Natural Laws and Human Prediction

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Abstract. Interrelations between dynamical and statistical laws in physics, on the one hand, and between classical and quantum mechanics, on the other hand, are discussed within the philosophy of separating the natural from the human, as a very specific part of Nature, and with emphasis on the new phenomenon of dynamical chaos.
The principal results of the studies of chaos in classical mechanics are presented in some detail, including the strong local instability and robustness of motion, continuity of both phase space and the motion spectrum, and the time reversibility but nonrecurrency of statistical evolution, within the general picture of chaos as a specific case of dynamical behavior.
Analysis of the apparently very deep and challenging contradictions of this picture with the quantum principles is given. The quantum view of dynamical chaos, as an attempt to resolve these contradictions guided by the correspondence principle and based upon the characteristic time scales of quantum evolution, is explained. The picture of quantum chaos as a new generic dynamical phenomenon is outlined together with a few other examples of such chaos: linear (classical) waves, the (many-dimensional) harmonic oscillator, the (completely integrable) Toda lattice, and the digital computer.
I conclude with discussion of the two fundamental physical problems: quantum measurement (ψ-collapse), and the causality principle, which both appear to be related to the phenomenon of dynamical chaos.

1 Philosophical Introduction: Separation of the Natural from the Human

The main purpose of this paper is the analysis of conceptual implications from the studies of a new phenomenon (or rather a whole new field of phenomena) known as *dynamical chaos* both in classical and especially in quantum mechanics. The concept of dynamical chaos resolves (or, at least, helps to do so) the two fundamental problems in physics and, hence, in all the natural sciences:

- are the dynamical and statistical laws of a different nature or does one of them, and which one, follow from the other;
- are classical and quantum mechanics of a different nature or is the latter the most universal and general theory currently available to describe the whole empirical evidence including the classical mechanics as the limiting case.

The essence of my debut philosophy is the separation of the human from the natural following Einstein's approach to the science – *building up a model of the real world.* Clearly, the human is also a part of the world, and moreover the most important part for us as human beings but not as physicists. The whole phenomenon of life is extremely specific, and one should not transfer its peculiarities into other fields of natural sciences as was erroneously (in my opinion) done in almost all major philosophical systems. One exception is positivism, which seems to me rather dull; it looks only at Nature but does not even want to see its internal mechanics. Striking examples of the former are Hegel's 'Philosophy of Nature' (Naturphilosophie) and its 'development', Engels' 'Dialectic of Nature'.

Another notorious confusion of such a 'human-oriented' physics was Wigner's claim that quantum mechanics is incompatible with the existence of self-reproducing systems (Wigner (1961)). The resolution of this 'paradox' is just in that Wigner assumed the Hamiltonian of such a system to be arbitrary, whereas it is actually highly specific (Schuster (1994)).

A more hidden human-oriented philosophy in physics, rather popular nowadays, is the information-based representation of natural laws, particularly when information is substituted for entropy (with opposite sign). In the most general way such a philosophy was recently presented by Kadomtsev (1994). That approach is possible and might be done in a self-consistent way, but one should be very careful to avoid many confusions. In my opinion, the information is an adequate conception for only the special systems that actually use and process the information like various automata, both natural (living systems) and man-made ones. In this case the information becomes a physical notion rather than a human view of natural phenomena. The same is also true in the theory of measurement, which is again a very specific physical process, the basic one in our studies of Nature but still not a typical one for Nature itself. This is crucially important in quantum mechanics as will be discussed in some detail below (Sections 2.4 and 3.1).

One of the major implications from studies of dynamical chaos is the conception of statistical laws as an intrinsic part of dynamics without any additional statistical hypotheses [for the current state of the theory see, e.g., Lichtenberg and Lieberman (1992) and recent collection of papers by Casati and Chirikov (1995) as well as the introduction to this collection by Casati and Chirikov (1995a)]. This basic idea can be traced back to Poincaré (1908) and Hadamard (1898), and even to Maxwell (1873); the principal condition for dynamical chaos being strong local instability of motion (Section 2.4). In this picture the statistical laws are considered as *secondary* with respect to more fundamental and general *primary* dynamical laws.

Yet, this is not the whole story. Surprisingly, the opposite is also true! Namely, under certain conditions the dynamical laws were found to be completely contained in the statistical ones. Nowadays this is called 'synergetics' (Haken (1987), Wunderlin (these proceedings)) but the principal idea goes back to Jeans (1929) who discovered the instability of gravitating gas (a typical example of a statistical system), which is the basic mechanism for the formation of galaxies and stars in modern cosmology, and eventually the Solar system, a classical example of a dynamical system. In this case the resulting dynamical laws proved to be secondary with respect to the primary statistical laws which include the former.

Thus, the whole picture can be represented as a chain of dynamical–statistical inclusions:

$$...?...\boxed{D \supset S} \supset D \supset S ...?... \qquad (1.1)$$

Both ends of this chain, if any, remain unclear. So far the most fundamental (elementary) laws of physics seem to be dynamical (see, however, the discussion of quantum measurement in Sections 3 and 4). This is why I begin chain (1.1) with some primary dymamical laws.

The strict inclusion on each step of the chain has a very important consequence allowing for the so-called numerical experiments, or computer simulation, of a broad range of natural processes. As a matter of fact the former (not laboratory experiments) are now the main source of new information in the studies of the secondary laws for both dynamical chaos and synergetics. This might be called *the third way of cognition*, in addition to laboratory experiments and theoretical analysis.

In what follows I restrict myself to the discussion of just a single ring of the chain as marked in (1.1). Here I will consider the dynamical chaos separately in classical and quantum mechanics. In the former case the chaos explains the origin and mechanism of random processes in Nature (within the classical approximation). Moreover, that deterministic randomness may occur (and is typical as a matter of fact) even for a minimal number of degress of freedom $N > 1$ (for Hamiltonian systems), thus enormously expanding the domain for the application of the powerful methods of statistical analysis.

In quantum mechanics the whole situation is much more tricky and still remains rather controversial. Here we encounter an intricate tangle of various apparent contradictions between the correspondence principle, classical chaotic behavior, and the very foundations of quantum physics. This will be the main topic of my discussions below (Section 3).

One way to untangle this tangle is the new general conception, *pseudochaos*, of which quantum chaos is the most important example. Another interesting example is the digital computer, also very important in view of the broad application of numerical experiments in the studies of dynamical systems. On the other hand, pseudochaos in computers will hopefully help us to understand quantum pseudochaos and to accept it as a sort of *chaos* rather than a sort of regular motion, as many researchers, even in this field, still do believe.

The new and surprising phenomenon of dynamical chaos, especially in quantum mechanics, holds out new hopes for eventually solving some old, longstanding, fundamental problems in physics. In Section 4, I will briefly discuss two of them:

- the causality principle (time ordering of cause and effect), and
- ψ-collapse in the quantum measurement.

The conception of dynamical chaos I am going to present here, which is not common as yet, was the result of the long-term Siberian–Italian (SI) collaboration including Giulio Casati and Italo Guarneri (Como), and Felix Izrailev and

Dima Shepelyansky (Novosibirsk) with whom I share the responsibility for our joint scientific results and the conceptual interpretation.

2 Scientific Results and Conceptual Implications: the Classical Limit

Classical dynamical chaos, as a part of classical mechanics, was historically the first to have been studied simply because in the time of Boltzmann, Maxwell, Poincaré and other founders, statistical mechanics and quantum mechanics did not exist. No doubt, the general mathematical theory of dynamical systems, including the ergodic theory as its modern part describing various statistical properties of the motion, has arisen from (and is still conceptually based on) classical mechanics (Kornfeld et al. (1982), Katok and Hasselblatt (1994)). Yet, upon construction, it is not necessarily restricted to the latter and can be applied to a much broader class of dynamical phenomena, for example, in quantum mechanics (Section 3).

2.1 What is a Dynamical System?

In classical mechanics, 'dynamical system' means an object whose motion in some *dynamical space* is *completely* determined by a given interaction and the *initial conditions.* Hence, the synonym *deterministic system.* The motion of such a system can be described in two seemingly different ways which, however, prove to be essentially equivalent.

The first one is through the *motion equations* of the form

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t), \tag{2.1}$$

which always have a unique solution

$$\mathbf{x} = \mathbf{x}(t, \mathbf{x}_0) \tag{2.2}$$

Here $\mathbf{x}$ is a finite-dimensional vector in the dynamical space and $\mathbf{x}_0$ is the initial condition $[\mathbf{x}_0 = \mathbf{x}(0)]$. A possible explicit time-dependence in the right-hand side of (2.1) is assumed to be a regular, e.g., periodic, one or, at least, one with a discrete spectrum.

The most important feature of dynamical (deterministic) systems is the *absence of any random parameters or any noise* in the motion equations. Particularly for this reason I will consider a special class of dynamical systems, the so-called *Hamiltonian (nondissipative) systems*, which are most fundamental in physics.

Dissipative systems, being very important in many applications, are neither fundamental (because the dissipation is introduced via a crude approximation of the very complicated interaction with some 'heat bath') nor purely dynamical in view of principally inevitable random noise in the heat bath (fluctuation–dissipation theorem). In a more accurate and natural way the dissipative systems can be described in the frames of the secondary dynamics ($S \supset D$ inclusion in (1.1)) when both dissipation and fluctuations are present from the beginning in the primary statistical laws.

A purely dynamical system is necessarily the *closed* one, which is the main object in fundamental physics. Thus, any coupling to the environment is completely neglected. I will come back to this important question below (Section 2.4).

In Hamiltonian mechanics the dynamical space, called *phase space*, is an even-dimensional one composed of N pairs of canonically conjugated 'coordinates' and 'momenta', each pair corresponding to one freedom of motion.

In the problem of dynamical chaos the initial conditions play a special role: they completely determine a particular trajectory, for a given interaction, or a particular realization of a dynamical process which may happen to be a very specific, nontypical, one. To get rid of such singularities another description is useful, namely the Liouville partial differential equation for the *phase space density*, or distribution function $f(\mathbf{x}, t)$:

$$\frac{\partial f}{\partial t} = \hat{L} f \tag{2.3}$$

with the solution

$$f = f(\mathbf{x}, t; f_0(\mathbf{x})). \tag{2.4}$$

Here $\hat{L}$ is a *linear* differential operator, and $f_0(\mathbf{x}) = f(\mathbf{x}, 0)$ is the initial density. For any smooth f_0 this description provides the generic behavior of a dynamical system via a continuum of trajectories. In the special case $f_0 = \delta(\mathbf{x} - \mathbf{x_0})$ the density describes a single trajectory like the motion equations (2.1).

In any case the phase space itself is assumed to be *continuous*, which is the most important feature of the classical picture of motion and the main obstacle in the understanding of quantum chaos (Section 3).

2.2 What is Dynamical Chaos?

Dynamical chaos can be characterized in terms of both the individual trajectories and the trajectory ensembles, or phase density. Almost all trajectories of a chaotic system are in a sense most complicated (they are *unpredictable* from observation of any preceding motion to use this familiar human term). Exceptional, e.g., periodic trajectories form a set of zero invariant measure, yet it might be everywhere dense.

An appropriate notion in the theory of chaos is the *symbolic trajectory* first introduced by Hadamard (1898). The theory of symbolic dynamics was developed further by Morse (1966), Bowen (1973), and Alekseev and Yakobson (1981). The symbolic trajectory is a projection of the true (exact) trajectory on to a discrete partition of the phase space at discrete instants of time t_n, e.g., such that $t_{n+1} - t_n = T$ fixed. In other words, to obtain a symbolic trajectory we first turn from the motion differential equations (2.1) to the difference equations over a certain time interval T:

$$\mathbf{x}(t_{n+1}) \equiv \mathbf{x}_{n+1} = M(\mathbf{x}_n, t_n). \tag{2.5}$$

This is usually called *mapping* or *map*: $\mathbf{x}_n \to \mathbf{x}_{n+1}$. Then, while running a (theoretically) *exact* trajectory we record each $\mathbf{x}_n$ to a *finite* accuracy: $\mathbf{x}_n \approx m_n$. For a finite partition each m_n can be chosen to be integer. Hence, the whole infinite symbolic trajectory

$$\sigma \equiv ...m_{-n}...m_{-1}m_0m_1...m_n... = S(\mathbf{x}_0; T), \tag{2.6}$$

can be represented by a *single* number σ, which is generally irrational and which is some function of the *exact* initial conditions. The symbolic trajectory may be also called a *coarse-grained trajectory*. I remind you that the latter is a *projection* of (not substitution for) the exact trajectory to represent in compact form the global dynamical behavior without unimportant microdetails.

A remarkable property of chaotic dynamics is that the set of its symbolic trajectories is *complete*; that is, it actually contains all possible sequences (2.6). Apparently, this is related to continuity of function $S(\mathbf{x}_0)$ (2.6). On the contrary, for a regular motion this function is everywhere discontinuous.

In a similar way the *coarse-grained phase density* $\bar{f}(m_n, t)$ is introduced, in addition to the exact, or *fine-grained density*, which is also a projection of the latter on to some partition of the phase space.

The coarse-grained density represents the global dynamical behavior, particularly the most important process of *statistical relaxation*, for chaotic motion, to some *steady state* $f_s(m_n)$ (statistical equilibrium) independent of the initial $f_0(\mathbf{x})$ if the steady state is *stable*. Otherwise, synergetics comes into play giving rise to a secondary dynamics. As the relaxation is an aperiodic process the spectrum of chaotic motion is *continuous*, which is another obstacle for the theory of quantum chaos (Section 3).

Relaxation is one of the characteristic properties of statistical behavior. Another is *fluctuation*. Chaotic motion is a generator of noise which is purely *intrinsic* by definition of the dynamical system. Such noise is a particular manifestation of the complicated dynamics as represented by the symbolic trajectories or by the difference

$$f(\mathbf{x}, t) - \bar{f}(m_n, t) \equiv \tilde{f}(\mathbf{x}, t). \tag{2.7}$$

The relaxation $\bar{f} \to f_s$, apparently asymmetric with respect to time reversal $t \to -t$, gave rise to a long-standing misconception of the notorious *time arrow*. Even now some very complicated mathematical constructions are still being erected (see, e.g., Misra et al. (1979), Goldstein et al. (1981)) in attempts to extract somehow statistical irreversibility from the reversible mechanics. In the theory of dynamical chaos there is no such problem. The answer turns out to be conceptual rather than physical: one should separate two similar but different notions, *reversibility* and *recurrency*. The exact density $f(\mathbf{x}, t)$ is always *time-reversible* but *nonrecurrent* for chaotic motion; that is, it will never come back to the initial $f_0(\mathbf{x})$ in *both directions of time* $t \to \pm\infty$. In other words, the relaxation, also present in f, is time-symmetric. The projection of f, coarse-grained $\overline{f}$, which is both nonrecurrent and irreversible, emphasizes nonrecurrency of the exact solution. The apparent violation of the statistical relaxation upon time reversal, as described by the exact $f(\mathbf{x}, t)$, represents in fact the growth of a big fluctuation which will eventually be followed by the same relaxation in the opposite direction of time. This apparently surprising symmetry of the statistical behavior was discovered long ago by Kolmogorov (1937). One can say that instead of an imagionary time arrow there exists a *process arrow* pointing always to the steady state. The following simple example would help, perhaps, to overcome this conceptual difficulty. Consider the hyperbolic one-dimensional (1D) motion:

$$x(t) = a \cdot \exp(\Lambda t) + b \cdot \exp(-\Lambda t), \tag{2.8}$$

which is obviously time-reversible yet remains *unstable* in both directions of time ($t \to \pm\infty$). Besides its immediate appeal, this example is closely related to the mechanism of chaos which is the motion instability.

2.3 A Few Physical Examples of Low-Dimensional Chaos

In this paper I restrict myself to finite-dimensional systems where the peculiarities of dynamical chaos are most clear (see Section 3.2 for some brief remarks on infinite systems). Consider now a few examples of chaos in minimal dimensionality.

Billiards (2 degrees of freedom). The ball motion here is chaotic for almost any shape of the boundary except special cases like circle, ellipse, rectangle and some other (see, e.g., Lichtenberg and Lieberman (1992), Kornfeld et al. (1982), Katok and Hasselblatt (1994)). However, the ergodicity (on the energy surface) is only known for singular boundaries. If the latter is smooth enough the structure of motion becomes a very complicated admixture of chaotic and regular domains of various sizes (the so-called divided phase space). Another version of billiards is the wave cavity in the geometric optics approximation. This provides a helpful bridge between classical and quantum chaos.

Perturbed Kepler motion is a particular case of the famous 3-body problem. Now we understand why it has not been solved since Newton: chaos is generally present in such a system. One particular example is the motion of comet Halley perturbed by Jupiter which was found to be chaotic with an estimated life time in the Solar system of the order of 10 Myrs (Chirikov and Vecheslavov (1989); 2 degrees of freedom in the model used, divided phase space).

Another example is a new, diffusive, mechanism of ionization of the Rydberg (highly excited) hydrogen atom in the external monochromatic electric field. It was discovered in laboratory experiments (Bayfield and Koch (1974)) and was explained by dynamical chaos in a classical approximation (Delone et al. (1983)).

In this system a given field plays the role of the third body. The simplest model of the diffusive photoelectric effect has 1.5 degrees of freedom (1D Kepler motion and the external periodic perturbation), and is also characterized by a divided phase space.

Budker's problem: charged particle confinement in an adiabatic magnetic trap (Chirikov (1987)). A simple model of two freedoms (axisymmetric magnetic field) is described by the Hamiltonian:

$$H = \frac{p^2}{2} + \frac{(1 + x^2)\, y^2}{2}. \tag{2.9}$$

Here magnetic field $B = \sqrt{1 + x^2}$; $p^2 = \dot{x}^2 + \dot{y}^2$; x describes the motion along magnetic line, and y does so accross the line (a projection of Larmor's rotation). At small pitch angles $\beta \approx |\dot{y}/\dot{x}|$ the motion is chaotic with the chaos border being at roughly

$$p \sim \frac{1}{|\ln \beta|} \tag{2.10}$$

and being very complicated, so-called critical, structure (Section 2.5).

Matinyan's problem: internal dynamics of the Yang–Mills (gauge) fields in classical approximation (Matinyan (1979), Matinyan (1981)). Surprisingly, this completely different physical system can be also represented by Hamiltonian (2.9) with a symmetrized 'potential energy':

$$U = \frac{(1 + x^2)\, y^2 + (1 + y^2)\, x^2}{2}. \tag{2.11}$$

Dynamics is always chaotic with a divided phase space similar to model (2.9) (Chirikov and Shepelyansky (1982)). Model (2.11) describes the so-called massive gauge field; that is, one with the quanta of nonzero mass. The massless field corresponds to the 'potential energy'

$$U = \frac{x^2 y^2}{2} \tag{2.12}$$

and looks ergodic in numerical experiments.

2.4 Instability and Chaos

Local instability of motion responsible for a very complicated dynamical behavior is described by the *linearized equations*:

$$\frac{d\mathbf{u}}{dt} = \mathbf{u} \cdot \frac{\partial \mathbf{v}(\mathbf{x}^0(t), t)}{\partial \mathbf{x}}. \tag{2.13}$$

Here $\mathbf{x}^0(t)$ is a reference trajectory satisfying (2.1), and $\mathbf{u} = \mathbf{x}(t) - \mathbf{x}^0(t)$ is the deviation of a close trajectory $\mathbf{x}(t)$. On average, the solution of (2.13) has the form

$$|\mathbf{u}| \sim \exp(\Lambda t), \tag{2.14}$$

where Λ is *Lyapunov's exponent.* The motion is (exponentially) unstable if $\Lambda > 0$. In the Hamiltonian system of N degrees of freedom there are $2N$ Lyapunov's exponents satisfying the condition $\sum \Lambda = 0$. The partial sum of all positive exponents $\Lambda_+ > 0$,

$$h = \sum \Lambda_+ \tag{2.15}$$

is called the (dynamical) *metric entropy.* Notice that it has the dimensions of frequency and characterises the instability rate.

The motion instability is only a necessary but not sufficient condition for chaos. Another important condition is *boundedness* of the motion, or its oscillatory (in a broad sense) character. The chaos is produced by the combination of these two conditions (also called stretching and folding). Let us again consider an elementary example of a 1D map

$$x_{n+1} = 2\,x_n \mod 1, \tag{2.16}$$

where operation mod 1 restricts (folds) x to the interval (0,1). This is not a Hamiltonian system but it can be interpreted as a 'half' of that; namely, as the dynamics of the oscillation phase. This motion is unstable with $\Lambda = \ln 2$ because the linearized equation is the same except for the fractional part (mod 1). The explicit solution for both reads

$$u_n = 2^n\,u_0,$$

$$x_n = 2^n\,x_0 \mod 1. \tag{2.17}$$

The first (linearized) motion is unbounded, like Hamiltonian hyperbolic motion, (2.8) and is perfectly regular. The second one is not only unstable but also chaotic just because of the additional operation mod 1, which makes the motion bounded, and which mixes up the points within a finite interval.

We may look at this example from a different viewpoint. Let us express the initial x_0 in the binary code as the sequence of two symbols, 0 and 1, and let us make the partition of the unit x interval also in two equal halves marked by the same symbols. Then, the symbolic trajectory will simply repeat x_0; that is, (2.6) takes the form

$$\sigma = x_0. \tag{2.18}$$

It implies that, as time goes on, the global motion will eventually depend on ever-diminishing details of the initial conditions. In other words, when we formally fix the *exact* x_0 we 'supply' the system with infinite complexity, which arises due to the strong motion instability. Still another interpretation is that the exact x_0 is the source of *intrinsic noise* amplified by the instability. For this noise to be *stationary* the string of x_0 digits has to be infinite, which is only possible in *continuous* phase space.

A nontrivial part of this picture of chaos is that the instability must be *exponential* because a power-law instability is insufficient for chaos. For example, the linear instability ($|u| \sim t$) is a generic property of perfectly regular motion of the completely integrable system whose motion equations are *nonlinear* and, hence, whose oscillation frequencies depend on the initial conditions (Born (1958), Casati et al. (1980)). The character of motion for a faster instability ($|\mathbf{u}| \sim t^\alpha,\ \alpha > 1$) is unknown.

On the other hand, the exponential instability ($h > 0$) is not invariant with respect to the change of time variable (Casati and Chirikov (1995a), Batterman (these proceedings); in this respect the only invariant statistical property is ergodicity, Kornfeld et al. (1982), Katok and Hasselblatt (1994)). A possible resolution of this difficulty is that the proper characteristic of motion instability, important for dynamical chaos, should be taken with respect to the oscillation phases whose dynamics determines the nature of motion. It implies that the proper time variable must change proportionally with the phases so that the oscillations become stationary (Casati and Chirikov (1995a)). A simple example is harmonic oscillation with frequency ω recorded at the instances of time $t_n = 2^n t_0$. Then, oscillation phase $x = \omega t/2\pi$ obeys map (2.16), which is chaotic. Clearly, the origin of chaos here is not in the dynamical system but in the recording procedure (random t_0). Now, if ω is a parameter (linear oscillator), then the oscillation is exponentially unstable (in new time n) but only with respect to the change of parameter ω, not of the initial x_0 ($x \to x + x_0$). In a slightly 'camouflaged' way, essentially the same effect was considered by Blümel (1994) with far-reaching conclusions on quantum chaos (Section 3.2).

Rigorous results concerning the relation between instability and chaos are concentrated in the Alekseev–Brudno theorem (see Alekseev and Yakobson (1981), Batterman (these proceedings), White (1993)), which states that the complexity per unit time of almost any symbolic trajectory is asymptotically equal to the metric entropy:

$$\frac{C(t)}{|t|} \to h, \qquad |t| \to \infty. \tag{2.19}$$

Here $C(t)$ is the so-called algorithmic complexity, or in more familiar terms, the information associated with a trajectory segment of length $|t|$.

The transition time from dynamical to statistical behavior according to (2.19) depends on the partition of the phase space, namely, on the size of a cell μ, which is inversely proportional to the biggest integer $M \geq m_n$ in symbolic trajectory (2.6). The transition is controlled by the *randomness parameter* (Chirikov (1985)):

$$r = \frac{h\,|t|}{\ln M} \sim \frac{|t|}{t_r}, \tag{2.20}$$

where t_r is the *dynamical time scale.* As both $|t|$, $M \to \infty$ we have a somewhat confusing situation, typical in the theory of dynamical chaos, in which two limits do not commute:

$$M \to \infty,\ |t| \to \infty \neq |t| \to \infty,\ M \to \infty. \tag{2.21}$$

For the left order ($M \to \infty$ first) parameter $r \to 0$, and we have *temporary determinism* ($|t| \lesssim t_r$), while for the right order $r \to \infty$, and we arrive at *asymptotic randomness* ($|t| \gtrsim t_r$).

Instead of the above double limit we may consider the *conditional limit*

$$|t|,\ M \to \infty, \qquad r = \text{const}, \tag{2.22}$$

which is also a useful method in the theory of chaotic processes. Particularly for $r \lesssim 1$, strong dynamical correlations persist in a symbolic trajectory, which allows for the prediction of trajectory from a finite-accuracy observation. This is no longer the case for $r \gtrsim 1$ when only a statistical description is possible. Nevertheless, the motion equations can still be used to completely derive all the statistical properties without any *ad hoc* hypotheses. Here the exact trajectory *does exist* as well but becomes the Kantian *thing-in-itself*, which can be neither predicted nor reproduced in any other way.

The mathematical origin of this peculiar property goes back to the famous Gödel theorem (Gödel (1931)), which states (in a modern formulation) that *most* theorems in a given mathematical system are unprovable, and which forms the basis of contemporary mathematical logic (see Chaitin (1987) for a detailed explanation and interesting applications of this relatively less-known mathematical achievement). A particular corollary, directly related to symbolic trajectories (2.6), is that almost all real numbers are uncomputable by any finite algorithm. Besides rational numbers some irrationals like π or e are also known to be computable. Hence, their total complexity, e.g., $C(\pi)$, is finite, and the complexity per digit is zero (cf. (2.19)).

The main object of my discussion here, as well as of the whole physics, is a closed system that requires neglection of the external perturbations. However, in case of strong motion instability this is no longer possible, at least dynamically. What is the impact of a weak perturbation on the statistical properties of a chaotic system? The rigorous answer was given by the robustness theorem due to Anosov (1962): not only do statistical properties remain unchanged but, moreover, the trajectories get only slightly deformed providing (and due to) the same strong motion instability. The explanation of this striking peculiarity is that the trajectories are simply transposed and, moreover, the less the stronger is instability.

In conclusion let me make a very general remark, far beyond the particular problem of chaotic dynamics. According to the Alekseev–Brudno theorem (2.19) the source of stationary (new) information is always chaotic. Assuming farther that any creative activity, science including, is such a source we come to an interesting conclusion that any such activity has to be (partly!) chaotic. This is the creative side of chaos.

2.5 Statistical Complexity

The theory of dynamical chaos does not need any statistical hypotheses, nor does it allow for arbitrary ones. Everything is to be deduced from the dynamical equations. Sometimes the statistical properties turn out to be quite simple and familiar (Lichtenberg and Lieberman (1992), Chirikov (1979)). This is usually the case if the chaotic motion is also ergodic (on the energy surface), like in some billiards and other simple models (Section 2.3). However, quite often, and even typically for a few-freedom chaos, the phase space is divided, and the chaotic component of the motion has a very complicated structure.

One beautiful example is the so-called Arnold diffusion driven by a weak ($\epsilon \to 0$) perturbation of a completely integrable system with $N > 2$ degrees of freedom (Lichtenberg and Lieberman (1992), Chirikov (1979)). The phase space of such a system is pierced by the everywhere-dense set of nonlinear resonances

$$\sum_n m_n \cdot \omega_n^0(I) \approx 0, \tag{2.23}$$

where m_n are integers, and ω_n^0 are the unperturbed frequences depending on dynamical variables (usually actions I). Each resonance is surrounded by a separatrix, the singular highly unstable trajectory with zero motion frequency. As a result, no matter how weak the perturbation ($\epsilon \to 0$) is, a narrow chaotic layer always arises around the separatrix. The whole set of chaotic layers is everywhere dense as is the set of resonances. For $N > 2$ the layers form a united connected chaotic component of the motion supporting the diffusion over the whole energy surface. Both the total measure of the chaotic component and the rate of Arnold diffusion are exponentially small ($\sim \exp(-C/\sqrt{\epsilon})$) and can be neglected in most cases; hence the term *KAM integrability* (Chirikov and Vecheslavov (1990)) for such a structure (after Kolmogorov, Arnold and Moser who rigorously analysed some features of this structure). This quasi-integrability has the nature and quality of adiabatic invariance. However, on a very big time scale this weak but universal instability may essentially affect the motion.

One notable example is celestial mechanics, particularly the stability of the Solar system (Wisdom (1987) Laskar (1989), Laskar (1990), Laskar (1994)). Surprisingly, this 'cradle' of classical determinism and the exemplar case of dynamical behavior proves to be unstable and chaotic. The instability time of the Solar system was found to be rather long ($\Lambda^{-1} \sim 10$ Myrs), and its life time is still many orders of magnitude larger. It has not been estimated as yet, and might well exceed the cosmological time ~ 10 Byrs.

Another interesting example of complicated statistics is the so–called *critical structure* near the chaos border which is a necessary element of divided phase space (Chirikov (1991)). The critical structure is a hierarchy of chaotic and regular domains on ever decreasing spatial and frequency scales. It can be universally described in terms of the *renormalization group*, which proved to be so efficient in other branches of theoretical physics. In turn, the renormalization group may be considered as an abstract dynamical system that describes the variation of the whole motion structure, for the original dynamical system, in dependence of its spatial and temporal scale. Logarithm of the latter plays a role of 'time' (renormtime) in that renormdynamics. At the chaos border the latter is determined by the motion frequencies. The simplest renormdynamics is a periodic variation of the structure or, for a renorm-map, the invariance of the structure with respect to the scale (MacKay (1983)). Surprisingly, this scale invariance includes the chaotic trajectories as well. The opposite limit—renormchaos—is also possible, and was found in several models (see Chirikov (1991)).

Even though the critical structure occupies a very narrow strip along the chaos border it may qualitatively change the statistical properties of the whole chaotic component. This is because a chaotic trajectory unavoidably enters from time to time the critical region and 'sticks' there for a time that is longer the closer it comes to the chaos border. The sticking results in a slow power-law correlation decay for large time, in a singular motion spectrum for low frequency, and even in the superdiffusion when the phase-density dispersion $\sigma^2 \sim t^\alpha$ ($\alpha > 1$) grows faster than time (Chirikov (1987), Chirikov (1991)).

3 Scientific Results and Conceptual Implications: Quantum Chaos

The mathematical theory of dynamical chaos—ergodic theory—is self-consistent. However, this is not the case for the physical theory unless we accept the philosophy of the two separate mechanics: classical and quantum. Even though such a view cannot be excluded at the moment it has a profound difficulty concerning the border between the two. Nor is it necessary according to recent intensive studies of quantum dynamics. Then, we have to understand the mechanics of dynamical chaos from a quantum point of view. Our guiding star will be the *correspondence principle* which requires the complete quantum theory of any classical phenomenon, in the quasiclassical limit, assuming that the whole classical mechanics is but a special part (the limiting case) of the currently most general and fundamental physical theory: quantum mechanics. Now it would be more correct to speek about quantum field theory but here I restrict myself to finite-dimensional systems only (see Sections 3.2 and 3.4).

3.1 The Correspondence Principle

In attempts to build up the quantum theory of dynamical chaos we immediately encounter a number of apparently very deep contradictions between the well-established properties of classical dynamical chaos and the most fundamental principles of quantum mechanics.

To begin with, quantum mechanics is commonly understood as a *fundamentally statistical theory*, which seems to imply *always some quantum chaos*, independent of the behavior in the classical limit. This is certainly true but in some restricted sense only. A novel developement here is the *isolation* of this fundamental quantum randomness as solely the characteristic of the very specific quantum process, measurement, and even as the particular part of that—the so-called *ψ-collapse* which, indeed, has so far no dynamical description (see Section 4 for further discussion of this problem).

No doubt, quantum measurement is absolutely necessary for the study of the microworld by us, the macroscopic human beings. Yet, the measurement is, in a sense, foreign to the proper microworld that might (and should) be described separately from the former. Explicitly (Casati and Chirikov (1995a)) or, more often, implicitly such a philosophy has become common in studies of chaos but not yet beyond this field of research (see, e.g., Shimony (1994)).

This approach allows us to single out the dynamical part of quantum mechanics as represented by a *specific dynamical variable* $\psi(t)$ in *Hilbert space*, satisfying some *deterministic equation of motion*, e.g., the Schrödinger equation. The more difficult and vague statistical part is left for a better time. Thus, we temporarily bypass (not resolve!) the first serious difficulty in the theory of quantum chaos (see also Section 4). The separation of the first part of quantum dynamics, which is very natural from a mathematical viewpoint, was first introduced and emphasized by Schrödinger, who, however, certainly underestimated the importance of the second part in physics.

However, another principal difficulty arises. As is well known, the energy (and frequency) spectrum of any quantum motion *bounded in phase space* is always *discrete*. And this is not the property of a particular equation but rather a consequence of the fundamental quantum principle—the *discreteness of phase space* itself, or in a more formal language, the noncommutative geometry of quantum phase space. Indeed, according to another fundamental quantum principle—the uncertainty principle—a single quantum state cannot occupy the phase space volume $V_1 \lesssim \hbar^N \equiv 1$ [in what follows I set $\hbar = 1$, particularly, not to confuse it with metric entropy h (2.15)]. Hence, the motion bounded in a domain of volume V is represented by $V/V_1 \sim V$ eigenstates, a property even stronger than the general discrete spectrum (almost periodic motion).

According to the existing ergodic theory such a motion is considered to be *regular*, which is something opposite to the known chaotic motion with a continuous spectrum and exponential instability (Section 2.2), again independent of the classical behavior. This seems to *never imply any chaos* or, to be more precise, any *classical-like chaos* as defined in the ergodic theory. Meanwhile, the correspondence principle requires *conditional chaos* related to the nature of motion in the classical limit.

3.2 Pseudochaos

Now the principal question to be answered reads: where is the expected quantum chaos in the ergodic theory? Our answer to this question (Chirikov et al. (1981), Chirikov et al. (1988); not commonly accepted as yet) was concluded from a simple observation (principally well known but never comprehended enough) that the sharp border between the discrete and continuous spectrum is physically meaningful in the limit $|t| \to \infty$ only, the condition actually assumed in the ergodic theory. Hence, to understand quantum chaos the existing ergodic theory needs modification by the introduction of a new 'dimension', the time. In other words, a new and central problem in the ergodic theory is the *finite-time statistical properties* of a dynamical system, both quantum as well as classical (Section 3.4).

Within a finite time the discrete spectrum is dynamically equivalent to the continuous one, thus providing much stronger statistical properties of the motion than was (and still is) expected in the ergodic theory for the case of a discrete spectrum. In short, motion with a discrete spectrum may exhibit *all the statistical properties* of classical chaos but only on some *finite time scales* (Section 3.3). Thus, the conception of a time scale becomes fundamental in our theory of quantum chaos (Chirikov et al. (1981), Chirikov et al. (1988)). This is certainly a *new dynamical phenomenon*, related but not identical at all to classical dynamical chaos. We call it *pseudochaos*; the term *pseudo* is used to emphasize the difference from the asymptotic (in time) chaos in the ergodic theory. Yet, from the physical point of view, we accept here that the latter, strictly speaking, does not exist in Nature. So, in the common philosophy of the universal quantum mechanics *pseudochaos is the only true dynamical chaos* (cf. the term 'pseudoeuclidian geometry' in special relativity). Asymptotic chaos is but a limiting pattern which is, nevertheless, important both in theory, to compare with the real chaos, and in applications, as a very good approximation in a macroscopic domain, as is the whole classical mechanics. Ford describes the former *mathematical chaos* as contrasted to the *real physical chaos* in quantum mechanics (Ford (1994)). Another curious but impressive term is *artificial reality* (Kaneko and Tsuda (1994)), which is, of course, a self-contradictory notion reflecting, particularly, confusion in the interpretation of surprising phenomena such as chaos.

The statistical properties of the discrete-spectrum motion are not completely new subjects of research, such research goes back to the time of intensive studies in the mathematical foundations of statistical mechanics *before* dynamical chaos was discovered or, better to say, understood (see, e.g., Kac (1959)). We call this early stage of the theory *traditional statistical mechanics* (TSM). It is equally applicable to both classical as well as quantum systems. For the problem under consideration here, one of the most important rigorous results with far-reaching consequences was the *statistical independence* of oscillations with incommensurable (linearly independent) frequencies ω_n, such that the only solution of the resonance equation,

$$\sum_n^N m_n \cdot \omega_n = 0, \tag{3.1}$$

in integers is $m_n \equiv 0$ for all n. This is a generic property of the real numbers; that is, the resonant frequencies (3.1) form a set of zero Lebesgue measure. If we define now $y_n = \cos(\omega_n t)$, the statistical independence of y_n means that trajectory $y_n(t)$ is ergodic in N-cube $|y_n| \leq 1$. This is a consequence of ergodicity of the phase trajectory $\phi_n(t) = \omega_n t \ \mathbf{mod}\ 2\pi$ in N-cube $|\phi_n| \leq \pi$.

Statistical independence is a basic property of a set to which the probability theory is to be applied. Particularly, the sum of statistically independent quantities,

$$x(t) = \sum_n^N A_n \cdot \cos(\omega_n t + \phi_n), \tag{3.2}$$

which is motion with a discrete spectrum, is the main object of this theory. However, the familiar statistical properties such as Gaussian fluctuations, postulated (directly or indirectly) in TSM, are reached in the limit $N \to \infty$ only, which is called the *thermodynamical limit.* In TSM this limit corresponds to infinite-dimensional models (Kornfeld et al. (1982), Katok and Hasselblatt (1994)), which provide a very good approximation for macroscopic systems, both classical and quantal.

However, what is really necessary for good statistical properties of sum (3.2) is a large number of frequencies $N_\omega \to \infty$, which makes the discrete spectrum continuous (in the limit). In TSM the latter condition is satisfied by setting $N_\omega = N$. The same holds true for quantum fields which are infinite-dimensional. In quantum mechanics another mechanism, independent of N, works in the quasiclassical region $q \gg 1$ where $q = I/\hbar \equiv I$ is some big quantum parameter, e.g., quantum number, and I stands for a characteristic action of the system. Indeed, if the quantum motion (3.2) [with $\psi(t)$ instead of $x(t)$] is determined by many ($\sim q$) eigenstates we can set $N_\omega = q$ independent of N. The actual number of terms in expansion (3.2) depends, of course, on a particular state $\psi(t)$ under consideration. For example, if it is just an eigenstate the sum reduces to a single term. This corresponds to the special peculiar trajectories of classical chaotic motion whose total measure is *zero*. Similarly, in quantum mechanics $N_\omega \sim q$ for *most states* if the system is *classically chaotic*. This important condition was found to be certainly *sufficient* for good quantum statistical properties (see Chirikov et al. (1981), Chirikov et al. (1988) and Section 3.3 below). Whether it is also the necessary condition remains as yet unclear.

Thus, with respect to the mechanism of the quantum chaos we essentially *come back* to TSM with an exchange of the number of freedoms N for the quantum parameter q. However, in quantum mechanics we are not interested, unlike in TSM, in the limit $q \to \infty$, which is simply the classical mechanics. Here, the central problem is the statistical properties for *large but finite* q. This problem does not exist in TSM describing macroscopic systems. Thus, with an old mechanism the new phenomena were understood in quantum mechanics.

3.3 Characteristic Time Scales in Quantum Chaos

The existing ergodic theory is asymptotic in time, and hence contains no time scales at all. There are two reasons for this. One is technical: it is much simplier to derive the asymptotic relations than to obtain rigorous finite-time estimates. Another reason is more profound. All statements in the ergodic theory hold true up to measure zero, that is, excluding some peculiar nongeneric sets of zero measure. Even this minimal imperfection of the theory did not seem completely satisfactory but has been 'swallowed' eventually and is now commonly tolerated even among mathematicians, to say nothing about physicists. In a finite-time theory all these exceptions acquire a *small but finite* measure which would be already 'unbearable' (for mathematicians). Yet, there is a standard mathematical trick, to be discussed below, for avoiding both these difficulties.

The most important time scale t_R in quantum chaos is given by the general estimate

$$\ln t_R \sim \ln q\,, \qquad t_R \sim q^\alpha \sim \rho_0 \leq \rho_H, \tag{3.3}$$

where $\alpha \sim 1$ is a system-dependent parameter. This is called the *relaxation time scale* refering to one of the principal properties of chaos: *statistical relaxation* to some steady state (statistical equilibrium). The physical meaning of this scale is principally simple and is directly related to the fundamental uncertainty principle ($\Delta t \cdot \Delta E \sim 1$) as implemented in the second equation in (3.3), where ρ_H is the *full* average energy level density (also called the Heisenberg time). For $t \lesssim t_R$ the discrete spectrum is not resolved, and the statistical relaxation follows the classical (limiting) behavior. This is just the 'gap' in the ergodic theory (supplemented with the additional, time, dimension) where pseudochaos, particularly quantum chaos, dwells. A more accurate estimate relates t_R to a *part* ρ_0 of the level density. This is the density of the so-called *operative eigenstates*; that is, only those that are actually present in a particular quantum state ψ and actually control its dynamics.

The formal trick mentioned above is to consider not the finite-time relations we really need but rather the special *conditional limit* (cf. (2.22)):

$$t,\, q \to \infty \qquad \tau = \frac{t}{t_R(q)} = const \tag{3.4}$$

Quantity τ is a new rescaled time which is, of course, nonphysical but very helpful technically. The *double* limit (3.4) (unlike the single one $q \to \infty$) is *not* the classical mechanics which holds true, in this representation, for $\tau \lesssim 1$ and with respect to the statistical relaxation only. For $\tau \gtrsim 1$ the behavior becomes essentially quantum (even in the limit $q \to \infty$!) and is called nowadays *mesoscopic phenomena*. Particularly, the quantum steady state is quite different from the classical statistical equilibrium in that the former may be *localized* (under certain conditions) that is *nonergodic* in spite of classical ergodicity.

Another important difference is in *fluctuations*, which are also a characteristic property of chaotic behavior. In comparison with classical mechanics quantum $\psi(t)$ plays, in this respect, an intermediate role between the classical trajectory (exact or symbolic) with big relative fluctuations ~ 1 and the coarse-grained classical phase space density with no fluctuations at all. Unlike both the fluctuations of $\psi(t)$ are $\sim {N_\omega^{-1}}^{/2}$, which are another manifistation of statistical independence, or *decoherence*, of even pure quantum state (3.2) in case of quantum chaos. In other words, chaotic $\psi(t)$ represents statistically a *finite ensemble* of $\sim N_\omega$ systems even though formally $\psi(t)$ describes a single system. Quantum fluctuations clearly demonstrate also the difference between physical time t and auxillary variable τ: in the double limit $(t,\, q \to \infty)$ the fluctuations vanish and one needs a new trick to recover them.

The relaxation time scale should be distinguished from the *Poincaré recurrence time* $t_P \gg t_R$, which is typically much longer, and which sharply increases with a decrease in the recurrence domain. Time scale t_P characterizes big fluctuations (for both the classical trajectory, but not the phase space density, and quantum ψ) of which recurrences is a particular case. Unlike this, t_R describes the average relaxation process.

Stronger statistical properties than relaxation and fluctuations are related in the ergodic theory to the exponential instability of motion. Their importance for statistical mechanics is not completely clear. Nevertheless, in accordance with the correspondence principle, those stronger properties are also present in quantum chaos as well, but on a *much shorter* time scale,

$$t_r \sim \frac{\ln q}{h}, \tag{3.5}$$

where h is classical metric entropy (2.15). This time scale was discovered and partly explained by Berman and Zaslavsky (1978) (see also Chirikov et al. (1981), Chirikov et al. (1988), Casati and Chirikov (1995a)). Being very short, t_r grows indefinitely as $q \to \infty$.

The simplest example of quantum dynamics on this scale is the stretching/squeezing of an initially narrow wave packet, with the conservation of the phase space volume like in classical mechanics, followed by the packet inflation (increasing phase space volume), and eventually by the complete destruction of the packet, its splitting into many irregular subpackets (Casati and Chirikov (1995a)).

In a quasiclassical region ($q \gg 1$), $t_r \ll t_R$ (3.3). This leads to an interesting conclusion that the quantum diffusion and relaxation are *dynamically stable* contrary to the classical behavior. It suggests, in turn, that the motion instability is not important *during* statistical relaxation. However, the *foregoing* correlation decay on a short time scale t_r is *crucial* for the statistical properties of quantum dynamics.

3.4 Examples of Pseudochaos in Classical Mechanics

Pseudochaos is a new generic dynamical phenomenon missed in the ergodic theory. No doubt, the most important particular case of pseudochaos is quantum chaos. Nevertheless, pseudochaos occurs in classical mechanics as well. Here are a few examples of classical pseudochaos, which may help us to understand the physical nature of quantum chaos, my primary goal in this paper. Besides, this unveils new features of classical dynamics as well.

Linear waves is the example of pseudochaos (see, e.g., Chirikov (1992)) that is closest to quantum mechanics. I remind you that here only a part of quantum dynamics is discussed, the one described, e.g., by the Schrödinger equation, which is a linear wave equation. For this reason quantum chaos is sometimes called wave chaos (Ŝeba (1990)). Classical electromagnetic waves are used in laboratory experiments as a physical model for quantum chaos (Stöckmann and Stein (1990), Weidenmüller et al. (1992)). The 'classical' limit corresponds here to the geometrical 'optics', and the 'quantum' parameter $q = L/\lambda$ is the ratio of a characteristic size L of the system to the wave length λ.

The linear oscillator (many-dimensional) is a particular case of waves (without dispersion). A broad class of quantum systems can be reduced to this model (Eckhardt (1988)). Statistical properties of linear oscillators, particularly in the thermodynamic limit ($N \to \infty$), were studied by Bogolyubov (1945) in the framework of TSM. On the other hand, the theory of quantum chaos suggests richer behavior for a large but finite N, particularly, the characteristic time scales for the harmonic oscillator motion (Chirikov (1986)) and the number of degrees of freedom N playing the role of the 'quantum' parameter.

Completely integrable nonlinear systems also reveal pseudochaotic behavior. An example of statistical relaxation in the Toda lattice had been presented in Ford et al. (1973) much before the problem of quantum chaos arose. Moreover, the strongest statistical properties in the limit $N \to \infty$, including one equivalent to the exponential instability (the so-called K-property) were rigorously proved just for the (infinite) completely integrable systems (see Kornfeld et al. (1982), Katok and Hasselblatt (1994)).

The digital computer is a very specific classical dynamical system whose dynamics is extremely important in view of the ever increasing application in numerical experiments covering now all branches of science and beyond. The computer is an 'overquantized' system in that *any* quantity here is *discrete*, whereas in quantum mechanics only the product of two conjugated variables is. The 'quantum' parameter here is $q = M$, which is the largest computer integer, and the short time scale (3.5) is $t_r \sim \ln M$, which is the number of digits in the computer word (Chirikov et al. (1981), Chirikov et al. (1988)). Owing to the discreteness, any dynamical trajectory in the computer eventually becomes periodic, an effect well known in the theory and practice of the so-called pseudorandom number generators. One should take all necessary precautions to exclude this computer artifact in numerical experiments. On the mathematical part, the periodic approximations in dynamical systems are also studied in ergodic theory, apparently without any relation to pseudochaos in quantum mechanics or computers.

Computer pseudochaos is the best answer to those who refuse accept the quantum chaos as, at least, a kind of chaos, and who still insist that only the classical-like (asymptotic) chaos deserves this name, the same chaos that was (and is) studied to a large extent just on computers; that is, the chaos inferred from a pseudochaos!

4 Conclusion: Old Challenges and New Hopes

The discovery and understanding of the new surprising phenomenon—dynamical chaos—opened up new horizons in solving many other problems including some long-standing ones. Unlike in previous sections, here I can give only a preliminary consideration of possible new approaches to such problems, together with some plausible conjectures (see also Casati and Chirikov (1995a)).

Let us begin with the problem directly related to quantum dynamics, namely the quantum measurement or, to be more correct, the specific stage of the latter:

ψ-collapse. This is just the part of quantum dynamics I bypassed above in the report on scientific results. This part still remains very vague to the extent that there is no common agreement even on the question of whether it is a real physical problem or an ill-posed one so that the Copenhagen interpretation of (or convention in) quantum mechanics gives satisfactory answers to all the *admissible* questions. In any event there exists as yet no dynamical description of the quantum measurement including ψ-collapse. The quantum measurement, as far as the result is concerned, is fundamentally a random process. However, there are good reasons to hope that this randomness can be interpreted as a particular manifestation of dynamical chaos (Cvitanović et al. (1992)).

The Copenhagen convention was (and still remains) very important as a phenomenological link between very specific quantum theory and laboratory experiments. Without this link studies of the microworld would be simply impossible. The Copenhagen philosophy perfectly matches the standard experimental setup of two measurements: the first one fixes the initial quantum state, and the second records the changes in the system. However, it is less clear how to deal with *natural processes* without any man-made measurements that is without the notorious *observer*. Since the beginning of quantum mechanics such a question has been considered ill-posed (meaning nasty). However, now there is a revival of interest in a deeper insight into this problem (see, e.g., Cvitanović et al. (1992)). Particularly, Gell-Mann and Hartle put a similar question, true, in the context of a very specific and global problem—the quantum birth of the Universe (Gell-Mann and Hartle (1989)). In my understanding, such a question arises as well in much simpler problems concerning any natural quantum processes. What is more important, the answer from Gell-Mann and Hartle (1989) does not seem satisfactory. Essentially, it is the substitution of the automaton (information gathering and utilizing system) for the standard human observer. Neither seems to be a generic construction in the microworld.

The theory of quantum chaos allows us to solve, at least (the simpler) half of the ψ-collapse problem. Indeed, the measurement device is by purpose a macroscopic system for which the classical description is a very good approximation. In such a system strong chaos with exponential instability is quite possible. The chaos in the classical measurment device is not only possible but unavoidable since the measurement system has to be, by purpose again, a highly unstable system where a microscopic intervention produces the macroscopic effect. The importance of chaos for the quantum measurement is that it destroys the coherence of the initial pure quantum state to be measured converting it into the incoherent mixture. In the present theories of quantum measurement this is described as the effect of external noise (see, e.g., Wheeler and Zureck (1983)). True, the noise is sufficient to destroy the quantum coherence, yet it is not necessary at all. Chaos theory allows us to get rid of the unsatisfactory effect of the external noise and to develop a purely dynamical theory for the loss of quantum coherence. Unfortunately, this is not yet the whole story. If we are satisfied with the *statistical* desciption of quantum dynamics (measurement including) then the decoherence is all we need. However, the *individual* behavior includes the second (main) part of ψ-collapse: namely, the *concentration* of ψ in a single state of the original superposition

$$\psi = \sum_n c_n \psi_n \rightarrow \psi_k, \qquad \sum_n |c_n|^2 = 1. \tag{4.1}$$

This is the proper ψ-collapse to be understood.

Also, it is another challenge to the correspondence principle. For quantum mechanics to be universal it must explain as well the very specific classical phenomenon of the *event* that does happen and remains for ever in the classical records, and is completely foreign to the proper quantum mechanics. It is just the effect of ψ-collapse.

All these problems could be resolved by a hypotetical phenomenon of *self-collapse*; that is, the collapse without any 'observer', human or automatic. Unfortunately, it seems that any physical explanation of ψ-collapse requires some changes in the existing quantum mechanics, and this is the main difficulty both technical and philosophical.

Now we come to the even more difficult problem of the *causality principle*: the universal time ordering of the events. This principle has been well confirmed by numerous experiments in all branches of physics. It is frequently used in the construction of various theories but, to my knowledge, no general relation of causality to the rest of physics was ever studied.

This principle looks like a statistical law (another time arrow), hence a new hope to understand the mechanism of causality via dynamical chaos. Yet, it directly enters the dynamics as the additional constraint on the interaction and/or the solutions of dynamical equations. A well-known and quite general example is in keeping the retarded solutions of a wave equation, only discarding advanced ones as 'nonphysical'. However, this is generally impossible for a bounded dynamics because of the boundary conditions. Still, causality holds true as well.

In some simple classical *dissipative* models, such as a driven damping oscillator, the dissipation was shown to imply causality (Youla et al. (1959), Dolph (1963), Zemanian (1965), Güttinger (1966), Nussenzveig (1972)). However, such results were formulated as the restriction on a class of systems showing causality rather than the foundations of the causality principle. Nevertheless, it was already some indication of a possible physical connection between dynamical causality and statistical behavior. To my knowledge, this connection was never studied further. To the contrary, the developement of the theory went the opposite way: taking for granted the causality to deduce all possible consequences, particularly various dispersion relations (Nussenzveig (1972)).

Causality relates two qualitatively different kinds of events: *causes* and *effects*. The former may be simply the initial conditions of motion, the point missed in the above-mentioned examples of the causality-dissipation relation. The initial conditions not only formally fix a particular trajectory but also are *arbitrary*, which is, perhaps, the key point in the causality problem. Also, this may shed some light on another puzzling peculiarity of *all* known dynamical laws: they discribe the motion up to arbitrary initial conditions only (cf. Weingartner (these proceedings)). It looks like the dynamical laws already include the causality implicitly even though they do not this explicitly. In any event, something arbitrary suggests chaos is around.

Again, we arrive at a tangle of interrelated problems. A plausible conjecture for how to resolve them might be as follows. An arbitrary cause indicates some statistical behavior, while the cause–effect relation points out a dynamical law. Then, we may conjecture that when the cause acts the transition from statistical to dynamical behavior occurs, which statistically separates the cause from the 'past' and dynamically fixes the effect in the 'future'. In this imaginary picture the 'past' and 'future' are related not to time but rather to cause and effect, respectively. Thus, the causality might be not time ordering (time arrow) but *cause–effect ordering*, or the *causality arrow*. The latter is very similar to the process arrow discussed in Section 2.2. Now, the central point is that the cause is arbitrary while the effect is not, whatever the time ordering.

This is, of course, but a raw guess to be developed, carefully analysed, and eventually confirmed or disproved experimentally.

Also, this picture seems to be closer to the statistical (secondary) dynamics [synergetics, or $S \supset D$ inclusion in (1.1)] rather than to dynamical chaos. Does it mean that the primary physical laws are statistical or, instead, that the chain of inclusions (1.1) is actually a closed ring with a 'feedback' coupling the secondary statistics to the primary dynamics?

We don't know.

In all this long lecture I have never given the definition of dynamical chaos, either classical or quantal, restricting myself to informal explanations (see Casati and Chirikov (1995a) for some current definitions of chaos). In a mathematical theory the definition of the main object of the theory precedes the results; in physics, expecially in new fields, it is quite often vice versa. First, one studies a new phenomenon such as dynamical chaos and only at a later stage, after understanding it sufficiently, we try to classify it, to find its proper place in the existing theories and eventually to choose the most reasonable definition. This time has not yet come.

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Comment on Boris Chirikov's Paper "Natural Laws and Human Prediction"

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I enjoyed all parts of this paper, but the part on which I should especially like to comment is the part dealing with the history of investigations of chaos in systems subject to the Hamiltonian equations of classical mechanics. I believe that this history has been described by Professor Chirikov, perhaps through modesty, in a way which does not fully bring out the importance of contributions to the study of chaos in such systems which were made by Professor Chirikov himself. Those classical authors which are cited in the paper, including especially Poincaré, had admittedly achieved an understanding of the possibilities of chaotic behaviour that may arise in Hamiltonian systems. On the other hand, their attempts at rigorous mathematical proof of the properties of such systems came up against some very severs difficulties. Necessarily, such proofs were attempted by means of perturbation theory, for sufficiently small departures from a regular (periodic-orbits) solution. Nevertheless, many formidable obstacles (including the famous "small divisors" problem, for example) opposed the development of their arguments into a "watertight" mathematical proof. Against this background, one of the vitally important contributions of the famous "KAM" papers of Kolmogorov (1954), Arnold (1963) and Moser (1962) referred to in section 2.5 of Professor Chirikov's paper was their success in overcoming all the obstacles, and in achieving a first rigorous demonstration, for sufficiently small values of a perturbation amplitude, of the properties of such classical systems.

Even in those regions of parameter space (involving e.g. near-coincidence of resonance frequencies) where the difficulties were most formidable, the KAM methods produced completely reliable results. As far as chaos was concerned, these results demonstrated beyond any doubt that it could arise in such a system. Nevertheless, they showed that regular behaviour of the system was enormously more common. Indeed, it was only in regions of parameter space whose total measure was of smaller order than any algebraic power of a perturbation amplitude that this regular behaviour was replaced by chaotic behaviour. The presence of those "microscopic" gaps in parameter space where chaotic behaviour could be shown to come about was of course of the greatest physical as well as mathematical interest. On the other hand, a group of "die-hard" mathematicians who had long argued that behaviour was an essentially unproven hypothesis could still claim that the demonstration of its absence except in a region of parameter space of such exceedingly small measure had at least identified it as just "a rarity". It has been against that background that the 1979 paper of Professor Chirikov (see Chirikov (1979)) has required to be seen as of the utmost importance. By using computational methods of extreme precision to derive accurate numerical solutions for a Hamiltonian system, he was able first of all to verify for small amplitudes the transition from regular to chaotic behaviour in those extremely narrow regions of parameter space that are predicted by the KAM theory. There his methods were deriving identical results to those based upon a perturbation-theory approach. Then, he investigated what happened to those extremely narrow regions when the computations were carried out with progressively increasing perturbation amplitudes. It was above all this investigation which convinced the exponents of classical mechanics that chaos is not "a mere curiosity" - and, above all, not just "a rarity". On the contrary, as the perturbation amplitude increased, there appeared a steep widening of the regions of parameter space within which computed solutions exhibited the behaviour characteristic of chaotic systems. With a further increase of amplitude, chaotic behaviour from being exceedingly rare had become extremely normal. For many systems, furthermore, the computations indicated a transition to globally chaotic behaviour, sometimes called global stochasticity. Some other work at that time, being carried out independently in the USA by J.M. Greene (see Greene (1979)), was leading to rather similar conclusions, which have of course been strongly reinforced in many subsequent investigations. Nevertheless, it is no exaggeration for the friends of Professor Chirikov to claim, and moreover to wish to emphasize on an occasion like this, that it was his work above all which led to a full recognition of how, for conservative dynamical systems in classical mechanics, chaotic behaviour is the rule rather than the exception. In relation to the subject of this Symposium (the relation between knowledge of laws governing natural phenomena and the possibilities of prediction of those phenomena) this conclusion has, needless to say, proved to be of fundamental importance.

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