\Delta H/H)_\varphi$ stands for an axial asymmetry of the magnetic field H. This asymmetry is necessary to let different resonant surfaces and corresponding stochastic layers intersect. The intersection of many resonant surfaces leads, due to a motion along these surfaces, to a particle diffusion across the magnetic field or into the mirrors of a geomagnetic trap. The picture of intersection is clear from a geometrical consideration if one imagines the interseption of 2-dimensional (in momentum or frequency space) resonant surfaces (I) by one of 2-dimensional energy surfaces. Similar consideration leads to the conclusion that for an axially symmetric magnetic field all, now only 2-frequences and 1-dimensional resonant lines ($q\Omega + \ell\bar{\omega} = 0$) are separated from each other, so that no Arnold' diffusion takes place. In the last case so-called eternal stability of a particle motion was proved by Arnold /6/. Because of the double exponencial dependence of the diffusion coefficient D_A upon the adiabaticity parameter $\mathcal{E}$ (2) there exists a rather sharp instability limit:

$$\mathcal{E}_c \approx \frac{1}{6} \approx 0.166 \qquad (3)$$

behind which the diffusion time becomes of cosmological scale! Following the paper /7/ I define the adiabaticity parameter as:

$$\mathcal{E} \simeq \frac{3}{2} \cdot \frac{V_{\parallel}}{\omega} \sqrt{H''/2H} \propto \sqrt{W} L^2 \qquad (4)$$

Here $V_{\parallel}$ stands for the longitudinal component of particle velocity and H'' is the second space derivative of the field strength along field line, the lagest (in the equatorial plane) value of ε being used in the expression (2). The last proportionality in (4) is valid if one assumes approximately that $v_{\parallel} \approx v$ where $\bar{W}, v$ are the energy and the full speed of a particle and L stands for the usual magnetic surface parameter which is, roughly speaking, the distance to the Earth centre in the equatorial plane measured in Earth radii.

In the case under consideration Arnold' diffusion results in a decrease of a particle life time by about one order of magnitude /2/. This effect was apparently observed in model experiments with electrons in a magnetic trap /8/. Similarly, one may expect a fall of particles intensity (a spectrum cut-off) in a geomagnetic trap under the conditions corresponding to the instability limit (3).

For a tentative check of this hypothesis I have used the data from Tverskoy's book /9/, namely, the spectra of particles of a given energy in respect to the parameter L. The dependence of the critical value of $\varepsilon = \varepsilon_c$ for protons at the cut-off upon the particles energy is given in Table and Figure.

Table

Protons group number	Average proton energy (Mev)	L cut-off	ε_c
I	0.I2	6.0	0.I2
2	0.I5	5.8	0.I3
3	0.22	5.4	0.I3
4	0.38	4.8	0.I4
5	0.74	4.3	0.I6
6	I.3	3.9	0.I8
7	I.5	3.8	0.I8
8	38	2.6	0.I3
9	58	2.5	0.I5
IO	90	2.3	0.I6

One can see that $\mathcal{E}_c$ varies less than by factor 2 as against 30-times change of the quantity $\sqrt{W}$ (4) in the range 0.I - I00 Mev. Even more striking feature of presented data is $\mathcal{E}_c$ absolute value (mean $\langle \mathcal{E}_c \rangle \approx 0.15$) which is rather close to the expected value (3). The variation of $\mathcal{E}_c$ with energy (and L) shows apparently some regularities, yet its discussion seems to be premature by the time being. The results presented above show only that Arnold' diffusion might play a role in the proton belt structure, and deserve, in my opinion, more careful comparison with the last experimental data. As for the electron belt, the maximal experimental value of $\mathcal{E}_c \approx 0.008$ is too small to provide any significance for the Arnold' diffusion (see, however, /II/).

As far as I know the first attempt to use nonadiabaticity criterion for an explanation of the Earth radiation belt structure was undertaken by Singer /I0/. After this, Pletnev /II/ assumed that magnetic moment adiabaticity might be violated under the influence of short-periodic variations of the magnetic field. Yet it was impossible at that time to evaluate quantitatively the adiabaticity limit because of the lack of theory. The present work should be considered as an attempt to develop this line of investigation.

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