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QUANTUM CHAOS

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QUANTUM CHAOS:
STATISTICAL RELAXATION IN DISCRETE SPECTRUM

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ABSTRACT

The controversial phenomenon of quantum chaos is discussed using the quantized standard map, or the kicked rotator, as a simple model. The relation to the classical dynamical chaos is tracked down on the basis of the correspondence principle. Various mechanisms of the quantum suppression of classical chaos are considered with an application to the excitation and ionization of Rydberg atoms in a microwave field. Several difinitions of the quantum chaos are discussed.

1. PROLOGUE: WHY QUANTUM CHAOS?

The main purpose of my lectures is to present our understanding of a new controversial phenomenon - the so called quantum chaos - which is attracting now much attention of scientists working in many different fields of research [1,2]. The picture of quantum chaos I am going to discuss has been developed in a long-term collaboration of the Italian-Soviet team including G.Casati (Milano); I.Guarneri (Pavia); B.Chirikov, F.Izrailev and D.Shepelyansky (Novosibirsk) [3,4]. Of course, this particular presentation is my own as well as the full responsibility for possible inaccuracies, misconceptions, and even mistakes.

So, why quantum chaos? Apparently because there is the classical dynamical chaos, and a common belief that

the quantum mechanics is universal, including particularly the classical mechanics together with its dynamical chaos. Hence, there must be a quantum theory of chaos, or the quantum chaos.

Now, the beauty of the classical chaos is, first of all, in its highly controversial nature relating the apparent opposites - deterministic (dynamical) motion and random (statistical) behaviour. Besides, the chaos turns out to be a rather widespread (generic) dynamical process which has been overlooked during a long period of time with a simplified approach to the Nature.

From the viewpoint of various applications the important property of chaos is a simple statistical description (of course, incomplete but essential) of very complicated motions. This is characteristic to any statistical theory but an exciting peculiarity of the dynamical chaos is in extension of a simple statistics down to a few freedoms which was a great surprise, indeed.

On the other hand, there exists a number of fundamental problems which are still waiting solution, and to which the conception of dynamical chaos seems to be relevant. I am not yet ready to discuss those problems at length. Still, I would like just to mention two of them: (i) ψ collapse in the quantum measurement which as yet has not been described by any dynamical theory, and (ii) the causality principle which still seems to be completely separated from the rest of physics. Further discussion of these problems can be found in [5,6].

2. A SIMPLE MODEL OF DIFFUSIVE PHOTOEFFECT IN RYDBERG ATOMS

The photoeffect in Hydrogen seems to be an appropriate example to discuss a new phenomenon, the quantum chaos. Amid other problems currently under study this one looks

apparently almost trivial. Yet, it turned out to be very complicated and surprising.

The puzzle arised 16 years ago in the experiments by Bayfield and Koch who observed a fast ionization of the Rydberg Hydrogen in a low-frequency microwave field [7]. The initial principal quantum number was $n_0 \sim 70$, and as many as about $\nu \sim 100$ quanta were required to ionize the atom. How is it possible at all?

The first insight has been got by Delone, Zon and Krainov [8]: they conjectured a new ionization mechanism, which is now called the diffusive ionization, as contrasted with the well-known direct multiphoton transition. It is interesting to mention that the immediate implication of this conjecture would be still much bigger number of one-photon transitions required, namely, $\sim \nu^2 \sim 10^4$(!) instead of $\nu \sim 10^2$ for the direct transition.

A partial resolution of this difficulty came from a simple observation that for a big quantum number $n_0 >> 1$ the classical mechanics should be applicable. The corresponding theory has been developed[9] which apparently settled the matter, at least, temporarily.

In spite of apparently 'trivial' character of the system some further simplification was necessary. In most studies that was the one-dimensional (1D) model specified by the Hamiltonian

$$H = -\frac{1}{2n^2} + \mathcal{E} z(n,\theta) \cos \omega t \qquad (1)$$

where $\mathcal{E}$, ω are the electric field strength and frequency; n , θ are the action-angle variables; z is the coordinate along the linearly polarized field, and the atomic units $e = m = \hbar = 1$ are used. In this model an extended electron orbit with big eccentricity is approximated by the straight-line orbit with singularity at $z = 0$. In what follows the so-called scaled variables $\omega_0 = \omega n_0^3$, $\mathcal{E}_0 = \mathcal{E} n_0^4$ etc will be convenient. These can be termed

also the classical variables as the classical motion cannot depend separately on quantum number n_0 .

The continuous model (1) is still rather complicated for both analytical studies as well as computer simulation, or numerical experiments as we use to say. Subsequently [10], a simple Kepler map has been devised: $(N, \varphi) \to (\bar{N}, \bar{\varphi})$ where

$$\bar{N} = N + k \sin\varphi$$
$$\bar{\varphi} = \varphi + 2\pi\omega \left(-2\omega\left(\bar{N}+\nu\right)\right)^{-\frac{3}{2}} \qquad (2)$$

Here φ is field's phase when the electron is in perihelion; the 'action' $N = (E-E_0)/\omega$ is the number of absorbed quanta; $E = -1/2n^2$ is atom's full energy, and $\nu = E_0/\omega = -n_0/2\omega_0$. Map (2) describes the change in canonically conjugated variables N , φ over a Kepler period of the electron.

The Kepler map can be still simplified by linearizing the second equation (2) which is the Kepler law. In this way we arrive at the so-called standard map

$$\bar{N} = N + k \sin\varphi; \quad \bar{\varphi} = \varphi + T\bar{N} \qquad (3)$$
$$k \approx 2.6\, \varepsilon_0 n_0/\omega_0^{5/3}; \quad T = 6\pi\omega_0^2/n_0; \quad K \equiv kT \approx 50\,\varepsilon_0\omega_0^{1/3}$$

which describes the dynamics of the Kepler map locally in N. Parameter K is the only one which completely determines the classical dynamics. Notice that upon introducing a new variable $N/n_0 = (1/2\omega_0)(1 - n_0^2/n^2)$ the classical maps (2) and (3) contain the scaled quantities only.

3. CLASSICAL CHAOS, OR RANDOM DYNAMICS

The standard map (3) has been studied in many details independent of the particular application in question (see, e.g. [11]). It proved to be a very convenient model, simple in appearance and highly nontrivial and rich in essence. In spite of intense studies it still remains inexhaustible and continues to supply new information on both classical and

quantum chaos. On the other hand, it approximately describes some real physical system as the present example shows.

The standard map is known to have chaotic component of motion for any finite $K > 0$. The chaotic motion means that almost all trajectories (of a chaotic component) of a purely dynamical system (without any random parameters), like standard map, are nevertheless random. In turn, the random trajectory means that it is highly irregular, complicated and unpredictable from any observation with arbitrarily high but finite accuracy. The ultimate origin of this randomness is in continuity of the phase space in classical mechanics. This property is very important for understanding the quantum chaos we are going to discuss below.

The mechanism of the chaos is explained by a strong local instability of motion. Namely, the main condition for the chaos is a positive (nonzero) Lyapunov exponent Λ for the solution of the linearized equations. In case of two-dimensional map which can have, at most, one $\Lambda > 0$ the latter is related to the information $I(t)$ associated with a typical trajectory. According to the Alekseev-Brudno theorem (see [12]).

$$\lim_{|t|\to\infty} \frac{I(t)}{|t|} = \Lambda \qquad (4)$$

This remarkable relation explains why chaotic trajectory is unpredictable. Indeed, to predict the next section of trajectory a new information is required which is not contained in the previous (infinite!) part of the trajectory. Notice that relation (4) does not depend on the observation accuracy. A nontrivial implication of this theorem is that a power-law instability (linear, for example) is insufficient for the true chaos.

The unpredectability is an asymptotic property as $|t| \to \infty$ (4). On a short time interval any dynamical trajectory is predictable, of course. Predictability is

controlled by the randomness parameter[14]

$$R = \frac{\Lambda |t|}{|\ln \mu|} \sim \frac{|t|}{t_p} \qquad (5)$$

where μ is the accuracy of observation, and t_p is the predictability time scale. The motion on the latter is also called the temporal determinism ($R \lesssim 1$) while for $R \gg 1$ the asymptotic randomness is built up.

Any dynamical description in terms of trajectories enevtually loses direct physical meaning for chaotic motion. Yet, the dynamical equations of motion can still be used to completely derive the statistical properties of the motion without any additional statistical hypotheses.

For the standard map chaotic components of motion exist for any $K > 0$ .However, the global statistical properties crucially depend on K . Namely, there is a critical $K = K_c \approx 1$ which separates strictly bounded ($K \leq K_c$) for any initial conditions and unbounded ($K > K_c$) for some initial conditions motions in N . In the former case a chaotic component has the form of a layer within which a fast statistical relaxation occurs. For any $K > K_c$ there exists an unbounded (in N) chaotic component of motion which is characterized by a homogeneous diffusion with the rate

$$D_N \equiv \frac{\langle (\Delta N)^2 \rangle}{\tau} = \frac{k^2}{2} C(K) \qquad (6)$$

Here τ is the number of map's iterations, and function $C(K)$ (describes the dynamical correlation of successive φ phases. Notice that the distribution function $f(N, \tau)$, which describes the diffusion process, is a coarse-grained one, averaged over φ .

All above relations can be, in principle, derived from the map (3) only. The main technical difficulty is the structure of the chaotic component which is highly complicated, unless K is very big. In the latter case the motion is ergodic to a good accuracy, and the

evaluation of $C(K)$ and D_N is fairly simple (see, e.g., review [15]).

The diffusion is an aperiodic process, and it implies a continuous spectrum of the motion. The same conclusion is directly inferred from the exponential instability which is incompatible with the discrete spectrum.

The diffusive evolution $f(N,\tau)$ is, of course, irreversible in time in apparent contradiction with the time-reversibility of the dynamical equations of motion. This is a particular case of the long-standing controversy in the foundations of statistical mechanics since the Boltzmann time. The dynamical chaos has extremely sharpened this paradox as the statistical (diffusive) irreversibility is now directly derived from the dynamical reversibility. So, what is the matter? The answer is now very simple, almost trivial: the irreversibility is the consequence of the particular description (very convenient and adequate!) via a corse-grained phase density. The exact (fine-grained) density, obeying the Liouville equation, is as reversible as the dynamical trajectory. The main difference between the two densities is in a big spatial oscillation in the latter whose scale is decreasing exponentially fast in the process of mixing with $\Lambda > 0$. Yet, the reversible density is, nevertheless, nonrecurrent for a chaotic motion. Thus, now there is no need to assume any special properties of the time like the notorious time arrow. The chaotic dynamics alone explains everything.

A rough estimate for the ionization time in the Hydrogen problem is

$$\tau_D \sim \frac{(\eta V)^2}{D_N} \sim \frac{\eta^2 \omega_0^{4/3}}{\varepsilon_0^2} \qquad (7)$$

where $\eta = 1 - n_0^2/n_c^2$, and n_c is ionization threshold, or cutoff, which depends on a particular experimental set-up. The most interesting peculiarity of this expression is in that the mean ionization rate $\tau_D^{-1} \sim \varepsilon_0^2$ as if it would be one-photon transition whereas, actually,

there are a lot of them ($\sim (\gamma\nu)^2 >> 1$). The explanation is the following. Unlike a direct multiphoton ionization, which starts immediately upon the interaction with the field, the diffusive ionization lags by $\sim \tau_D$ while for $\tau \to 0$ the ionization probability $P_I \sim \exp(-A/\tau)$ is exponentially small.

Kepler map (2) is a very simple model of electron's dynamics in a Rydberg atom. Yet, it brings about new difficulties. One is in that the relation between map's discrete time τ and continuous time t depends on electron's trajectory as $dt/d\tau \approx n^3$. A surprising implication is that even a steady-state density $f_s(N)$, which, by definition, does not depend on time, turns out to be different in both times:

$$f_s^{(t)} \sim f_s^{(\tau)} \cdot \left(1 - \frac{N}{|N|}\right)^{-\frac{3}{2}} \qquad (8)$$

In quantum mechanics this difficulty, not resolved as yet, becomes crucial because electron's trajectory has no longer any physical meaning.

4. QUANTUM PSEUDOCHAOS AND THE CORRESPONDENCE PRINCIPLE

A recent breakthrough in quantum dynamics [1,2] was due to, first, an intensive use of the concept of classical chaos and of its implications in quantum mechanics via the correspondence principle, and, second, a new philosophy of quantum dynamics. I mean the restriction of the latter to the Schrödinger (or similar) equation neglecting, or rather leaving for better times, a very intricate and vague problem of the quantum measurement. In this philosophy the wave function $\psi(t)$ is simply a very specific dynamical variable which describes the proper dynamics of a quantum system.

In the Kepler model (2), to be quantized below, integer N specifies what I call a photonic state which is

a group of neighbouring unperturbed Hydrogen levels separated from the next (or previous) photonic state by one field quantum, that is by ω in energy. Thus, $|\psi(N)|^2$ gives the probability within the energy interval $(N \pm \frac{1}{2})\omega$. All neighbouring photonic states are coupled by one-photon transitions.

The quantization of the Kepler map can be performed as follows. The 'momentum' N is integer, hence, the operator $\hat{N} = -i\partial/\partial\varphi$. Quantity $\nu \pmod 1 \approx \text{const}$ plays a role of 'quasimomentum', and it is approximately conserved.

Any map can be represented by a time-dependent Hamiltonian. For the Kepler map, for example,

$$\hat{H} = \hat{H}_0(\hat{N}) + k \cdot \cos\hat{\varphi} \cdot \delta_1(\tau)$$
$$\hat{H}_0 = -2\pi\left(-2\omega(\hat{N}+\nu)\right)^{-\frac{1}{2}} \longrightarrow \frac{T\hat{N}^2}{2} \qquad (9)$$

where $\delta_1(\tau)$ is δ-function of period 1. The latter H_0 expression in Eq.(9) is for the standard map. Now the quantization comes simply to substituting the operators for dynamical variables in Hamiltonian (9). Using the standard technique we arrive at a quantum map

$$\psi \to \overline{\psi} = \exp\left(-i\int d\tau \hat{H}\right)\psi \equiv \hat{R}\hat{F}\psi \qquad (10)$$

where

$$\hat{F} = e^{-ik\cos\varphi}; \quad \hat{R} = e^{-i\frac{T}{2}\hat{N}^2} \qquad (11)$$

are the operators of a 'kick' (in coordinate representation), and of the rotation (in momentum representation and for the standard map), respectively. The expression for $\hat{R}$ operator shows that an important quantum parameter is $T/4\pi \pmod 1$.

The transition to the classical limit corresponds to $k \to \infty$ and $T \to 0$ while the classical parameter $K = kT = \text{const}$. The latter condition is very important

in analyzing quantum effects in the quasiclassical region: the classical dynamics should be fixed. Otherwise, one cannot discriminate the quantum features of the process. Quantum map (10) shows that the original guess $n_0 \gg 1$ is necessary but insufficient for the quasiclassical treatment of the problem. Instead, one needs $k \sim n_0 \frac{\varepsilon_0}{\omega_0^{5/3}} \gg 1$. Under this condition the perturbation couples $\sim k$ photonic states per iteration.

In the oposite limiting case ($k \ll 1$) all photon transitions are suppressed, even if $n_0 \gg 1$, no matter what is system's behaviour in the classical limit. This is a purely quantum effect which is also called the perturbative localization. The latter term emphasizes that the perturbation is inefficient, and the exact eigenfunctions are close to the unperturbed ones. In relation to the problem of chaos this property was first discussed in[16], and we call the condition $k \sim 1$ Shuryak's border. In the Hydrogen problem this border is

$$\varepsilon_0^{(Sh)} \sim \frac{\omega_0^{5/3}}{n_0} \qquad (12)$$

For a fixed ω_0 the critical $\varepsilon_0^{(Sh)} \to 0$ as $n_0 \to \infty$ in accordance with the correspondence principle.

This border is well-known and is widely used in atomic and molecular physics (see, e.g.[8]). What is less known that it is not the only, and even not most important, quantum limitation of the classical dynamics in quasiclassical region. Meanwhile, such limitations are very essential to know as classical methods become rather popular in atomic and molecular physics (see, e.g.[17]). Indeed, the fundamental correspondence principle seems to require the transition between quantum and classical mechanics, including peculiar chaotic phenomena.

In our problem we would expect the diffusion in N for sufficiently big k , and for classical $K > 1$.

This is, indeed, the case[18]. Moreover, the quantum diffusion mimics all peculiarities of the classical case[19]. Yet, it only mimics those and besides on a finite time interval only, as was discovered already in[18]. Numerical experiments revealed that in a time $\sim \tau_R$ the transition of the distribution function

$$f(N,\tau) \sim \exp\left(-\frac{N^2}{2\tau D_N}\right) \to \exp\left(-\frac{2|N|}{l_s}\right) \qquad (13)$$

occurs from an expanding Gaussian one to an exponential steady state. The latter is a purely quantum formation without any classical counterpart. The process (13) is called the quantum localization of classical diffusion, or in brief, the diffusion localization. A remarkable relation

$$\tau_R \sim l_s \approx D_N \qquad (14)$$

holds between quantum localization length l_s and classical diffusion rate D_N (see [4,19]). Again, in accordance with the correspondence principle $l_s \sim k^2 \to \infty$ as $k \to \infty$.

The physical cause of localization is the discreteness of the quasienergy spectrum whose density $\rho \sim \tau_R \sim l_s$. Notice the finite quasienergy level density which is determined by the so-called operative eigenfunctions only, that is by those actually present in the initial quantum state $\psi(N,0)$. The effective number of these is finite because of the localization of eigenfunctions inevitably related to the diffusion localization. It is interesting to mention that eigenfunction localization length $l \approx l_s/2$ turns out to be quite different from l_s according to numerical experiments[19, 20]. A qualitative explanation of this surprising result is in very big fluctuations around the average exponential dependence.

The diffusion completely stops in time $\sim \tau_R$, and we call τ_R the relaxation time scale. It has

nothing to do with the so-called Poincarè recurrence time. The latter is much bigger than τ_R and it sharply depends on the recurrence region.

The present quantum example shows that statistical relaxation can also occur in the discrete spectrum. The main difference from the classical relaxation is in that the quantum distribution $f(N,\tau) = |\psi(N,\tau)|^2$ is never constant but rather oscillates about the classical equilibrium distribution, and even sometime (on a very rare occasion!) can come back to the initial distribution. In other words quantum distribution behaves, in a sense, like a classical trajectory or like the classical phase density for a regular motion. A more precise statement is that an individual quantum system, as described by its ψ function, corresponds statistically to a finite ensemble of $\sim l_s \sim k^2$ 'particles' with respect to ψ fluctuations.

Thus, the quantum chaotic motion, or briefly the quantum chaos, has a discrete spectrum which in a classical system would correspond to the opposite limiting case of regular motion. Yet, it reveals some statistical properties including diffusion and relaxation but only on a finite time scale τ_R. For this reason we use to speak about the temporary quantum pseudochaos.

Another peculiarity of the quantum chaos is its dynamical stability[21], that is the quantum Lyapunov exponent $\Lambda_q \equiv 0$. This is a direct consequence of the discreteness of phase space in quantum mechanics. But how about the correspondence principle? It turns out that a temporary instability is still possible, indeed. This was discovered by Berman and Zaslavsky[22]. The physical meaning of the instability is the exponentially fast spreading of a narrow wave packet which follows for a while the classical unstable trajectory according to the Ehrenfest theorem. Hence, the corresponding time

scale

$$\tau_{BZ} \sim \frac{|\ell_n T|}{\Lambda} \qquad (15)$$

is logarithmically short. Yet, $\tau_{BZ} \to \infty$ as $T \to 0$ in accordance with the correspondence principle.

There is an interesting analogy between the diffusion localization and well-known Anderson's localization in solids[23]. In my opinion, this analogy cannot explain the diffusion localization as it would require the quantum phases $TN^2/2$ mod 2π (see Eq.(11)) to be random whereas they are obviously not. Yet, the analogy can by used in the opposite direction: the diffusion localization with nonrandom phases implies that the random potential is only sufficient but not necessary condition for the Anderson localization. Indeed, it was proved that even quasiperiodic potential can provide localization (see, e.g.[13]).

In the standard map the diffusion stops and the quantum steady state is formed no matter how big is quantum parameter $k \to \infty$. This is not the case in the Kepler map with a finite number $|\nu|$ of photonoc states up to the continuum. Hence, a new important parameter $\lambda = \ell_s / |\nu|$ appears which I call the ergodicity parameter. The diffusion localization would influence (suppress) ionization only if $\lambda \lesssim 1$. The condition $\lambda \sim 1$ is called delocalization border.

Numerical experiments well confirmed the above predictions, including the validity of $1D$ model [3,24]. Recently, first indications of the diffusion localization in Hydrogen were reported also in laboratory experiments [25]. The localization occurs for $\omega_0 \gtrsim 1$ only. The case $\omega_0 < 1$ is much more difficult for analytical studies while the result is much simpler and not so interesting, namely, the classical behaviour holds as soon as $n_0 >> 1$ (see, e.g.[26]).

5. EPILOGUE: WHAT IS THE QUANTUM CHAOS?

The quantum chaos, unlike its classical counterpart, is a rather new topic of research. There is, as yet, even no common definition of the quantum chaos. Here four versions currently in use are listed.

The Quantum Chaos is:

(i) quantum dynamics of classically chaotic systems;

(ii) partial imitation of the classical chaos;

(iii) dynamically stable statistical relaxation;

(iv) the relaxation in discrete spectrum.

The most popular is the first definition which is also logically simplest one. In my opinion, however, it is completely inadequate from the physical point of view. Indeed, that 'quantum chaos' may happen to be a perfectly regular motion, for example, below Shuryak's border (12). Corrently, I advocate the fourth definition as the best. A common disadvantage of all the above definitions but the first one is in that such a chaos is also possible in a classical system. Yet, in quantum mechanics it is the maximal chaos available apart from very exotic examples (see Section 2 in [4] and [27]).

Another interesting question is whether one really needs any property stronger than relaxation (even in a discrete spectrum) to develop statistical mechanics?

Let me conclude with a principal question due to Ford: Is there any quantum chaos? Now I would answer this question in the affirmative, even though the quantum chaos is certainly rather different from the classical one. Then, the next question is quite natural: Is there any classical chaos? I would say no, to the extent that the quantum mechanics is a universal theory. In any event, the classical chaos remains a very important conception, at least, as a limiting pattern to compare with real physical systems.

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