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Editors
T. Grozdanov
P. Grujić
P. Krstić

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PROGRESS REPORTS

QUANTUM LIMITATIONS OF CLASSICAL DYNAMICS IN QUASICLASSICAL REGION

Boris V.Chirikov
Institute of Nuclear Physics
630090 Novosibirsk , USSR

ABSTRACT

Quantum limitations of a classical-like chaotic motion in simple few-dimensional models are discussed with special emphasis on the phenomenon of diffusion localization.Discussion includes the spectral properties of quantum chaos , the role of external noise and of the measurement as well as a classical model for quantum dynamics .

1. THE CORRESPONDENCE PRINCIPLE

In the discussion of applications of the classical dynamics in atomic and molecular physics I will relay upon the fundamental correspondence principle .In a narrow sense this principle had been formulated by Niels Bohr at the dawn of quantum mechanics as a practical method for solving quantum problems before the complete quantum theory was built . However, in the broad sense the correspondence principle must hold in any new fundamental theory which simply means immutability of all firmly established previously scientific laws. Future development of the science may only restrict, as a rule, or sometimes even broaden, the domain of their validity but would never refute them alltogether.

Unlike , say , the theory of relativity in quantum mechanics the transition to the old, classical, theory is rather singular and complicated . Whence the problem of quantum behaviour in the quasiclassical region which becomes especially difficult if the so-called dynamical chaos occurs in the classical limit. The chaos means that the motion of a purely dynamical system without any random parameters or any noise becomes, under certain conditions, very irregular and unpredictable .Three peculiarities of dynamical chaos are essential for what follows :

i) a continuous component in the motion spectrum;

ii) exponential local instability of trajectories as the principal condition for chaos;

iii) continuety of the phase space as the chaos ultimate origin.

It is well known by now that none of these properties persists in quantum mechanics . Then, how does the correspondence principle "work"? And what is going on in the quasiclassical region? Presently, those questions are widely discussed using a number of fairly simple quantum models as particular examples (see, e.g., Ref.1). Below I am going to dwell on some recent results as well as on moot points in this field .

The principal conclusion is that in quantum mechanics the so-called <u>pseudochaos</u> is only possible which mimics some (important) features of the true chaos (randomness) in classical mechanics. Ford's principal question[2]: "Quantum chaos, is there any?" I would answer:"A lot but pseudochaos !"

One may put the question another way[37]: are there two <u>different</u> mechanics, classical and quantum ones, or we need only to understand the quasiclassical transition? I certainly tend to the second possibility.

The ideas presented below have been developed in the process of a long and close international collaboration among a group of physicists including , at different stages, G.Casati and I.Guarneri (Italy); J.Ford (USA) ; F.Vivaldi (England) ; B.V.Chirikov, F.M.Izrailev and D.L. Shepelyansky (USSR) . I has greatly benefited from this co-

llaboration as, I hope, my colleagues also did.

2. MODELS

One of the simplest models is specified in the classical limit by the so-called <u>standard map</u> (SM): $(J,\theta) \to (\bar{J},\bar{\theta})$ where

$$\bar{J} = J + k \cdot \sin\theta; \quad \bar{\theta} = \theta + T \cdot \bar{J} \tag{1}$$

Here J, θ are the action-phase variables; k is the parameter of perturbation, and T its period[3,4]. This map has been studied thoroughly but still not yet completely. Physically, SM may be interpreted either as a rotator driven by the external periodic perturbation ("kicks") or as a local (in J) description of the dynamics in a conservative system of two freedoms on an energy surface, using Poincare's section method. In quantum case the second interpretation is questionable. To the best of my knowledge, it has been never studied in any detail. Nevertheless, I am going to consider such an interpretation as well relaying upon the correspondence principle.

Particularly, to study global motion properties on the energy surface one may transform the unbounded (in J) phase cylinder of model (1) to a finite torus by "closuring-up" the former: $J \to J$ mod L with $L = 2\pi m/T$ and m integer[5,6].

Below I will consider also a more physical, and much more complicated, model for the photoeffect in a Ridberg atom [8]. In one-dimensional (1D) approximation the Hamiltonian of this model has the form :

$$H = \frac{p^2}{2} - \frac{1}{z} + \varepsilon z \cdot \cos(\omega t); \quad z > 0 \tag{2}$$

where ε is the strength of homogeneous electric field, and where ω is its frequency in atomic units: $|e| = m = \hbar = 1$. For $\omega \gtrsim 1/n^3$ (the Kepler frequency) the problem can be approximately reduced to SM (1) with the parameters [9]:

$$k \approx 2.6 \frac{\mathcal{E}}{\omega^{5/3}}\ ; \quad T \approx 6\pi\omega^2 n^5 \qquad (3)$$

Here $n \gg 1$ is the principal quantum number of Hydrogen atom, while now $\mathcal{J}$ in Eq. (1) enumerates the "photonic states" of period ω in energy. Such states present a good approximation [8] as soon as the group Δn of atomic levels, corresponding to each photonic state, becomes relatively narrow: $\Delta n \sim k \ll \omega_0 = \omega n^3$. The latter inequality greatly improves with n, and the excitation spectrum takes the characteristic shape of a sequence of narrow peaks. The quasiclassical transition corresponds to $n \to \infty$ while the reduced frequency $\omega_0 = \omega n^3$ and reduced field $\mathcal{E}_0 = \mathcal{E} n^4$ are fixed.

In Ref.10 the question was raised if 1D-approximation was really valid since the motion in the second freedom proved to be unstable. However, a more detailed analysis [11] revealed that in case of chaotic $\mathcal{J}$ motion the instability is too slow to influence any appriciable change in $\mathcal{J}$ dynamics.

Quantization of SM (1) is determined by a simple condition that $\mathcal{J}$ is to be integer ($\hbar = 1$). Literally, it would imply periodicity of $\psi(\theta)$ as in the rotator model, for instance. However, this is not necessarily the case. SM describes also a particle in spatially periodic field. Then $\mathcal{J}$ may take on any value. Yet, the fractional part $j = \{\mathcal{J}\}$, the quasimomentum, is strictly conserved and, hence, can be removed. As the states with different j evolve independently of each other, the excitation dynamics remains essentially unchanged (cf. Ref.7). A similar situation takes place for the photonic states in Hydrogen where $\mathcal{J} = \left(\frac{1}{n_0^2} - \frac{1}{n^2}\right)/2\omega$, n_0 being the quantum number of the initial state, and j is now determined approximately only due to discreteness of n. In what follows I consider $\mathcal{J}$ to be integer, then operator $\hat{\mathcal{J}} = -i\,\partial/\partial\theta$, and the quantized standard map (QSM) has a form [12]:

$$\overline{\psi} = \exp\left(i\frac{T}{2}\frac{\partial^2}{\partial\theta^2}\right)\cdot\exp\left(-ik\cdot\cos\theta\right)\psi \qquad (4)$$

Notice that QSM depends on both parameters k and T, while in the classical limit $(k\to\infty,\ T\to 0)$ SM dynamics is completely determined by the only parameter $K = kT$. This is achieved by introducing a new variable $p = T\mathcal{J}$. The same can be done in quantum case as well [13], yet the second parameter T persists owing to $\mathcal{J}$ quantization. Parameter T may be also considered as an effective quantum constant [13], the map (4) remaining unchanged, of course. Thus, the motion structure is more rich in quantum case while (chaotic) trajectories prove to be richer in clas - sical case (see Sections 3 and 4).

In case of the rotator model the map (4) is exact , and it completely describes the evolution of the former . However, this is not the case at all for the model of photoeffect in Hydrogen . Here the corresponding map is much simpler as compared to the exact Schrödinger equation for Hamiltonian (2) . Yet, the map is applicable only locally in $\mathcal{J}$ as parameter $T(n)$ depends on $\mathcal{J}(n)$ (see Eq.(3)) . In Ref.9 the global map has been constructed :

$$\overline{\psi} = \exp\left(-i\hat{H}_0\right)\cdot\exp\left(-ik\cdot\cos\theta\right)\psi \qquad (5)$$

$$\hat{H}_0 = +\frac{\pi\sqrt{2}}{\sqrt{\omega}}\left(\mathcal{J}_0 - \hat{\mathcal{J}}\right)^{-1/2}; \quad \mathcal{J}_0 = \frac{1}{2\omega n_0^2}$$

Even though , $\psi(\mathcal{J})$ provides a very abbreviated description for the Hydrogen quantum state, being a rather peculiar projection of the latter on the photonic states . Roughly, the description is cut by a factor of

ωn^3 which rapidly grows with n. One may conclude from the results in Ref.9 that each photonic state repre - sents the total probability $(|\psi(\mathcal{J})|)$ in its energy range (ω). Yet, this question has to be studied further.

Another difficulty is in that the map for $\psi(\mathcal{J})$ is determined over the Kepler period which depends on $\mathcal{J}$. Hence, $\psi(\mathcal{J})$ in Eq. (5) represents system's state at various moments of time. This question also requires the additional analysis. Nevertheless, numerical experiments[9)] do verify that abbreviated description (5), at least time averaged, satisfactorily agrees with the direct solution of Schrödinger's equation.

3. CLASSICAL DYNAMICAL CHAOS

The classical dynamics of the models described in Section 2 has been studied in detail and, basically, is understood [3,8,18)]. In model (1) an unbounded diffusion in $\mathcal{J}$ sets in under $K = kT \gtrsim 1$ whose rate is

$$D_{\mathcal{J}} = \frac{\langle (\Delta \mathcal{J})^2 \rangle}{\tau} = \frac{k^2}{2} C(K) \qquad (6)$$

where τ is the number of SM iterations. Dependence $C(K)$ characterizes the correlation between successive θ values which decays as K grows : $C(K) = 1 + O(K^{-1/2})$. For $K \to 1$ the diffusion rate falls down with $C(K) \approx 0.6 (K-1)^3$.

In case of SM on a torus the exponential relaxation to homogeneous distribution occurs with a characteristic time

$$\tau_R = \frac{L^2}{2\pi^2 D_{\mathcal{J}}} \qquad (7)$$

where L is torus circumference in $\mathcal{J}$.

Notice that for $K \gtrsim 1$ not too big a chaos border is present which drastically changes the motion statistical properties [15]. Particulary, the relaxation proceeds according to some power law rather that to exponential one .

The ultimate origin of dynamical chaos lies in con - tinuety of phase space while the chaos mechanism is related to the exponential instability of close trajectories whose rate for model (1) is

$$\Lambda \approx \ln \frac{K}{2} \ , \quad K > 4 \qquad (8)$$

The quantity Λ is called Lyapunov's exponent . The exponential instability is a simple and ,perhaps, the most convenient criterion for dynamical chaos in numerical ex - periments. Notice that Λ is determined bythe behaviour of close trajectories, i.e. by the linearized equations of motion .

Comparing Eqs. (6-8) one can conclude that, generally, there is no direct relation between the local instability and relaxation as was assumed in Ref.16. Partly, it is due to specific perturbation in SM (kicks). In case of a ra - ther general continuous perturbation the mean diffusion rate does relate to Λ , indeed [17,18] :

$$D_\omega \approx 40\,\Lambda^3 \qquad (9)$$

However, the question if a similar relation would hold in many-dimensional systems with several Lyapunov's exponents is still open. The same remark is true for the relaxation in phase θ which preceeds the diffusion process , and which is but the initial packet spreading with a characteristic time Λ^{-1} .

4. QUANTUM LOCALIZATION OF DYNAMICAL CHAOS

The correspondence principle suggests that QSM (4) should also reveal the diffusion, in quasiclassical re - gion, provided the global chaos occurs in the classical limit ($K \gtrsim 1$). To begin with, what is the quasiclassical region for QSM? The common belief is that quantum numbers (here $\mathcal{J}$) must be big. However, the dynamics of model (4) does not depend on $\mathcal{J}$ in that each kick drives transitions within the same interval $\Delta\mathcal{J} \approx \pm k$ for any $\mathcal{J}$. It is again related to homogeneity of model (4) in $\mathcal{J}$. Instead, the quasiclassical parameters here are quantities k and $1/T$ which both must be big in quasiclassical region.

In this region a quantum diffusion, close to classical one, is observed, indeed, as had been first discovered in Ref. 12 and then repeatedly confirmed. Particularly, the rate of quantum diffusion follows all the details of dependence (6)[14]. However, it continues within a certain finite time interval $\tau \lesssim \tau_D$ only. Afterwards, the diffusion gradually slows down, and finally stops completely. A quantum steady-state, or stationary oscillation, builds up which is totally different from the classical unbounded diffusion.

The latter phenomenon had been discovered also in Ref. 12, and was explained actually from the uncertainty principle alone[18]. The basic idea was in that the diffusion time scale τ_D of quantum evolution is determined by mean density ρ of quasienergy (QE) levels:

$$\tau_D \sim \rho \sim D_{\mathcal{J}} \qquad (10)$$

The latter estimate is a result of the self-consistent calculation for ρ based upon the number of unperturbed states involved in the diffusion during the time τ_D.

The cease of diffusion implies localization of the quasienergy eigenfunctions (QEEs) . According to numerical experiments [14] the localization is exponential owing to homogeneity of QSM in $\mathcal{J}$. The localization length is

$$\ell \approx \frac{D_{\mathcal{J}}}{2} \tag{11}$$

This remarkable law relates the essentially quantum characteristic ℓ to system's properties in the classical limit (6) .

Moreover, it was found that the localization length ℓ_s for the stationary oscillation is different from ℓ , namely : $\ell_s \approx 2\ell$ according to Refs. 4 and 14. Even though in the latter papers a certain explanation is given for this surprise fact, based upon very big QEE fluctuations, the quantitative theory is still to be developed.

There is a completely different type of localization under condition, in QSM model, as follows

$$k \lesssim 1 \tag{12}$$

In this case any transitions between unperturbed states are suppressed, so that the QEEs are close to unperturbed ones . This is the domain of applicability for the quantum perturbation theory, hence a graphic term due to Smilansky - the <u>perturbative localization</u>. Condition (12) is well known and, in a sense, trivial. In the problem of quantum chaos it was , apparently, first analyzed in Ref. 19 and is usually called the <u>quantum stability border</u>.

The inequality opposite to Eq.(12) is a necessary condition for quantum diffusion whose localization mechanism is completely different. The latter localization is a direct consequence of the discreteness of QE spectrum with a <u>finite</u> density ρ . This density is determined by those QEEs only which are actually excited in a given initial state. I shall call them the <u>operative QEEs</u> . Notice that the spectrum of all QEEs is generally everywhere dense as QE is usually defined mod $2\pi/T$. Whence, the notion of quasicontinuum which is sometimes believed to provide an irreversible relaxation. Generally, such a conclusion is wrong just because the time evolution depends on the operative QEEs only which leads to the diffusion localization.

The discreteness of QE spectrum is inferred, by derivation in Refs.4 and 18, from the fundamental principle of the discreteness of phase space in quantum mechanics or, equivalently, from quantization of the action. However, this conclusion is not rigorous. Moreover, QSM does have a continuous spectrum for rational $T/4\pi$. That corresponds to the so-called quantum resonance [12,20] again due to QSM homogeneity in $\mathcal{J}$. Clearly, it is not a generic phenomenon even in QSM .

Estimate (10) for the diffusion scale τ_D is still a point of disagreement among researchers. In Ref.21, for example, this scale is apparently confused with another one

$$\tau_E \sim \frac{|\ln T|}{2\Lambda} \tag{13}$$

The latter, indeed, has been discovered in Ref.22 and explained by a rapid spreading of the initial wave packet (see also Ref.18). This process precedes quantum diffusion which persists on a much longer scale τ_D . At $\tau \lesssim \tau_E$ an initially narrow packet moves along the classical trajectory in accordance with the Ehrenfest theorem,whence the term Ehrenfest's scale. In first paper [21] the authors reported on the observation, in a numerical experiment , of this interesting process in a satisfactory agreement with the classical prediction .

In any event, the quantum dynamics turns out to be stable for $\tau \gtrsim \tau_E$. This is clearly demonstrated by the numerical experiment with velocity reversal [23,24] .

In the classical limit the reversal causes a little impact on the diffusion owing to a rapid growth of computational errors while in a quantum system the "antidiffusion" sets in, up to the complete restoration of the initial state. It is especially striking in case of the photoeffect in Hydrogen [24] where the recurrence even from the continuous spectrum does occur as well .

In Ref. 25 one more estimate for the diffusion scale was obtained : $\tau_D \propto T^{-1/3}$ (K = 1.5) . It is intermediate between Eq.(10) ($\tau_D \propto T^{-2}$ and Eq. (13). The authors proceeded from the properties of the critical structure near K =1. However, the analysis in Ref.14 shows that critical phenomena are insignificant for all values of T parameter in Ref. 25 (see Eq.(20)below). Apparently, the above contradictions are related to some specific definitions of τ_D both in Ref.25 and 21 . In my opinion, this question has been settled in a recent paper [26] where the law of quantum diffusion decay was found:

$$\mathcal{D}(\tau) = \frac{\mathcal{D}(0)}{\left(1 + \frac{\tau}{\tau_D}\right)^{1+\beta}} \qquad (14)$$

Here $\mathcal{D}(0) = \mathcal{D}_y$ is the rate of classical diffusion (6), and $\beta \approx 0.2$ depends on the fluctuations ("repulsion") in QE spectrum (see Section 7 below) . The value of β was obtained by comparison of Eq.(14) with the data of numerical experiments. Relation (14) provides also an exact definition for τ_D . Using the expression for the energy of stationary oscillation [4] $E_s = \langle y^2\rangle/2 \approx \mathcal{D}_y^2/2$ (initial $E(0)$ =0), one arrives at the relation [26]

$$\tau_D \approx \beta \mathcal{D}_y \qquad (15)$$

in accordance with estimate (10). In the absence of QE-level repulsion (β = 0, see Ref.27) the energy would grow logarithmically but only up to about the same limit

$E_s \approx \mathcal{D}_y^2/2$ which apparently does not depend on spectral fluctuations. Whence, $\tau_D \sim \mathcal{D}_y/\ln \mathcal{D}_y$. Comparison with Eq.(15) suggests that β may depend on $\mathcal{D}_y$, say, $\beta \sim 1/\ln \mathcal{D}_y$. As a matter of fact $1/\ln \mathcal{D}_y$ values in Ref. 26 all lie in the interval 0.15 - 0.25 .

There is a far reaching correspondence (not yet completely comprehended) between the localization of excitation in momentum space (in energy) driven by an external periodic perturbation and the Anderson spatial localization in an irregular static potential [28,29]. One may say that the former is a dynamical counterpart of Anderson's statistical localization. In the latter theory the potential is assumed to be random. In QSM model this would correspond to randomness of unperturbed QE values $E_n^o = Tn^2/2$ mod 2π . Yet, this sequence is known to be nonrandom. Particularly, it would imply that Anderson's localization requires a certain weak irregularity only, rather than randomness, of the potential. Indeed, recent mathematical studies revealed that the localization is possible even in the quasiperiodic potential of two incommensurable periods only .

5. DIFFUSIVE PHOTOEFFECT

The photoeffect in a Ridberg atom turned out to be a very curious phenomenon. It had been discovered in laboratory experiments by Bayfield and Koch in 1974 but was more or less comprehended only recently (see review[8]).The most intensive ionization occurs around the classical Kepler frequency $\omega \sim 1/n_o^3$ where the absorbtion of $\sim \frac{n_o}{2} \gg 1$ photons is required. As this ionization has a diffusive nature the total number of photonic transitions (absorbtion and emission) is still much bigger and amounts up to $\sim n_o^2$! Nevertheless, the rate (inverse time) of ionization

$$\frac{1}{t_I} \sim \varepsilon^2 \frac{n_o}{\omega^{4/3}} \qquad (16)$$

is proportional to the square of electric field like for a direct one-photon transition. Notice, however, that unlike the latter the diffusive ionization lags by $\sim t_I$.

At high frequency $(\omega_0 = \omega n^3 \gtrsim 1)$ the photoeffect dynamics is approximately described by QSM (4) with parameters (3). At low frequency $(\omega_0 \lesssim 1)$ such a simple approximation is no longer valid . Yet, the classical description proves to be applicable [30] provided $n \gg 1$. On the other hand, as is shown in Ref. 31 the ionization border is well in agreement with a modified quantum criterion (12). Coincidence of the classical and quantum ionization borders seems surprising and gives rise to some doubts .

At high frequency , according to QSM model, the localization occurs which generally suppresses the photoeffect. However, the total number of photonic states ($n_0/2\omega_0$, above the initial state) is <u>finite</u>. Therefore, the localization is decisive if it comprises the descrete spectrum only, i.e. if its length (in $\mathcal{J}$) $\ell_{\mathcal{J}} \lesssim n_0/2\omega_0$,or if [9]

$$\mathcal{E}_0 \lesssim \frac{\omega_0^{7/6}}{\sqrt{6 n_0}} \equiv \mathcal{E}_q \qquad (17)$$

The right-hand side of this inequality is called the <u>delocalization border</u> . Remarkably, in Ref.8 the same condition has been derived in a quite different way.At $\mathcal{E}_0 \gtrsim \mathcal{E}_q$ the photoeffect is described classically .

In any case , the diffusive ionization requires the classical global chaos under condition

$$K = kT \approx 50\, \mathcal{E}_0\, \omega_0^{1/3} \gtrsim 1 \qquad (18)$$

and the quantum transitions to occur for

$$k \approx \frac{2.6\,\varepsilon_0 n_0}{\omega_0^{5/3}} \gtrsim 1 \qquad (19)$$

The latter condition (see Eq.(12)) is decisive at a very high frequency only ($\omega_0 \gtrsim n_0$) which is apparently of no interest.

In papers[32] a different mechanism of the quantum localization is discussed which based upon the critical phenomena in chaotic motion. True, those would change many of the above estimates but only at $K \approx 1$ or ,more precisely, for[14]

$$\Delta K \approx K - 1 \lesssim \frac{(2K)^{2/3}}{k^{1/3}} \qquad (20)$$

Finally, a new conjecture was put forth in a recent paper[13] that the localization observed in numerical experiments is related to a big value of parameter $T \sim 1$ which violates the quasiclassical approximation (Section 4).Actually, this question was already studied in Ref.14 with the conclusion that estimate (11,6)for the localization length remains valid upon substitution

$$K = 2k \cdot \sin\left(\frac{T}{2}\right) \qquad (21)$$

Notice that parameter T is determined mod 4π (see Eq. (4)).

Another important question: if localization persists in a many-dimensional case? Preliminary evaluations suggested that generally it does not. Particularly, for the photoeffect in the 2D-model the delocalization border (17) was expected to considerably fall off[8]. However , numerical experiments[11] refuted this prediction. The localization in n proved to be practically independent of the motion in the second freedom. In Ref.11 this result was explained by a low oscillation frequency in this second freedom, ultimately due to the Coulomb degeneration. As a result the level density ϱ is enhanced in narrow energy intervals only, so that the motion remains essentially one-dimensional .

This example shows that the phenomenon of quantum localization may occur not only in 1D-case. Apparently, the same is true for the Anderson localization as well. Also, there exists a plausible, not yet checked though, conjecture[14] that the excitation of a many-dimensional internally chaotic system by a sufficiently weak external periodic perturbation should be localized .

6. DELOCALIZATION BY NOISE

Localization is caused by the <u>discreteness</u> of QE spectrum. Therefore, any external perturbation of a continuous spectrum (noise) would destroy the complete localization. This phenomenon was investigated in Ref.33 on model (1,4) with an additional noise: $k \cdot \cos\theta \rightarrow k \cdot \cos\theta + \tilde{k} \cdot \varphi(\theta)$ where φ is some random function, and $\tilde{k} \ll 1$. The localization breaks down completely under condition that the noise provides transitions between unperturbed states within localization time τ_D , or for

$$\mathcal{D}_\gamma \cdot \tilde{\mathcal{D}} \gtrsim 1 \qquad (22)$$

where $\tilde{\mathcal{D}} \sim \tilde{k}^2$ is the rate of noise-driven diffusion. Under this condition the classical diffusion in quantum system goes on indefinitely. In the quasiclassical transition $\mathcal{D}_\gamma \rightarrow \infty$, hence an arbitrarily weak noise destroys localization and restores the classical behaviour. However, the question, if any noise could also provide the classical exponential instability of motion, remains as yet unclear.

A similar effect is caused by the intermediate measurements. Till now, the evolution of a quantum system "by itself" has been only considered. In other words, the two measurements only were presupposed: the first one which fixes the initial state, and the final one which records the result of evolution. Unlike classical mechanics the intermediate measurements substantially change the quantum evolution. It is not a weak noise at all !

Particularly, for restoration of classical diffusion the complete quantum measurement is to be done, loosely speaking, at least once per time τ_D. Even much more often measurements are required to "keep" a quantum packet on the classical trajectory - once per τ_E (13). In the latter case the special measurement, that of a coherent state minimizing uncertainties $\Delta\theta_m \cdot \Delta J_m \sim 1$, must be provided under the additional condition

$$\Delta J_m \ll D_J \left(\frac{\tau_m}{\tau_D}\right)^{1/2}; \qquad D_m \sim \frac{(\Delta J)_m^2}{\tau_m} \ll D_J \qquad (23)$$

Otherwise, the diffusion rate would be determined by the noise. Here τ_m is the mean period between successive measurements, and D_m is the "measurement-induced" diffusion rate. Particularly, for minimal $\tau_m = 1$ the measurement accuracy $\Delta J_m \ll D_J^{1/2}$. On the other hand, the second Eq.(1) implies that the optimal accuracy is reached for $\Delta\theta_m \sim T\Delta J_m$, whence $\Delta J_m \sim 1/\Delta\theta_m \sim T^{-1/2} \gg 1$.

One more interesting question: if such collapses of ψ wave would occur somehow in a natural way, without special measurements?

7. SPECTRAL PROPERTIES OF QUANTUM CHAOS

At $\tau \gg \tau_D$ the evolution of a quantum system free of noise does no longer resemble the classical dynamical chaos in any way as the former is quasiperiodic and has a discrete spectrum. In the classical limit such an evolution is called regular and considered to be the opposite limiting case to the chaos. Are any remnants of the classical chaos left nevertheless as $\tau \to \infty$?It turns out that some are, indeed, and they are in specific statistical properties of QE spectrum as well as of the QEEs. I am going to dwell a while on the spectrum .

As is well known the energy levels in a "complicated" quantum system "repel " (see, e.g., Ref.34). Particulary, the distribution of spacings s between neighbouring levels has a specific form:

$$p(s) = As^{\beta}e^{-Bs^2} \qquad (24)$$

which is called the Wigner-Dyson (WD) distribution. Here A, B are normalizing constants; average $\langle S \rangle = 1$,and repulsion parameter β = 1;2;4 depends on system's symmetry. It has been recently shown[35] that in "simple" models the distribution (24) corresponds to the dynamical chaos in the classical limit while for the regular motion the repulsion is absent, $\beta = 0$, and $p(s) = exp(-s)$. Thus WD-distribution, and, particularly, the β value may serve as a criterion for quantum chaos.

In Ref.5 all these statistical properties of energy levels were extended onto the quasienergy spectrum. However, for the full spectrum of QSM parameter $\beta = 0$ as was earlier proved in the counterpart problem on Anderson's localization in an infinite 1D-lattice. It is explained by the fact that most of localised QEEs do not overlap which is precisely the mechanism of repulsion. The statistical properties drastically change[5] in QSM on a torus, i.e. with a finite number of states L (Section 2). Here a new parameter appears, namely

$$\lambda = \frac{\ell}{L} \qquad (25)$$

where ℓ is QEE localization length (11). If $\lambda \gg 1$ all QEEs completely overlap, and under condition $K \gg 1$ (classical chaos) and $k \gg 1$ (12), it is natural to expect the WD-distribution . Numerical experiments confirmed this conjecture. Naturally, Eq.(24) was called the limiting statistics[5]. Another limiting statistics occurs as $\ell \to \infty$ ($\lambda \to 0$).

What is the intermediate statistics at $\lambda \sim 1$? This problem was studied in Ref.6 where the following one-parameter family of distributions has been proposed:

$$p(s,\beta) = A s^{\beta} \exp\left\{-\frac{\pi^2}{16}\beta s^2 - \left(C - \frac{\beta}{2}\right)\frac{\pi}{2} s\right\} \qquad (26)$$

Now $A(\beta)$ and $C(\beta)$ are the functions of a continuous parameter β ($0 \leq \beta \leq 2$) determined by normalization and by $\langle S \rangle = 1$. Moreover, the dependence $\beta(\lambda)$ has been found in Ref.6 or, better to say, the method for calculating β from the QEEs. Particularly, $\beta \approx 4\lambda$ for $\lambda \ll 1$. Numerical experiments[6] satisfactorily agree with these dependences.

The intermediate statistics allows also to qualitatively understand the mechanism of repulsion for the operative QEEs which determine the diffusion suppression (14). Those QEEs do partially overlap, hence an intermediate value $\beta \approx 0.2$. However, it appears to be too small ($\lambda \approx 0.05$).

8. CLASSICAL MODEL OF QUANTUM DYNAMICS

As peculiarities of quantum dynamics are ultimately related to discreteness of the action variables one may attempt to construct some classical model of the quantum system[18]. To this end we simply took the integer part of $|J|$: $k \cdot \sin\theta \to [k \cdot \sin\theta]$ (1). Surprisingly, such a "little" change in classical model makes it to represent, at least qualitatively, many QSM properties. First, the "quantum" stability border $k < 1$ emerges (cf. Eq.(12)).

Second, the "quantum" resonance occurs[36] at rational $T/2\pi = r/q$, although its condition differs from that in QSM by a factor of 2: $T/4\pi = r/q$ (4). Apparently, the most important fact is in that the classical model describes qualitativly, up to a factor of ~ 2, the quantum localization of diffusion as well .

In QSM with finite number of states L the phase θ is also discrete and has L different values only. This is just the situation which always takes place in the digital computer. It is tempting to use these computer rounding-off "errors" for simulating quantum effects ! Unfortunately, that is generally impossible since any dynamical trajectory in computer is going to be periodical after all while a generic quantum evolution is almost periodic. Indeed, thorough investigations[36] revealed that the full agreement with QSM requires an additional term in the discrete model which the former is hardly simpler than the exact quantum map (4). Nevertheless, further studies into the discrete classical dynamics is certainly of interest, I believe .

REFERENCES

1. Chaotic Behaviour in Quantum Systems, Casati, G.,Ed., Plenum, New York, 1985 .
2. Ford, J., "Quantum Chaos: Is there any?",Directions in Chaos, Hao Bai-Lin, Ed.,Singapore,1987 .
3. Chirikov, B.V.,Phys.Reports 52, 263 (1979).
4. Chirikov, B.V.,Izrailev,F.M. and Shepelyansky, D.L., Physica D,September 1988.
5. Izrailev,F.M.,Phys.Rev.Lett.56,541 (1986).
6. Izrailev,F.M.,"Intermediate Statistics of Quasi-Energy Spectrum and Quantum Localization of Classical Chaos", Preprint INP 88-45,Novosibirsk,1988.
7. Yukawa,T.,"Semi-Classical Limit of the Periodically

Kicked Rotator", KEK-Preprint-87-145,1988.
8. Casati,G. et al.,Phys.Reports 154, 77 (1987).
9. Casati,G. et al.,Phys.Rev. A36, 3501 (1987) .
10. Leopold,J.G. and Richards,D.,J.Phys. B 19, 1125 (1986)
11. Casati,G. et al.,Phys.Rev.Lett, 59,2927(1987).
12. Casati,G. et al.,Lecture Notes in Physics 93,334 (1979)
13. Jensen,R.V. et al.,"High Frequency Microwave Ionization of Excited Hydrogen Atoms",J.Phys. B Lett. (in press).
14. Shepelyansky, D.L.,Physica D28, 103 (1987)
15. Chirikov,B.V. and Shepelyansky,D.L.,Physica D13,395 (1984); "Chaos Border and Statistical Anomalies",preprint INP 86-174,Novosibirsk,1986 .
16. Kuzmin,M.V. et al.,Chem.Phys.Lett.124,522 (1986).
17. Rechester,A.B. et al.,Phys.Rev.Lett. 42,1247 (1979).
18. Chirikov,B.V.,Izrailev,F.M. and Shepelyansky,D.L.,Sov. Sci.Rev. C 2, 209 (1981).
19. Shuryak,E.V.,Zh.Eksp.Teor.Fiz. 71,2039 (1976).
20. Izrailev,F.M. and Shepelyansky,D.L.,Teor. i Mat.Fiz. 43, 417 (1980).
21. Toda,M. and Ikeda,K., Phys.Lett. A 124 , 165 (1987). Adachi,S. et al.,Recovery of Liouville Dynamics in Quantum Mechanically Suppressed Chaotic Behaviour, Preprint RIFP-736, Kyoto,1988 .
22. Berman,G.P. and Zaslavsky, G.M., Physica A 91,450(1978)
23. Shepelyansky, D.L.,Physica D 8, 208 (1983).
24. Casati,G. et al.,Phys.Rev.Lett. 56,2437 (1986).
25. Fishman,S. et al.,Phys.Rev. A 36, 289 (1987).
26. Berman,G.P. and Izrailev,F.M.,"Concerning the Dynamics of Correlation Function and of Diffusion Limitation in the Domain of Quantum Chaos",Preprint 497F,Krasnoyarsk, 1988 (in Russian).
27. Chirikov,B.V., Usp.Fiz. Nauk 139,360 (1983).
28. Fishman,S. et al.,Phys.Rev. A 29, 1639 (1984).

29. Shepelyansky,D.L., Phys.Rev.Lett. 56, 677 (1986).
30. Van Leeuwen,K. et al.,Phys.Rev.Lett.55, 2231(1985).
31. Blümel,R. and Smilansky,U.,Phys.Rev.Lett. 58,2531(1987)
32. MacKay,R.S. et al.,Physica D 13, 55 (1984);
"The Relation between Quantum and Classical Thresholds for Multiphoton Ionization of Excited Atoms",preprint IFSR 209,Austin, 1987;
Geisel,T. et al.,Phys.Rev.Lett. 57,2883 (1986).
33. Ott,E. and Antonsen,T.,Phys.Rev.Lett.53, 2187(1984).
34. Brody,T.A. et al.,Rev.Mod.Phys. 53,385 (1981).
35. Boihas,O. and Giannoni,M.,Lecture Notes in Physics 209,1 (1984).
36. Berman,G.P. et al.,Physica D 17, 183 (1985);
"Quantum Chaos and Diffusion Peculiarities in Wigner's Representation",Preprint INP 87-118,Novosibirsk,1987.
37. Clough,S.,New Scientists, 24 March 1988,p.37 .