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Law and Prediction in the Light of Chaos Research

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Preface

The contributions in this book are based on the conference "*Law and Prediction in Natural Science in the Light of our New Knowledge from Chaos Research*" held at the *Institut für Wissenschaftstheorie, Internationales Forschungszentrum*, Salzburg, July 1994. The papers given at the conference have been revised and partially extended, taking into account the discussion and scientific communication at the conference. Great emphasis has been given to the discussions among the participants. Preliminary versions of the papers were distributed among the participants one month before the conference. Each participant had about an hour for the presentation of his paper and an hour for discussion afterwards. Since important additions and clarifications have emerged from the discussions, important parts of them have been included in the proceedings.

There were people who were invited to the conference but could not attend because of other commitments (Prigogine, Lighthill, Nicolis and Haken). They agreed to send their papers or comments. Peter Schuster who attended the conference was not able to send his paper.

The editors would like to thank the publisher, especially Prof. Wolf Beiglböck, Gabriele Köbrunner Krisch for writing the manuscript and transcribing the discussions from the tape and Helmut Prendinger for the conversion into LaTeX. Last but not least the editors would like to express their gratitude in the name of all participants of the conference to those institutions that generously sponsored this research conference: The Austrian *Bundesministerium für Wissenschaft und Forschung*, Vienna and the *Internationales Forschungszentrum*, Salzburg.

Paul Weingartner

Gerhard Schurz

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Abstract. Interrelations between dynamical and statistical laws in physics, on the one hand, and between classical and quantum mechanics, on the other hand, are discussed within the philosophy of separating the natural from the human, as a very specific part of Nature, and with emphasis on the new phenomenon of dynamical chaos.
The principal results of the studies of chaos in classical mechanics are presented in some detail, including the strong local instability and robustness of motion, continuity of both phase space and the motion spectrum, and the time reversibility but nonrecurrency of statistical evolution, within the general picture of chaos as a specific case of dynamical behavior.
Analysis of the apparently very deep and challenging contradictions of this picture with the quantum principles is given. The quantum view of dynamical chaos, as an attempt to resolve these contradictions guided by the correspondence principle and based upon the characteristic time scales of quantum evolution, is explained. The picture of quantum chaos as a new generic dynamical phenomenon is outlined together with a few other examples of such chaos: linear (classical) waves, the (many-dimensional) harmonic oscillator, the (completely integrable) Toda lattice, and the digital computer.
I conclude with discussion of the two fundamental physical problems: quantum measurement (ψ-collapse), and the causality principle, which both appear to be related to the phenomenon of dynamical chaos.

1 Philosophical Introduction: Separation of the Natural from the Human

The main purpose of this paper is the analysis of conceptual implications from the studies of a new phenomenon (or rather a whole new field of phenomena) known as *dynamical chaos* both in classical and especially in quantum mechanics. The concept of dynamical chaos resolves (or, at least, helps to do so) the two fundamental problems in physics and, hence, in all the natural sciences:

- are the dynamical and statistical laws of a different nature or does one of them, and which one, follow from the other;
- are classical and quantum mechanics of a different nature or is the latter the most universal and general theory currently available to describe the whole empirical evidence including the classical mechanics as the limiting case.

The essence of my debut philosophy is the separation of the human from the natural following Einstein's approach to the science – *building up a model of the real world*. Clearly, the human is also a part of the world, and moreover the most important part for us as human beings but not as physicists. The whole

phenomenon of life is extremely specific, and one should not transfer its peculiarities into other fields of natural sciences as was erroneously (in my opinion) done in almost all major philosophical systems. One exception is positivism, which seems to me rather dull; it looks only at Nature but does not even want to see its internal mechanics. Striking examples of the former are Hegel's 'Philosophy of Nature' (Naturphilosophie) and its 'development', Engels' 'Dialectic of Nature'.

Another notorious confusion of such a 'human-oriented' physics was Wigner's claim that quantum mechanics is incompatible with the existence of self-reproducing systems (Wigner (1961)). The resolution of this 'paradox' is just in that Wigner assumed the Hamiltonian of such a system to be arbitrary, whereas it is actually highly specific (Schuster (1994)).

A more hidden human-oriented philosophy in physics, rather popular nowadays, is the information-based representation of natural laws, particularly when information is substituted for entropy (with opposite sign). In the most general way such a philosophy was recently presented by Kadomtsev (1994). That approach is possible and might be done in a self-consistent way, but one should be very careful to avoid many confusions. In my opinion, the information is an adequate conception for only the special systems that actually use and process the information like various automata, both natural (living systems) and man-made ones. In this case the information becomes a physical notion rather than a human view of natural phenomena. The same is also true in the theory of measurement, which is again a very specific physical process, the basic one in our studies of Nature but still not a typical one for Nature itself. This is crucially important in quantum mechanics as will be discussed in some detail below (Sections 2.4 and 3.1).

One of the major implications from studies of dynamical chaos is the conception of statistical laws as an intrinsic part of dynamics without any additional statistical hypotheses [for the current state of the theory see, e.g., Lichtenberg and Lieberman (1992) and recent collection of papers by Casati and Chirikov (1995) as well as the introduction to this collection by Casati and Chirikov (1995a)]. This basic idea can be traced back to Poincaré (1908) and Hadamard (1898), and even to Maxwell (1873); the principal condition for dynamical chaos being strong local instability of motion (Section 2.4). In this picture the statistical laws are considered as *secondary* with respect to more fundamental and general *primary* dynamical laws.

Yet, this is not the whole story. Surprisingly, the opposite is also true! Namely, under certain conditions the dynamical laws were found to be completely contained in the statistical ones. Nowadays this is called 'synergetics' (Haken (1987), Wunderlin (these proceedings)) but the principal idea goes back to Jeans (1929) who discovered the instability of gravitating gas (a typical example of a statistical system), which is the basic mechanism for the formation of galaxies and stars in modern cosmology, and eventually the Solar system, a classical example of a dynamical system. In this case the resulting dynamical laws proved to be secondary with respect to the primary statistical laws which include the former.

Thus, the whole picture can be represented as a chain of dynamical–statistical inclusions:

$$...?...\boxed{D \supset S} \supset D \supset S ...?... \tag{1.1}$$

Both ends of this chain, if any, remain unclear. So far the most fundamental (elementary) laws of physics seem to be dynamical (see, however, the discussion of quantum measurement in Sections 3 and 4). This is why I begin chain (1.1) with some primary dymamical laws.

The strict inclusion on each step of the chain has a very important consequence allowing for the so-called numerical experiments, or computer simulation, of a broad range of natural processes. As a matter of fact the former (not laboratory experiments) are now the main source of new information in the studies of the secondary laws for both dynamical chaos and synergetics. This might be called *the third way of cognition*, in addition to laboratory experiments and theoretical analysis.

In what follows I restrict myself to the discussion of just a single ring of the chain as marked in (1.1). Here I will consider the dynamical chaos separately in classical and quantum mechanics. In the former case the chaos explains the origin and mechanism of random processes in Nature (within the classical approximation). Moreover, that deterministic randomness may occur (and is typical as a matter of fact) even for a minimal number of degress of freedom $N > 1$ (for Hamiltonian systems), thus enormously expanding the domain for the application of the powerful methods of statistical analysis.

In quantum mechanics the whole situation is much more tricky and still remains rather controversial. Here we encounter an intricate tangle of various apparent contradictions between the correspondence principle, classical chaotic behavior, and the very foundations of quantum physics. This will be the main topic of my discussions below (Section 3).

One way to untangle this tangle is the new general conception, *pseudochaos*, of which quantum chaos is the most important example. Another interesting example is the digital computer, also very important in view of the broad application of numerical experiments in the studies of dynamical systems. On the other hand, pseudochaos in computers will hopefully help us to understand quantum pseudochaos and to accept it as a sort of *chaos* rather than a sort of regular motion, as many researchers, even in this field, still do believe.

The new and surprising phenomenon of dynamical chaos, especially in quantum mechanics, holds out new hopes for eventually solving some old, long-standing, fundamental problems in physics. In Section 4, I will briefly discuss two of them:

- the causality principle (time ordering of cause and effect), and
- ψ-collapse in the quantum measurement.

The conception of dynamical chaos I am going to present here, which is not common as yet, was the result of the long-term Siberian–Italian (SI) collaboration including Giulio Casati and Italo Guarneri (Como), and Felix Izrailev and Dima Shepelyansky (Novosibirsk) with whom I share the responsibility for our joint scientific results and the conceptual interpretation.

2 Scientific Results and Conceptual Implications: the Classical Limit

Classical dynamical chaos, as a part of classical mechanics, was historically the first to have been studied simply because in the time of Boltzmann, Maxwell, Poincaré and other founders, statistical mechanics and quantum mechanics did not exist. No doubt, the general mathematical theory of dynamical systems, including the ergodic theory as its modern part describing various statistical properties of the motion, has arisen from (and is still conceptually based on) classical mechanics (Kornfeld et al. (1982), Katok and Hasselblatt (1994)). Yet, upon construction, it is not necessarily restricted to the latter and can be applied to a much broader class of dynamical phenomena, for example, in quantum mechanics (Section 3).

2.1 What is a Dynamical System?

In classical mechanics, 'dynamical system' means an object whose motion in some *dynamical space* is *completely* determined by a given interaction and the *initial conditions*. Hence, the synonym *deterministic system*. The motion of such a system can be described in two seemingly different ways which, however, prove to be essentially equivalent.

The first one is through the *motion equations* of the form

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t), \tag{2.1}$$

which always have a unique solution

$$\mathbf{x} = \mathbf{x}(t, \mathbf{x}_0) \tag{2.2}$$

Here $\mathbf{x}$ is a finite-dimensional vector in the dynamical space and $\mathbf{x}_0$ is the initial condition $[\mathbf{x}_0 = \mathbf{x}(0)]$. A possible explicit time-dependence in the right-hand side of (2.1) is assumed to be a regular, e.g., periodic, one or, at least, one with a discrete spectrum.

The most important feature of dynamical (deterministic) systems is the *absence of any random parameters or any noise* in the motion equations. Particularly for this reason I will consider a special class of dynamical systems, the so-called *Hamiltonian (nondissipative) systems*, which are most fundamental in physics.

Dissipative systems, being very important in many applications, are neither fundamental (because the dissipation is introduced via a crude approximation of the very complicated interaction with some 'heat bath') nor purely dynamical in view of principally inevitable random noise in the heat bath (fluctuation–dissipation theorem). In a more accurate and natural way the dissipative systems can be described in the frames of the secondary dynamics ($S \supset D$ inclusion in (1.1)) when both dissipation and fluctuations are present from the beginning in the primary statistical laws.

A purely dynamical system is necessarily the *closed* one, which is the main object in fundamental physics. Thus, any coupling to the environment is completely neglected. I will come back to this important question below (Section 2.4).

In Hamiltonian mechanics the dynamical space, called *phase space*, is an even-dimensional one composed of N pairs of canonically conjugated 'coordinates' and 'momenta', each pair corresponding to one freedom of motion.

In the problem of dynamical chaos the initial conditions play a special role: they completely determine a particular trajectory, for a given interaction, or a particular realization of a dynamical process which may happen to be a very specific, nontypical, one. To get rid of such singularities another description is useful, namely the Liouville partial differential equation for the *phase space density*, or distribution function $f(\mathrm{x}, t)$:

$$\frac{\partial f}{\partial t} = \hat{L} f \tag{2.3}$$

with the solution

$$f = f(\mathrm{x}, t; f_0(\mathrm{x})). \tag{2.4}$$

Here $\hat{L}$ is a *linear* differential operator, and $f_0(\mathrm{x}) = f(\mathrm{x}, 0)$ is the initial density. For any smooth f_0 this description provides the generic behavior of a dynamical system via a continuum of trajectories. In the special case $f_0 = \delta(\mathrm{x} - \mathrm{x_0})$ the density describes a single trajectory like the motion equations (2.1).

In any case the phase space itself is assumed to be *continuous*, which is the most important feature of the classical picture of motion and the main obstacle in the understanding of quantum chaos (Section 3).

2.2 What is Dynamical Chaos?

Dynamical chaos can be characterized in terms of both the individual trajectories and the trajectory ensembles, or phase density. Almost all trajectories of a chaotic system are in a sense most complicated (they are *unpredictable* from observation of any preceding motion to use this familiar human term). Exceptional, e.g., periodic trajectories form a set of zero invariant measure, yet it might be everywhere dense.

An appropriate notion in the theory of chaos is the *symbolic trajectory* first introduced by Hadamard (1898). The theory of symbolic dynamics was developed further by Morse (1966), Bowen (1973), and Alekseev and Yakobson (1981). The symbolic trajectory is a projection of the true (exact) trajectory on to a discrete partition of the phase space at discrete instants of time t_n, e.g., such

that $t_{n+1} - t_n = T$ fixed. In other words, to obtain a symbolic trajectory we first turn from the motion differential equations (2.1) to the difference equations over a certain time interval T:

$$\mathrm{x}(t_{n+1}) \equiv \mathrm{x}_{n+1} = M(\mathrm{x}_n, t_n). \tag{2.5}$$

This is usually called *mapping* or *map*: $\mathrm{x}_n \rightarrow \mathrm{x}_{n+1}$. Then, while running a (theoretically) *exact* trajectory we record each x_n to a *finite* accuracy: $\mathrm{x}_n \approx m_n$. For a finite partition each m_n can be chosen to be integer. Hence, the whole infinite symbolic trajectory

$$\sigma \equiv ...m_{-n}...m_{-1}m_0m_1...m_n... = S(\mathrm{x_0};\, T), \tag{2.6}$$

can be represented by a *single* number σ, which is generally irrational and which is some function of the *exact* initial conditions. The symbolic trajectory may be also called a *coarse-grained trajectory*. I remind you that the latter is a *projection* of (not substitution for) the exact trajectory to represent in compact form the global dynamical behavior without unimportant microdetails.

A remarkable property of chaotic dynamics is that the set of its symbolic trajectories is *complete*; that is, it actually contains all possible sequences (2.6). Apparently, this is related to continuity of function $S(\mathrm{x_0})$ (2.6). On the contrary, for a regular motion this function is everywhere discontinuous.

In a similar way the *coarse-grained phase density* $\overline{f}(m_n, t)$ is introduced, in addition to the exact, or *fine-grained density*, which is also a projection of the latter on to some partition of the phase space.

The coarse-grained density represents the global dynamical behavior, particularly the most important process of *statistical relaxation*, for chaotic motion, to some *steady state* $f_s(m_n)$ (statistical equilibrium) independent of the initial $f_0(\mathrm{x})$ if the steady state is *stable*. Otherwise, synergetics comes into play giving rise to a secondary dynamics. As the relaxation is an aperiodic process the spectrum of chaotic motion is *continuous*, which is another obstacle for the theory of quantum chaos (Section 3).

Relaxation is one of the characteristic properties of statistical behavior. Another is *fluctuation*. Chaotic motion is a generator of noise which is purely *intrinsic* by definition of the dynamical system. Such noise is a particular manifestation of the complicated dynamics as represented by the symbolic trajectories or by the difference

$$f(\mathrm{x}, t) - \overline{f}(m_n, t) \equiv \tilde{f}(\mathrm{x}, t). \tag{2.7}$$

The relaxation $\overline{f} \rightarrow f_s$, apparently asymmetric with respect to time reversal $t \rightarrow -t$, gave rise to a long-standing misconception of the notorious *time arrow*. Even now some very complicated mathematical constructions are still being erected (see, e.g., Misra et al. (1979), Goldstein et al. (1981)) in attempts to extract somehow statistical irreversibility from the reversible mechanics. In the theory of dynamical chaos there is no such problem. The answer turns out to be conceptual rather than physical: one should separate two similar but different notions, *reversibility* and *recurrency*. The exact density $f(\mathbf{x}, t)$ is always *time-reversible* but *nonrecurrent* for chaotic motion; that is, it will never come back to the initial $f_0(\mathbf{x})$ in *both directions of time* $t \to \pm\infty$. In other words, the relaxation, also present in f, is time-symmetric. The projection of f, coarse-grained $\overline{f}$, which is both nonrecurrent and irreversible, emphasizes nonrecurrency of the exact solution. The apparent violation of the statistical relaxation upon time reversal, as described by the exact $f(\mathbf{x}, t)$, represents in fact the growth of a big fluctuation which will eventually be followed by the same relaxation in the opposite direction of time. This apparently surprising symmetry of the statistical behavior was discovered long ago by Kolmogorov (1937). One can say that instead of an imagionary time arrow there exists a *process arrow* pointing always to the steady state. The following simple example would help, perhaps, to overcome this conceptual difficulty. Consider the hyperbolic one-dimensional (1D) motion:

$$x(t) = a \cdot \exp(\Lambda t) + b \cdot \exp(-\Lambda t), \tag{2.8}$$

which is obviously time-reversible yet remains *unstable* in both directions of time ($t \to \pm\infty$). Besides its immediate appeal, this example is closely related to the mechanism of chaos which is the motion instability.

2.3 A Few Physical Examples of Low-Dimensional Chaos

In this paper I restrict myself to finite-dimensional systems where the peculiarities of dynamical chaos are most clear (see Section 3.2 for some brief remarks on infinite systems). Consider now a few examples of chaos in minimal dimensionality.

Billiards (2 degrees of freedom). The ball motion here is chaotic for almost any shape of the boundary except special cases like circle, ellipse, rectangle and some other (see, e.g., Lichtenberg and Lieberman (1992), Kornfeld et al. (1982), Katok and Hasselblatt (1994)). However, the ergodicity (on the energy surface) is only known for singular boundaries. If the latter is smooth enough the structure of motion becomes a very complicated admixture of chaotic and regular domains of various sizes (the so-called divided phase space). Another version of billiards is the wave cavity in the geometric optics approximation. This provides a helpful bridge between classical and quantum chaos.

Perturbed Kepler motion is a particular case of the famous 3-body problem. Now we understand why it has not been solved since Newton: chaos is generally present in such a system. One particular example is the motion of comet Halley perturbed by Jupiter which was found to be chaotic with an estimated life time in the Solar system of the order of 10 Myrs (Chirikov and Vecheslavov (1989); 2 degrees of freedom in the model used, divided phase space).

Another example is a new, diffusive, mechanism of ionization of the Rydberg (highly excited) hydrogen atom in the external monochromatic electric field. It was discovered in laboratory experiments (Bayfield and Koch (1974)) and was explained by dynamical chaos in a classical approximation (Delone et al. (1983)).

In this system a given field plays the role of the third body. The simplest model of the diffusive photoelectric effect has 1.5 degrees of freedom (1D Kepler motion and the external periodic perturbation), and is also characterized by a divided phase space.

Budker's problem: charged particle confinement in an adiabatic magnetic trap (Chirikov (1987)). A simple model of two freedoms (axisymmetric magnetic field) is described by the Hamiltonian:

$$H = \frac{p^2}{2} + \frac{(1 + x^2)\,y^2}{2}. \tag{2.9}$$

Here magnetic field $B = \sqrt{1 + x^2}$; $p^2 = \dot{x}^2 + \dot{y}^2$; x describes the motion along magnetic line, and y does so accross the line (a projection of Larmor's rotation). At small pitch angles $\beta \approx |\dot{y}/\dot{x}|$ the motion is chaotic with the chaos border being at roughly

$$p \sim \frac{1}{|\ln \beta|} \tag{2.10}$$

and being very complicated, so-called critical, structure (Section 2.5).

Matinyan's problem: internal dynamics of the Yang–Mills (gauge) fields in classical approximation (Matinyan (1979), Matinyan (1981)). Surprisingly, this completely different physical system can be also represented by Hamiltonian (2.9) with a symmetrized 'potential energy':

$$U = \frac{(1 + x^2)\,y^2 + (1 + y^2)\,x^2}{2}. \tag{2.11}$$

Dynamics is always chaotic with a divided phase space similar to model (2.9) (Chirikov and Shepelyansky (1982)). Model (2.11) describes the so-called massive gauge field; that is, one with the quanta of nonzero mass. The massless field corresponds to the 'potential energy'

$$U = \frac{x^2 y^2}{2} \tag{2.12}$$

and looks ergodic in numerical experiments.

2.4 Instability and Chaos

Local instability of motion responsible for a very complicated dynamical behavior is described by the *linearized equations*:

$$\frac{d\mathbf{u}}{dt} = \mathbf{u} \cdot \frac{\partial \mathbf{v}(\mathbf{x}^0(t), t)}{\partial \mathbf{x}}. \tag{2.13}$$

Here $\mathbf{x}^0(t)$ is a reference trajectory satisfying (2.1), and $\mathbf{u} = \mathbf{x}(t) - \mathbf{x}^0(t)$ is the deviation of a close trajectory $\mathbf{x}(t)$. On average, the solution of (2.13) has the form

$$|\mathbf{u}| \sim \exp(\Lambda t), \tag{2.14}$$

where Λ is *Lyapunov's exponent*. The motion is (exponentially) unstable if $\Lambda > 0$. In the Hamiltonian system of N degrees of freedom there are $2N$ Lyapunov's exponents satisfying the condition $\sum \Lambda = 0$. The partial sum of all positive exponents $\Lambda_+ > 0$,

$$h = \sum \Lambda_+ \tag{2.15}$$

is called the (dynamical) *metric entropy*. Notice that it has the dimensions of frequency and characterises the instability rate.

The motion instability is only a necessary but not sufficient condition for chaos. Another important condition is *boundedness* of the motion, or its oscillatory (in a broad sense) character. The chaos is produced by the combination of these two conditions (also called stretching and folding). Let us again consider an elementary example of a 1D map

$$x_{n+1} = 2x_n \mod 1, \tag{2.16}$$

where operation mod 1 restricts (folds) x to the interval (0,1). This is not a Hamiltonian system but it can be interpreted as a 'half' of that; namely, as the dynamics of the oscillation phase. This motion is unstable with $\Lambda = \ln 2$ because the linearized equation is the same except for the fractional part (mod 1). The explicit solution for both reads

$$u_n = 2^n u_0,$$

$$x_n = 2^n x_0 \mod 1. \tag{2.17}$$

The first (linearized) motion is unbounded, like Hamiltonian hyperbolic motion, (2.8) and is perfectly regular. The second one is not only unstable but also chaotic just because of the additional operation mod 1, which makes the motion bounded, and which mixes up the points within a finite interval.

We may look at this example from a different viewpoint. Let us express the initial x_0 in the binary code as the sequence of two symbols, 0 and 1, and let us make the partition of the unit x interval also in two equal halves marked by the same symbols. Then, the symbolic trajectory will simply repeat x_0; that is, (2.6) takes the form

$$\sigma = x_0. \tag{2.18}$$

It implies that, as time goes on, the global motion will eventually depend on ever-diminishing details of the initial conditions. In other words, when we formally fix the *exact* x_0 we 'supply' the system with infinite complexity, which arises due to the strong motion instability. Still another interpretation is that the exact x_0 is the source of *intrinsic noise* amplified by the instability. For this noise to be *stationary* the string of x_0 digits has to be infinite, which is only possible in *continuous* phase space.

A nontrivial part of this picture of chaos is that the instability must be *exponential* because a power-law instability is insufficient for chaos. For example, the linear instability ($|u| \sim t$) is a generic property of perfectly regular motion of the completely integrable system whose motion equations are *nonlinear* and, hence, whose oscillation frequencies depend on the initial conditions (Born (1958), Casati et al. (1980)). The character of motion for a faster instability ($|u| \sim t^\alpha$, $\alpha > 1$) is unknown.

On the other hand, the exponential instability ($h > 0$) is not invariant with respect to the change of time variable (Casati and Chirikov (1995a), Batterman (these proceedings); in this respect the only invariant statistical property is ergodicity, Kornfeld et al. (1982), Katok and Hasselblatt (1994)). A possible resolution of this difficulty is that the proper characteristic of motion instability, important for dynamical chaos, should be taken with respect to the oscillation phases whose dynamics determines the nature of motion. It implies that the proper time variable must change proportionally with the phases so that the oscillations become stationary (Casati and Chirikov (1995a)). A simple example is harmonic oscillation with frequency ω recorded at the instances of time $t_n = 2^n t_0$. Then, oscillation phase $x = \omega t/2\pi$ obeys map (2.16), which is chaotic. Clearly, the origin of chaos here is not in the dynamical system but in the recording procedure (random t_0). Now, if ω is a parameter (linear oscillator), then the oscillation is exponentially unstable (in new time n) but only with respect to the change of parameter ω, not of the initial x_0 ($x \to x + x_0$). In a slightly 'camouflaged' way, essentially the same effect was considered by Blümel (1994) with far-reaching conclusions on quantum chaos (Section 3.2).

Rigorous results concerning the relation between instability and chaos are concentrated in the Alekseev–Brudno theorem (see Alekseev and Yakobson (1981), Batterman (these proceedings), White (1993)), which states that the complexity per unit time of almost any symbolic trajectory is asymptotically equal to the metric entropy:

$$\frac{C(t)}{|t|} \to h, \quad |t| \to \infty. \tag{2.19}$$

Here $C(t)$ is the so-called algorithmic complexity, or in more familiar terms, the information associated with a trajectory segment of length $|t|$.

The transition time from dynamical to statistical behavior according to (2.19) depends on the partition of the phase space, namely, on the size of a cell μ, which is inversely proportional to the biggest integer $M \geq m_n$ in symbolic trajectory (2.6). The transition is controlled by the *randomness parameter* (Chirikov (1985)):

$$r = \frac{h\,|t|}{\ln M} \sim \frac{|t|}{t_r}, \tag{2.20}$$

where t_r is the *dynamical time scale*. As both $|t|$, $M \to \infty$ we have a somewhat confusing situation, typical in the theory of dynamical chaos, in which two limits do not commute:

$$M \to \infty,\ |t| \to \infty \neq |t| \to \infty,\ M \to \infty. \tag{2.21}$$

For the left order ($M \to \infty$ first) parameter $r \to 0$, and we have *temporary determinism* ($|t| \lesssim t_r$), while for the right order $r \to \infty$, and we arrive at *asymptotic randomness* ($|t| \gtrsim t_r$).

Instead of the above double limit we may consider the *conditional limit*

$$|t|,\ M \to \infty, \qquad r = \text{const}, \tag{2.22}$$

which is also a useful method in the theory of chaotic processes. Particularly for $r \lesssim 1$, strong dynamical correlations persist in a symbolic trajectory, which allows for the prediction of trajectory from a finite-accuracy observation. This is no longer the case for $r \gtrsim 1$ when only a statistical description is possible. Nevertheless, the motion equations can still be used to completely derive all the statistical properties without any *ad hoc* hypotheses. Here the exact trajectory *does exist* as well but becomes the Kantian *thing-in-itself*, which can be neither predicted nor reproduced in any other way.

The mathematical origin of this peculiar property goes back to the famous Gödel theorem (Gödel (1931)), which states (in a modern formulation) that *most* theorems in a given mathematical system are unprovable, and which forms the basis of contemporary mathematical logic (see Chaitin (1987) for a detailed explanation and interesting applications of this relatively less-known mathematical achievement). A particular corollary, directly related to symbolic trajectories (2.6), is that almost all real numbers are uncomputable by any finite algorithm. Besides rational numbers some irrationals like π or e are also known to be computable. Hence, their total complexity, e.g., $C(\pi)$, is finite, and the complexity per digit is zero (cf. (2.19)).

The main object of my discussion here, as well as of the whole physics, is a closed system that requires neglection of the external perturbations. However, in case of strong motion instability this is no longer possible, at least dynamically. What is the impact of a weak perturbation on the statistical properties of a chaotic system? The rigorous answer was given by the robustness theorem due to Anosov (1962): not only do statistical properties remain unchanged but, moreover, the trajectories get only slightly deformed providing (and due to) the same strong motion instability. The explanation of this striking peculiarity is that the trajectories are simply transposed and, moreover, the less the stronger is instability.

In conclusion let me make a very general remark, far beyond the particular problem of chaotic dynamics. According to the Alekseev–Brudno theorem (2.19) the source of stationary (new) information is always chaotic. Assuming farther that any creative activity, science including, is such a source we come to an interesting conclusion that any such activity has to be (partly!) chaotic. This is the creative side of chaos.

2.5 Statistical Complexity

The theory of dynamical chaos does not need any statistical hypotheses, nor does it allow for arbitrary ones. Everything is to be deduced from the dynamical equations. Sometimes the statistical properties turn out to be quite simple and familiar (Lichtenberg and Lieberman (1992), Chirikov (1979)). This is usually the case if the chaotic motion is also ergodic (on the energy surface), like in some

billiards and other simple models (Section 2.3). However, quite often, and even typically for a few-freedom chaos, the phase space is divided, and the chaotic component of the motion has a very complicated structure.

One beautiful example is the so-called Arnold diffusion driven by a weak ($\epsilon \to 0$) perturbation of a completely integrable system with $N > 2$ degrees of freedom (Lichtenberg and Lieberman (1992), Chirikov (1979)). The phase space of such a system is pierced by the everywhere-dense set of nonlinear resonances

$$\sum_n m_n \cdot \omega_n^0(I) \approx 0, \tag{2.23}$$

where m_n are integers, and ω_n^0 are the unperturbed frequences depending on dynamical variables (usually actions I). Each resonance is surrounded by a separatrix, the singular highly unstable trajectory with zero motion frequency. As a result, no matter how weak the perturbation ($\epsilon \to 0$) is, a narrow chaotic layer always arises around the separatrix. The whole set of chaotic layers is everywhere dense as is the set of resonances. For $N > 2$ the layers form a united connected chaotic component of the motion supporting the diffusion over the whole energy surface. Both the total measure of the chaotic component and the rate of Arnold diffusion are exponentially small ($\sim \exp(-C/\sqrt{\epsilon})$) and can be neglected in most cases; hence the term *KAM integrability* (Chirikov and Vecheslavov (1990)) for such a structure (after Kolmogorov, Arnold and Moser who rigorously analysed some features of this structure). This quasi-integrability has the nature and quality of adiabatic invariance. However, on a very big time scale this weak but universal instability may essentially affect the motion.

One notable example is celestial mechanics, particularly the stability of the Solar system (Wisdom (1987) Laskar (1989), Laskar (1990), Laskar (1994)). Surprisingly, this 'cradle' of classical determinism and the exemplar case of dynamical behavior proves to be unstable and chaotic. The instability time of the Solar system was found to be rather long ($\Lambda^{-1} \sim 10$ Myrs), and its life time is still many orders of magnitude larger. It has not been estimated as yet, and might well exceed the cosmological time ~ 10 Byrs.

Another interesting example of complicated statistics is the so-called *critical structure* near the chaos border which is a necessary element of divided phase space (Chirikov (1991)). The critical structure is a hierarchy of chaotic and regular domains on ever decreasing spatial and frequency scales. It can be universally described in terms of the *renormalization group*, which proved to be so efficient in other branches of theoretical physics. In turn, the renormalization group may be considered as an abstract dynamical system that describes the variation of the whole motion structure, for the original dynamical system, in dependence of its spatial and temporal scale. Logarithm of the latter plays a role of 'time' (renormtime) in that renormdynamics. At the chaos border the latter is determined by the motion frequencies. The simplest renormdynamics is a periodic variation of the structure or, for a renorm-map, the invariance of the structure with respect to the scale (MacKay (1983)). Surprisingly, this scale invariance includes the chaotic trajectories as well. The opposite limit—renormchaos—is also possible, and was found in several models (see Chirikov (1991)).

Even though the critical structure occupies a very narrow strip along the chaos border it may qualitatively change the statistical properties of the whole chaotic component. This is because a chaotic trajectory unavoidably enters from time to time the critical region and 'sticks' there for a time that is longer the closer it comes to the chaos border. The sticking results in a slow power-law correlation decay for large time, in a singular motion spectrum for low frequency, and even in the superdiffusion when the phase-density dispersion $\sigma^2 \sim t^\alpha$ $(\alpha > 1)$ grows faster than time (Chirikov (1987), Chirikov (1991)).

3 Scientific Results and Conceptual Implications: Quantum Chaos

The mathematical theory of dynamical chaos—ergodic theory—is self-consistent. However, this is not the case for the physical theory unless we accept the philosophy of the two separate mechanics: classical and quantum. Even though such a view cannot be excluded at the moment it has a profound difficulty concerning the border between the two. Nor is it necessary according to recent intensive studies of quantum dynamics. Then, we have to understand the mechanics of dynamical chaos from a quantum point of view. Our guiding star will be the *correspondence principle* which requires the complete quantum theory of any classical phenomenon, in the quasiclassical limit, assuming that the whole classical mechanics is but a special part (the limiting case) of the currently most general and fundamental physical theory: quantum mechanics. Now it would be more correct to speek about quantum field theory but here I restrict myself to finite-dimensional systems only (see Sections 3.2 and 3.4).

3.1 The Correspondence Principle

In attempts to build up the quantum theory of dynamical chaos we immediately encounter a number of apparently very deep contradictions between the well-established properties of classical dynamical chaos and the most fundamental principles of quantum mechanics.

To begin with, quantum mechanics is commonly understood as a *fundamentally statistical theory*, which seems to imply *always some quantum chaos*, independent of the behavior in the classical limit. This is certainly true but in some restricted sense only. A novel developement here is the *isolation* of this fundamental quantum randomness as solely the characteristic of the very specific quantum process, measurement, and even as the particular part of that—the so-called *ψ-collapse* which, indeed, has so far no dynamical description (see Section 4 for further discussion of this problem).

No doubt, quantum measurement is absolutely necessary for the study of the microworld by us, the macroscopic human beings. Yet, the measurement is, in a sense, foreign to the proper microworld that might (and should) be described separately from the former. Explicitly (Casati and Chirikov (1995a)) or, more often, implicitly such a philosophy has become common in studies of chaos but not yet beyond this field of research (see, e.g., Shimony (1994)).

This approach allows us to single out the dynamical part of quantum mechanics as represented by a *specific dynamical variable* $\psi(t)$ in *Hilbert space*, satisfying some *deterministic equation of motion*, e.g., the Schrödinger equation. The more difficult and vague statistical part is left for a better time. Thus, we temporarily bypass (not resolve!) the first serious difficulty in the theory of quantum chaos (see also Section 4). The separation of the first part of quantum dynamics, which is very natural from a mathematical viewpoint, was first introduced and emphasized by Schrödinger, who, however, certainly underestimated the importance of the second part in physics.

However, another principal difficulty arises. As is well known, the energy (and frequency) spectrum of any quantum motion *bounded in phase space* is always *discrete*. And this is not the property of a particular equation but rather a consequence of the fundamental quantum principle—the *discreteness of phase space* itself, or in a more formal language, the noncommutative geometry of quantum phase space. Indeed, according to another fundamental quantum principle—the uncertainty principle—a single quantum state cannot occupy the phase space volume $V_1 \lesssim \hbar^N \equiv 1$ [in what follows I set $\hbar = 1$, particularly, not to confuse it with metric entropy h (2.15)]. Hence, the motion bounded in a domain of volume V is represented by $V/V_1 \sim V$ eigenstates, a property even stronger than the general discrete spectrum (almost periodic motion).

According to the existing ergodic theory such a motion is considered to be *regular*, which is something opposite to the known chaotic motion with a continuous spectrum and exponential instability (Section 2.2), again independent of the classical behavior. This seems to *never imply any chaos* or, to be more precise, any *classical-like chaos* as defined in the ergodic theory. Meanwhile, the correspondence principle requires *conditional chaos* related to the nature of motion in the classical limit.

3.2 Pseudochaos

Now the principal question to be answered reads: where is the expected quantum chaos in the ergodic theory? Our answer to this question (Chirikov et al. (1981), Chirikov et al. (1988); not commonly accepted as yet) was concluded from a simple observation (principally well known but never comprehended enough) that the sharp border between the discrete and continuous spectrum is physically meaningful in the limit $|t| \to \infty$ only, the condition actually assumed in the ergodic theory. Hence, to understand quantum chaos the existing ergodic theory needs modification by the introduction of a new 'dimension', the time. In other words, a new and central problem in the ergodic theory is the *finite-time statistical properties* of a dynamical system, both quantum as well as classical (Section 3.4).

Within a finite time the discrete spectrum is dynamically equivalent to the continuous one, thus providing much stronger statistical properties of the motion than was (and still is) expected in the ergodic theory for the case of a discrete spectrum. In short, motion with a discrete spectrum may exhibit *all the statistical properties* of classical chaos but only on some *finite time scales* (Section 3.3). Thus, the conception of a time scale becomes fundamental in our theory of quantum chaos (Chirikov et al. (1981), Chirikov et al. (1988)). This is certainly a *new dynamical phenomenon*, related but not identical at all to classical dynamical chaos. We call it *pseudochaos*; the term *pseudo* is used to emphasize the difference from the asymptotic (in time) chaos in the ergodic theory. Yet, from the physical point of view, we accept here that the latter, strictly speaking, does not exist in Nature. So, in the common philosophy of the universal quantum mechanics *pseudochaos is the only true dynamical chaos* (cf. the term 'pseudoeuclidian geometry' in special relativity). Asymptotic chaos is but a limiting pattern which is, nevertheless, important both in theory, to compare with the real chaos, and in applications, as a very good approximation in a macroscopic domain, as is the whole classical mechanics. Ford describes the former *mathematical chaos* as contrasted to the *real physical chaos* in quantum mechanics (Ford (1994)). Another curious but impressive term is *artificial reality* (Kaneko and Tsuda (1994)), which is, of course, a self-contradictory notion reflecting, particularly, confusion in the interpretation of surprising phenomena such as chaos.

The statistical properties of the discrete-spectrum motion are not completely new subjects of research, such research goes back to the time of intensive studies in the mathematical foundations of statistical mechanics *before* dynamical chaos was discovered or, better to say, understood (see, e.g., Kac (1959)). We call this early stage of the theory *traditional statistical mechanics* (TSM). It is equally applicable to both classical as well as quantum systems. For the problem under consideration here, one of the most important rigorous results with far-reaching consequences was the *statistical independence* of oscillations with incommensurable (linearly independent) frequencies ω_n, such that the only solution of the resonance equation,

$$\sum_{n}^{N} m_n \cdot \omega_n = 0, \tag{3.1}$$

in integers is $m_n \equiv 0$ for all n. This is a generic property of the real numbers; that is, the resonant frequencies (3.1) form a set of zero Lebesgue measure. If we define now $y_n = \cos(\omega_n t)$, the statistical independence of y_n means that trajectory $y_n(t)$ is ergodic in N-cube $|y_n| \leq 1$. This is a consequence of ergodicity of the phase trajectory $\phi_n(t) = \omega_n t \mod 2\pi$ in N-cube $|\phi_n| \leq \pi$.

Statistical independence is a basic property of a set to which the probability theory is to be applied. Particularly, the sum of statistically independent quantities,

$$x(t) = \sum_{n}^{N} A_n \cdot \cos(\omega_n t + \phi_n), \tag{3.2}$$

which is motion with a discrete spectrum, is the main object of this theory. However, the familiar statistical properties such as Gaussian fluctuations, postulated (directly or indirectly) in TSM, are reached in the limit $N \to \infty$ only, which is called the *thermodynamical limit*. In TSM this limit corresponds to infinite-dimensional models (Kornfeld et al. (1982), Katok and Hasselblatt (1994)), which provide a very good approximation for macroscopic systems, both classical and quantal.

However, what is really necessary for good statistical properties of sum (3.2) is a large number of frequencies $N_\omega \to \infty$, which makes the discrete spectrum continuous (in the limit). In TSM the latter condition is satisfied by setting $N_\omega = N$. The same holds true for quantum fields which are infinite-dimensional. In quantum mechanics another mechanism, independent of N, works in the quasiclassical region $q \gg 1$ where $q = I/\hbar \equiv I$ is some big quantum parameter, e.g., quantum number, and I stands for a characteristic action of the system. Indeed, if the quantum motion (3.2) [with $\psi(t)$ instead of $x(t)$] is determined by many ($\sim q$) eigenstates we can set $N_\omega = q$ independent of N. The actual number of terms in expansion (3.2) depends, of course, on a particular state $\psi(t)$ under consideration. For example, if it is just an eigenstate the sum reduces to a single term. This corresponds to the special peculiar trajectories of classical chaotic motion whose total measure is *zero*. Similarly, in quantum mechanics $N_\omega \sim q$ for *most states* if the system is *classically chaotic*. This important condition was found to be certainly *sufficient* for good quantum statistical properties (see Chirikov et al. (1981), Chirikov et al. (1988) and Section 3.3 below). Whether it is also the necessary condition remains as yet unclear.

Thus, with respect to the mechanism of the quantum chaos we essentially *come back* to TSM with an exchange of the number of freedoms N for the quantum parameter q. However, in quantum mechanics we are not interested, unlike in TSM, in the limit $q \to \infty$, which is simply the classical mechanics. Here, the central problem is the statistical properties for *large but finite* q. This problem does not exist in TSM describing macroscopic systems. Thus, with an old mechanism the new phenomena were understood in quantum mechanics.

3.3 Characteristic Time Scales in Quantum Chaos

The existing ergodic theory is asymptotic in time, and hence contains no time scales at all. There are two reasons for this. One is technical: it is much simplier to derive the asymptotic relations than to obtain rigorous finite-time estimates. Another reason is more profound. All statements in the ergodic theory hold true up to measure zero, that is, excluding some peculiar nongeneric sets of zero measure. Even this minimal imperfection of the theory did not seem completely satisfactory but has been 'swallowed' eventually and is now commonly tolerated even among mathematicians, to say nothing about physicists. In a finite-time theory all these exceptions acquire a *small but finite* measure which would be already 'unbearable' (for mathematicians). Yet, there is a standard mathematical trick, to be discussed below, for avoiding both these difficulties.

The most important time scale t_R in quantum chaos is given by the general estimate

$$\ln t_R \sim \ln q, \qquad t_R \sim q^\alpha \sim \rho_0 \leq \rho_H, \tag{3.3}$$

where $\alpha \sim 1$ is a system-dependent parameter. This is called the *relaxation time scale* refering to one of the principal properties of chaos: *statistical relaxation* to some steady state (statistical equilibrium). The physical meaning of this scale is principally simple and is directly related to the fundamental uncertainty principle ($\Delta t \cdot \Delta E \sim 1$) as implemented in the second equation in (3.3), where ρ_H is the *full* average energy level density (also called the Heisenberg time). For $t \lesssim t_R$ the discrete spectrum is not resolved, and the statistical relaxation follows the classical (limiting) behavior. This is just the 'gap' in the ergodic theory (supplemented with the additional, time, dimension) where pseudochaos, particularly quantum chaos, dwells. A more accurate estimate relates t_R to a *part* ρ_0 of the level density. This is the density of the so-called *operative eigenstates*; that is, only those that are actually present in a particular quantum state ψ and actually control its dynamics.

The formal trick mentioned above is to consider not the finite-time relations we really need but rather the special *conditional limit* (cf. (2.22)):

$$t,\, q \to \infty \qquad \tau = \frac{t}{t_R(q)} = const \tag{3.4}$$

Quantity τ is a new rescaled time which is, of course, nonphysical but very helpful technically. The *double* limit (3.4) (unlike the single one $q \to \infty$) is *not* the classical mechanics which holds true, in this representation, for $\tau \lesssim 1$ and with respect to the statistical relaxation only. For $\tau \gtrsim 1$ the behavior becomes essentially quantum (even in the limit $q \to \infty$!) and is called nowadays *mesoscopic phenomena*. Particularly, the quantum steady state is quite different from the classical statistical equilibrium in that the former may be *localized* (under certain conditions) that is *nonergodic* in spite of classical ergodicity.

Another important difference is in *fluctuations*, which are also a characteristic property of chaotic behavior. In comparison with classical mechanics quantum $\psi(t)$ plays, in this respect, an intermediate role between the classical trajectory (exact or symbolic) with big relative fluctuations ~ 1 and the coarse-grained classical phase space density with no fluctuations at all. Unlike both the fluctuations of $\psi(t)$ are $\sim N_\omega^{-1/2}$, which are another manifistation of statistical independence, or *decoherence*, of even pure quantum state (3.2) in case of quantum chaos. In other words, chaotic $\psi(t)$ represents statistically a *finite ensemble* of $\sim N_\omega$ systems even though formally $\psi(t)$ describes a single system. Quantum fluctuations clearly demonstrate also the difference between physical time t and auxillary variable τ: in the double limit ($t,\, q \to \infty$) the fluctuations vanish and one needs a new trick to recover them.

The relaxation time scale should be distinguished from the *Poincaré recurrence time* $t_P \gg t_R$, which is typically much longer, and which sharply increases with a decrease in the recurrence domain. Time scale t_P characterizes big fluctuations (for both the classical trajectory, but not the phase space density, and quantum ψ) of which recurrences is a particular case. Unlike this, t_R describes the average relaxation process.

Stronger statistical properties than relaxation and fluctuations are related in the ergodic theory to the exponential instability of motion. Their importance for statistical mechanics is not completely clear. Nevertheless, in accordance with the correspondence principle, those stronger properties are also present in quantum chaos as well, but on a *much shorter* time scale,

$$t_r \sim \frac{\ln q}{h}, \tag{3.5}$$

where h is classical metric entropy (2.15). This time scale was discovered and partly explained by Berman and Zaslavsky (1978) (see also Chirikov et al. (1981), Chirikov et al. (1988), Casati and Chirikov (1995a)). Being very short, t_r grows indefinitely as $q \to \infty$.

The simplest example of quantum dynamics on this scale is the stretching/squeezing of an initially narrow wave packet, with the conservation of the phase space volume like in classical mechanics, followed by the packet inflation (increasing phase space volume), and eventually by the complete destruction of the packet, its splitting into many irregular subpackets (Casati and Chirikov (1995a)).

In a quasiclassical region ($q \gg 1$), $t_r \ll t_R$ (3.3). This leads to an interesting conclusion that the quantum diffusion and relaxation are *dynamically stable* contrary to the classical behavior. It suggests, in turn, that the motion instability is not important *during* statistical relaxation. However, the *foregoing* correlation decay on a short time scale t_r is *crucial* for the statistical properties of quantum dynamics.

3.4 Examples of Pseudochaos in Classical Mechanics

Pseudochaos is a new generic dynamical phenomenon missed in the ergodic theory. No doubt, the most important particular case of pseudochaos is quantum chaos. Nevertheless, pseudochaos occurs in classical mechanics as well. Here are a few examples of classical pseudochaos, which may help us to understand the physical nature of quantum chaos, my primary goal in this paper. Besides, this unveils new features of classical dynamics as well.

Linear waves is the example of pseudochaos (see, e.g., Chirikov (1992)) that is closest to quantum mechanics. I remind you that here only a part of quantum dynamics is discussed, the one described, e.g., by the Schrödinger equation, which is a linear wave equation. For this reason quantum chaos is sometimes called wave chaos (Šeba (1990)). Classical electromagnetic waves are used in laboratory experiments as a physical model for quantum chaos (Stöckmann and Stein (1990), Weidenmüller et al. (1992)). The 'classical' limit corresponds here to the geometrical 'optics', and the 'quantum' parameter $q = L/\lambda$ is the ratio of a characteristic size L of the system to the wave length λ.

The linear oscillator (many-dimensional) is a particular case of waves (without dispersion). A broad class of quantum systems can be reduced to this model (Eckhardt (1988)). Statistical properties of linear oscillators, particularly in the thermodynamic limit ($N \to \infty$), were studied by Bogolyubov (1945) in the framework of TSM. On the other hand, the theory of quantum chaos suggests richer behavior for a large but finite N, particularly, the characteristic time scales for the harmonic oscillator motion (Chirikov (1986)) and the number of degrees of freedom N playing the role of the 'quantum' parameter.

Completely integrable nonlinear systems also reveal pseudochaotic behavior. An example of statistical relaxation in the Toda lattice had been presented in Ford et al. (1973) much before the problem of quantum chaos arose. Moreover, the strongest statistical properties in the limit $N \to \infty$, including one equivalent to the exponential instability (the so-called K-property) were rigorously proved just for the (infinite) completely integrable systems (see Kornfeld et al. (1982), Katok and Hasselblatt (1994)).

The digital computer is a very specific classical dynamical system whose dynamics is extremely important in view of the ever increasing application in numerical experiments covering now all branches of science and beyond. The computer is an 'overquantized' system in that *any* quantity here is *discrete*, whereas in quantum mechanics only the product of two conjugated variables is. The 'quantum' parameter here is $q = M$, which is the largest computer integer, and the short time scale (3.5) is $t_r \sim \ln M$, which is the number of digits in the computer word (Chirikov et al. (1981), Chirikov et al. (1988)). Owing to the discreteness, any dynamical trajectory in the computer eventually becomes periodic, an effect well known in the theory and practice of the so-called pseudorandom number generators. One should take all necessary precautions to exclude this computer artifact in numerical experiments. On the mathematical part, the periodic approximations in dynamical systems are also studied in ergodic theory, apparently without any relation to pseudochaos in quantum mechanics or computers.

Computer pseudochaos is the best answer to those who refuse accept the quantum chaos as, at least, a kind of chaos, and who still insist that only the classical-like (asymptotic) chaos deserves this name, the same chaos that was (and is) studied to a large extent just on computers; that is, the chaos inferred from a pseudochaos!

4 Conclusion: Old Challenges and New Hopes

The discovery and understanding of the new surprising phenomenon—dynamical chaos—opened up new horizons in solving many other problems including some long-standing ones. Unlike in previous sections, here I can give only a preliminary consideration of possible new approaches to such problems, together with some plausible conjectures (see also Casati and Chirikov (1995a)).

Let us begin with the problem directly related to quantum dynamics, namely the quantum measurement or, to be more correct, the specific stage of the latter: ψ*-collapse*. This is just the part of quantum dynamics I bypassed above in the report on scientific results. This part still remains very vague to the extent that there is no common agreement even on the question of whether it is a real physical problem or an ill-posed one so that the Copenhagen interpretation of (or convention in) quantum mechanics gives satisfactory answers to all the *admissible* questions. In any event there exists as yet no dynamical description of the quantum measurement including ψ-collapse. The quantum measurement, as far as the result is concerned, is fundamentally a random process. However, there are good reasons to hope that this randomness can be interpreted as a particular manifestation of dynamical chaos (Cvitanović et al. (1992)).

The Copenhagen convention was (and still remains) very important as a phenomenological link between very specific quantum theory and laboratory experiments. Without this link studies of the microworld would be simply impossible. The Copenhagen philosophy perfectly matches the standard experimental setup of two measurements: the first one fixes the initial quantum state, and the second records the changes in the system. However, it is less clear how to deal with *natural processes* without any man-made measurements that is without the notorious *observer*. Since the beginning of quantum mechanics such a question has been considered ill-posed (meaning nasty). However, now there is a revival of interest in a deeper insight into this problem (see, e.g., Cvitanović et al. (1992)). Particularly, Gell-Mann and Hartle put a similar question, true, in the context of a very specific and global problem—the quantum birth of the Universe (Gell-Mann and Hartle (1989)). In my understanding, such a question arises as well in much simpler problems concerning any natural quantum processes. What is more important, the answer from Gell-Mann and Hartle (1989) does not seem satisfactory. Essentially, it is the substitution of the automaton (information gathering and utilizing system) for the standard human observer. Neither seems to be a generic construction in the microworld.

The theory of quantum chaos allows us to solve, at least (the simpler) half of the ψ-collapse problem. Indeed, the measurement device is by purpose a macroscopic system for which the classical description is a very good approximation. In such a system strong chaos with exponential instability is quite possible. The chaos in the classical measurment device is not only possible but unavoidable since the measurement system has to be, by purpose again, a highly unstable system where a microscopic intervention produces the macroscopic effect. The importance of chaos for the quantum measurement is that it destroys the coherence of the initial pure quantum state to be measured converting it into the incoherent mixture. In the present theories of quantum measurement this is described as the effect of external noise (see, e.g., Wheeler and Zureck (1983)). True, the noise is sufficient to destroy the quantum coherence, yet it is not necessary at all. Chaos theory allows us to get rid of the unsatisfactory effect of the external noise and to develop a purely dynamical theory for the loss of quantum coherence. Unfortunately, this is not yet the whole story. If we are satisfied with the *statistical* desciption of quantum dynamics (measurement including) then the decoherence is all we need. However, the *individual* behavior includes the second (main) part of ψ-collapse: namely, the *concentration* of ψ in a single state of the original superposition

$$\psi = \sum_n c_n \psi_n \to \psi_k, \qquad \sum_n |c_n|^2 = 1. \tag{4.1}$$

This is the proper ψ-collapse to be understood.

Also, it is another challenge to the correspondence principle. For quantum mechanics to be universal it must explain as well the very specific classical phenomenon of the *event* that does happen and remains for ever in the classical records, and is completely foreign to the proper quantum mechanics. It is just the effect of ψ-collapse.

All these problems could be resolved by a hypotetical phenomenon of *self-collapse*; that is, the collapse without any 'observer', human or automatic. Unfortunately, it seems that any physical explanation of ψ-collapse requires some changes in the existing quantum mechanics, and this is the main difficulty both technical and philosophical.

Now we come to the even more difficult problem of the *causality principle*: the universal time ordering of the events. This principle has been well confirmed by numerous experiments in all branches of physics. It is frequently used in the construction of various theories but, to my knowledge, no general relation of causality to the rest of physics was ever studied.

This principle looks like a statistical law (another time arrow), hence a new hope to understand the mechanism of causality via dynamical chaos. Yet, it directly enters the dynamics as the additional constraint on the interaction and/or the solutions of dynamical equations. A well-known and quite general example is in keeping the retarded solutions of a wave equation, only discarding advanced ones as 'nonphysical'. However, this is generally impossible for a bounded dynamics because of the boundary conditions. Still, causality holds true as well.

In some simple classical *dissipative* models, such as a driven damping oscillator, the dissipation was shown to imply causality (Youla et al. (1959), Dolph (1963), Zemanian (1965), Güttinger (1966), Nussenzveig (1972)). However, such results were formulated as the restriction on a class of systems showing causality rather than the foundations of the causality principle. Nevertheless, it was already some indication of a possible physical connection between dynamical causality and statistical behavior. To my knowledge, this connection was never studied further. To the contrary, the developement of the theory went the opposite way: taking for granted the causality to deduce all possible consequences, particularly various dispersion relations (Nussenzveig (1972)).

Causality relates two qualitatively different kinds of events: *causes* and *effects*. The former may be simply the initial conditions of motion, the point missed in the above-mentioned examples of the causality-dissipation relation. The initial conditions not only formally fix a particular trajectory but also are *arbitrary*, which is, perhaps, the key point in the causality problem. Also, this may shed some light on another puzzling peculiarity of *all* known dynamical laws: they discribe the motion up to arbitrary initial conditions only (cf. Weingartner (these proceedings)). It looks like the dynamical laws already include the causality implicitly even though they do not this explicitly. In any event, something arbitrary suggests chaos is around.

Again, we arrive at a tangle of interrelated problems. A plausible conjecture for how to resolve them might be as follows. An arbitrary cause indicates some statistical behavior, while the cause–effect relation points out a dynamical law. Then, we may conjecture that when the cause acts the transition from statistical to dynamical behavior occurs, which statistically separates the cause from the 'past' and dynamically fixes the effect in the 'future'. In this imaginary picture the 'past' and 'future' are related not to time but rather to cause and effect, respectively. Thus, the causality might be not time ordering (time arrow) but *cause–effect ordering*, or the *causality arrow*. The latter is very similar to the process arrow discussed in Section 2.2. Now, the central point is that the cause is arbitrary while the effect is not, whatever the time ordering.

This is, of course, but a raw guess to be developed, carefully analysed, and eventually confirmed or disproved experimentally.

Also, this picture seems to be closer to the statistical (secondary) dynamics [synergetics, or $S \supset D$ inclusion in (1.1)] rather than to dynamical chaos. Does it mean that the primary physical laws are statistical or, instead, that the chain of inclusions (1.1) is actually a closed ring with a 'feedback' coupling the secondary statistics to the primary dynamics?

We don't know.

In all this long lecture I have never given the definition of dynamical chaos, either classical or quantal, restricting myself to informal explanations (see Casati and Chirikov (1995a) for some current definitions of chaos). In a mathematical theory the definition of the main object of the theory precedes the results; in physics, expecially in new fields, it is quite often vice versa. First, one studies a new phenomenon such as dynamical chaos and only at a later stage, after understanding it sufficiently, we try to classify it, to find its proper place in the existing theories and eventually to choose the most reasonable definition. This time has not yet come.

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Comment on Boris Chirikov's Paper "Natural Laws and Human Prediction"

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I enjoyed all parts of this paper, but the part on which I should especially like to comment is the part dealing with the history of investigations of chaos in systems subject to the Hamiltonian equations of classical mechanics. I believe that this history has been described by Professor Chirikov, perhaps through modesty, in a way which does not fully bring out the importance of contributions to the study of chaos in such systems which were made by Professor Chirikov himself. Those classical authors which are cited in the paper, including especially Poincaré, had admittedly achieved an understanding of the possibilities of chaotic behaviour that may arise in Hamiltonian systems. On the other hand, their attempts at rigorous mathematical proof of the properties of such systems came up against some very severs difficulties. Necessarily, such proofs were attempted by means of perturbation theory, for sufficiently small departures from a regular (periodic-orbits) solution. Nevertheless, many formidable obstacles (including the famous "small divisors" problem, for example) opposed the development of their arguments into a "watertight" mathematical proof. Against this background, one of the vitally important contributions of the famous "KAM" papers of Kolmogorov (1954), Arnold (1963) and Moser (1962) referred to in section 2.5 of Professor Chirikov's paper was their success in overcoming all the obstacles, and in achieving a first rigorous demonstration, for sufficiently small values of a perturbation amplitude, of the properties of such classical systems.

Even in those regions of parameter space (involving e.g. near-coincidence of resonance frequencies) where the difficulties were most formidable, the KAM methods produced completely reliable results. As far as chaos was concerned, these results demonstrated beyond any doubt that it could arise in such a system. Nevertheless, they showed that regular behaviour of the system was enormously more common. Indeed, it was only in regions of parameter space whose total measure was of smaller order than any algebraic power of a perturbation amplitude that this regular behaviour was replaced by chaotic behaviour. The presence of those "microscopic" gaps in parameter space where chaotic behaviour could be shown to come about was of course of the greatest physical as well as mathematical interest. On the other hand, a group of "die-hard" mathematicians who had long argued that behaviour was an essentially unproven hypothesis could still claim that the demonstration of its absence except in a region of parameter space of such exceedingly small measure had at least identified it as just "a rarity". It has been against that background that the 1979 paper of Professor Chirikov (see Chirikov (1979)) has required to be seen as of

the utmost importance. By using computational methods of extreme precision to derive accurate numerical solutions for a Hamiltonian system, he was able first of all to verify for small amplitudes the transition from regular to chaotic behaviour in those extremely narrow regions of parameter space that are predicted by the KAM theory. There his methods were deriving identical results to those based upon a perturbation-theory approach. Then, he investigated what happened to those extremely narrow regions when the computations were carried out with progressively increasing perturbation amplitudes. It was above all this investigation which convinced the exponents of classical mechanics that chaos is not "a mere curiosity" - and, above all, not just "a rarity". On the contrary, as the perturbation amplitude increased, there appeared a steep widening of the regions of parameter space within which computed solutions exhibited the behaviour characteristic of chaotic systems. With a further increase of amplitude, chaotic behaviour from being exceedingly rare had become extremely normal. For many systems, furthermore, the computations indicated a transition to globally chaotic behaviour, sometimes called global stochasticity. Some other work at that time, being carried out independently in the USA by J.M. Greene (see Greene (1979)), was leading to rather similar conclusions, which have of course been strongly reinforced in many subsequent investigations. Nevertheless, it is no exaggeration for the friends of Professor Chirikov to claim, and moreover to wish to emphasize on an occasion like this, that it was his work above all which led to a full recognition of how, for conservative dynamical systems in classical mechanics, chaotic behaviour is the rule rather than the exception. In relation to the subject of this Symposium (the relation between knowledge of laws governing natural phenomena and the possibilities of prediction of those phenomena) this conclusion has, needless to say, proved to be of fundamental importance.

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Natural Laws and the Physics of Complex Systems Comments on the Report by B.Chirikov

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Abstract. In his report Professor Chirikov raises a number of important scientific and epistemological issues stemming from recent developments of chaos theory. In this short comment I would like to take up three items which I regard of special interest from the perspective of the implications of chaos research in different branches of science :

- The status of biological processes with respect to the laws of physics ;
- The status of the statistical description in general;
- The statistical properties of complex systems giving rise to bifurcations and chaos.

1 Biological processes and the laws of physics

This point, although present in the very title of the report, is subsequently only briefly treated in Section 1. Chirikov insists on the high specificity of the phenomenon of life. This is certainly true, but in this context it is worth drawing attention on the phenomenon of *self-organization* (Nicolis (1977)), whereby individual subunits achieve, through their cooperative interactions, states characterized by new, *emergent properties* transending the properties of their constitutive parts. Self-organization processes are ubiquitous in physics and chemistry, where large classes of systems obeying to nonlinear evolution laws and subjected to a constraint give rise spontaneously and under well-defined laboratory conditions to complex behavior in the form of abrupt transitions, a multiplicity of states, periodic or aperiodic oscillations, regular space patterning or spatio-temporal chaos. Many of these phenomena are observed in *in vitro* experiments on biochemical reactions. They also present appealing analogies with well-known manifestations of life such as biological rhythms, regulation at enzymatic level or at the level of the immune response, morphogenesis during embryonic development, or propagation of information through the nerve impulse (see, for instance Peliti (1991)). This poses the question of genericity and universality of life on a new basis and raises a number of concrete and challenging questions for future investigations.

2 Statistical description

In my view the inclusion of statistical laws into the dynamical ones or vice versa (Chirikov's eq. (1.1)) is not the real issue. At the deterministic level of description

the evolution laws (Chirikov's eq. (2.1))

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t) \tag{1}$$

can be embedded in phase space. Phase space density f obeys then to the Liouville equation

$$\frac{\partial f}{\partial t} = -div\ \mathbf{v} f = \hat{L} f \tag{2}$$

whose characteristics are, in turn, nothing but eqs. (1). This close correspondence of the two descriptions also holds for discrete time dynamical systems for which eq. (2) must be replaced by the Frobenius-Perron equation.

The real separation between deterministic and statistical views starts when the eigenvalue problem of $\hat{L}$ (or of the Frobenius-Perron operator $\hat{P}$) is addresed, since in this case one must specify the space of functions in which this problem is to be embedded. Depending on the smootness of the admissible functions one may derive, then, from the statistical description properties that were not built in an obvious manner into the deterministic description. A concrete example will be mentioned in Sec. 3 of this comment.

In many instances there exists an additional motivation for undertaking a statistical description. Macroscopic systems are usually coupled to a complex environment inflicting on them a variety of perturbations, which in many instances can be assimilated to an (external) *noise* process. In addition they themselves generate spontaneously variablity resembling in many respects to a noise process - the thermodynamic *fluctuations*. To account for these phenomena eqs. (1) must be augmented and one is led to a Langevin-type dynamics (Nicolis (1977), Gardiner (1983))

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t) + \mathbf{F}(\mathbf{x}, t) \tag{3}$$

where $\mathbf{F}$ is the *random force*. Eqs (3) have been analyzed in detail in the literature in the double limit of weak, white noise. As it turns out, the deterministic description is recovered as the most probable path of the full stochastic process. In this special sense the stochastic description would therefore appear to contain the deterministic one as mentioned briefly by Chirikov in his Sec. 1, although I do not see what "synergetics" has to do with this particular point. Still, some additional comments are in order :

- Basically fluctuations are nothing but deterministic chaos in the high-dimensional ($N \sim 10^{23}$) phase space of a macroscopic system. Eqs. (3) are therefore a shortcut to a full-fledged Liouville equation approach.
- In deriving the properties of the random force $\mathbf{F}$ use has been made of the deterministic properties, notably through the fluctuation-dissipation theorem (Callen and Welton (1951)).

3 Statistical properties of complex systems

In the last years the full solution of the eigenvalue problem of the Liouville equation associated to systems giving rise to bifurcations and chaos has been achieved. The most transparent case is that of *dissipative* systems, for which even 1-dimensional dynamics generates complexity resembling the one one is accustomed to find in many-body systems of interest in statistical mechanics. A simple example is provided by the discrete time chaotic map

$$x_{n+1} = rx_n \quad (mod\ 1), \quad r > 1 \tag{4}$$

for which a full spectral decomposition of the Frobenius-Perron operator has been obtained. As it turns out (Gaspard (1992), Antoniou and Tasaki (1993)):

- for $r = 2$, the eigenvalues are

$$S_k = 2^{-k} \quad k = 0, 1, 2, \ldots \tag{5}$$

  The spectrum is thus discrete, even though chaos is an aperiodic process. This is at variance with the statement made by Chirikov in his Sec. 2.2.
- The right eigenfunctions are the Bernoulli polynomials while the left ones are δ-functions and derivatives thereof (notice that $\hat{P}$ is not self-adjoint here).

Similar analysis has been carried out for continuous time dynamical systems undergoing pitchfork bifurcation. It has been shown that at the bifurcation point the spectrum of the Liouville operator becomes continuous, but remains discrete and confined to the negative real axis before and after bifurcation. Furthermore the symmetry-breaking character of the pitchfork bifurcation shows up through the appearance of degeneracies in the spectrum in the critical and post-critical cases (Gaspard et al. (1995)).

Common to both studies mentioned above is the observation that the probabilistic description is stable in the sense that the probability density is driven irreversibly to its invariant form. This is to be contrasted with the instability of motion inherent in the deterministic prediction.

In conlusion the connection between statistical and deterministic description is quite intricate indeed. For certain (perhaps even for most) types of our predictions statistical description is operationally more meaningful, since it reflects the finite precision of measurement process and bypasses the fundamental limitations associated with the instability of motion.

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Discussion of Boris Chirikov's Paper

Batterman, Chirkov, Noyes, Schurz, Suppes, Weingartner

Noyes: I would really like to hear what you have to say about the wave function collapse. This is a problem I have thought quite a bit about. I'd like to know what kind of a position you take on it. You did not have much time to mention it in your talk so I would like you to say something about it now.

Chirikov: I actually have no beforehand solution of this problem, I simply see that there is such a problem and it would be interesting to solve it somehow. But this is a very subtle problem, a very old one, from the beginning of quantum mechanics. And so the main question you need to solve for yourself or try to convince other people is whether this ψ-collapse is a real physical problem. In other words: is it a physical problem or a philosophical problem? You know that there is no such problem in the common (Copenhagen) interpretation of quantum mechanics. Rather, it is a convention necessary to do real research in quantum physics. You need to understand how to relate this ψ to a result of experiment, and how to interpret the experiment and derive a particular fundamental law of physics. In my opinion, I don't know an answer of course and cannot make any strong argument in support, but nevertheless my opinion is that it might be a physical problem. You should distinguish two types of physical processes. One is what you have in your laboratory. You have a particular device and you fix the initial conditions-which is not your immediate physical problem. The problem of initial conditions, also very interesting is another part of physics because you choose certain initial conditions by the quantum measurement. Anyway, you make some complete quantum measurement which fixes the ψ-function exactly. Then, to study something you make another measurement, and from the statistical results you derive some fundamental law. This is O.K.: all you need to know is that the modulus of the ψ-function squared is the probability of particular results. This was very important at the beginning of quantum mechanics to have a clear idea how to interpret experimental results, and how to recalculate from them a fundamental law of quantum interaction. But you may consider a different type of processes, I would say. Something just happens around, and nobody is interested to measure what has happened. But something very important does happen, for example, the first living molecule does appear. How would you describe this? It is not clear. You need this ψ-collapse without special measurement in the usual sense. Some physicists, including myself, think that it must be something which might be called the self-collapse that is a collapse without special measurement, something that from time to time produces the event. In the standard quantum mechanics there is no such conception. You have probabilities of everything but nothing happens. But many things do happen, and the problem is how to understand it from quantum mechanics? It seems to me that we need the mechanism of such a self-collapse, some dynamical

theory of the ψ-collapse. Again there are two different situations. In many cases you are satisfied with the statistical description of events. So, all you need is decoherence of the ψ-function. You begin with a pure state, not a mixture. One statistical effect of the ψ-collapse is in that you obtain an incoherent mixture of probabilities instead of the coherent superposition of probability amplitudes. This problem is solved in the theory of dynamical chaos. The principal result of this theory - statistical relaxation - means a general decoherence. Even the simple diffusion implies already decorrelation and decoherence, otherwise you would not have this characteristic linear dependence on time of second moment of distribution function. So, principally the problem of quantum decoherence is solved. Of course, it might be very difficult technically but principally it is solved. What remains unsolved is if you are not satisfied with the statistical output of the process but are interested in individual events, for example, the living molecule with the particular chirality, by the way, for some reason. Then the decoherence is not sufficient. You need to describe or to find a mechanism what is called the probability redistribution. It means that not only the different initial states in superposition become uncorrelated, but all the probability goes to a particular state in a particular event. This is, of course, a much more difficult problem to be solved. There are some attempts to find how it may happen but, of course, they do not rely upon the decoherence within the existing quantum mechanics. Particularly, the quantum chaos is a part of quantum mechanics, it is nothing beside the Schrödinger equation, a special solution of this equation. This redistribution of probabilities, if it is a real process which is the question at the moment, if there is no other explanation of the whole problem, requires unfortunately (or fortunately, I don't know) some changes in quantum mechanics because the Schrdinger equation does not describe this. And it this the main difficulty. So what is your opinion?

Noyes: For me, this problem can only be discussed in the framework of a physical cosmology which sets the boundary conditions in such a way that they are not arbitrary. My own cosmological model does just this in a way that is briefly discussed in Chapter 5 of my contribution to this conference. However, for the problem of the origin of biomolecular chirality which you mentioned, my cosmology coincides with the conventional view that earth-type planets are formed from the debris of supernovae, and hence are formed in a specific and necessarily chiral environment. This idea is due to Ed Rubenstein and is discussed briefly in "Supernovae and Life" by E. Rubenstein, W.A. Bonner, H.P. Noyes and G.S. Brown in *Nature* **306**, 118 (1983). When a star goes supernova it leaves behind a neutron star which traps the magnetic field and can be detected observationally as a pulsar. Out to the radius where rigid body motion would exceed the velocity of light, the ionized plasma is locked into this rotating magnetic field and emits synchrotron radiation. This chiral radiation has opposite chirality above and below the plane of rotation. Bill Bonner has shown that 200 nanometer chiral radiation, which is plentiful in the pulsar spectrum, decomposes a racemic mixture of leucine molecules leaving behind a 4% enatiomeric excess of left- or right- handed molecules depending on the chirality of the radiation. Thus any organic molecules in the interstellar medium from which the planets are formed necessarily are handed from the start. We now know that interstellar organic molecules cling to interstellar dust grains in forms that support this mechanism for the production of pre-planetary and planetary biomolecular chirality. Thus this particular problem is solved by embedding it in a well understood cosmological evolutionary scenario. This moves the statistical problem from the quantum mechanical to the macroscopic level, which is part of the problem my contribution to this conference addresses. No "observer" is needed.

Chirikov: I agree, perhaps, it is not the best example. Nevertheless, we need not only statistical results, but also individual ones to understand the events around. As to the cosmology, perhaps you know that Gell-Mann has studied recently this problem in cosmology, I give you a reference. The problem is the quantum birth of the universe. And the question for him is: who was the observer at that time? So he tried to develop a kind of automatic observer, a very particular type of information system. But, in my opinion, you don't need to study such a great problem to understand this. The electron diffraction on two slits is quite sufficient to understand all these difficulties, and to find a solution.

Weingartner: I have two other questions: The one concerns that example of you with the comet Halley. And you said that this just is a chaotic behavior. Now my question is whether this behaviour is chaotic only in the sense that it is a simple bifurcation that it is oscillating this way or does the comet break out from the plane of the ellipse?

Chirikov: All this is known more or less in some detail because the comet Halley is not only a famous event but the only comet for which the most information was updated during many years. So, it is most simple to calculate its trajectory and everything not only presently but over 2000 years or so. And then it was found from numerical simulation, very simple by the way, that the orbit is chaotic. It is within a chaotic component of motion in the sense that everything, period, excentricity, inclination, fluctuate chaotically. So, for example, the period and semi-major axis are diffusing and, moreover, in both directions of time. One interesting thing is that we are interested not so much in forward diffusion because this comet has no future. It will simply disappear, melt and evaporate. What is much more interesting - the diffusion backward in time because, then, you can estimate how long it is within the solar system. The answer - 10 million years - is not clear how to interprete. This is very small compared to the cosmological scale, and so you need to understand the origin of, at least, this particular comet: where it came from, how it appeared within the solar system. Maybe not all of you know that recently it was found that not only particular parts of the solar system like comets and asteroids are chaotic but the whole solar system is also chaotic. Planetary motion is chaotic. But we are not in immediate danger because even the instability scale involving Lyapunov's exponent is about 5 million years, so we have enough time!

Weingartner: When you mentioned this exponential dependency, Lyapunov exponent then, you mentioned also that this is a sign not only for the diverging adjacent points for instance, but also for a loss of information. I always thought that this could be interpreted with Shannon's definition of information but you mentioned that it is not like this. Do you have a reason why you cannot interpret it that way?

Chirikov: You must understand me: I am not an expert in this mathematical theory. But what I actually mean, Shannon's conception is the statistical information for a given ensemble. It is calculated via the distribution function, for example, of many trajectories. Suppose, that within this ensemble there is a symbolic trajectory: 0,0,0,... The statistical definition would not distinguish it from any other trajectory but it is obvious that the former has not the average information, its information is zero. Kolmogorov and other researchers developed a new conception - the information on an individual trajectory. And this information can be interpreted in two opposite ways. One is how much information you need to predict. You need some information from somewhere if you want to predict. And since this is an information flow, information per unit time, you cannot do prediction with any finite algorithm, for example. You need a permanent flow of information from the observation or whatever. Another way is how much information you can obtain if you follow the trajectory, record it. Particularly, you can obtain the information about the initial conditions, actual initial conditions, because in this picture, in this theory you assume the trajectory itself is exact, it is not a beam of trajectories, not a distribution function. You have a single trajectory.

Batterman: I have a question about what you were saying about the correspondence principle. One of your transparencies, you mentioned something called "conditional chaos". I was just wondering whether by that you mean what is sometimes called "quantum pseudo-chaos" or whether you mean something else.

Chirikov: Conditional chaos is the chaos which arises under certain special conditions. On both sides of this transparency there are the unconditional statements: never chaos, and always chaos. But the correspondence principle requires quantum chaos in some sense, some quantum chaos, if and only if there is chaos in the classical limit. Another way to describe this situation is the conditional double limit which is, perhaps, not a common term. You have several limits and take them simultaneously but under a certain special relation between the variables.

Batterman: That raises an interesting questions about the view that quantum mechanics is the true and fundamental theory, and that classical mechanics is completely replaced or superseded. Because, it seems to me that one needs to make sense of the notion that quantum chaos is conditional upon what happens in the classical phase space. At least in the quasi-classical limit, it looks like reference to classical structures are necessary to explain or account for certain apparently quantum mechanical phenomena (such as the statistics of spectra). If this is so, then in what sense is quantum mechanics basic? How has it superseded classical mechanics? Why shouldn't one say that quantum mechanics is, in part, dependent upon classical mechanics?

Chirikov: This is one of the most deep questions. The other way around, I never considered the possibility that quantum mechanics would be not the basic theory. Even though there are such theories which try to derive the quantum properties from classical ones.

Batterman: Right. Using Maslov's techniques and so on.

Chirikov: No, no. This is technical. When I say that there is a single mechanics I mean that generally everything must be described or can be described by the quantum equations while the classical mechanics is an approximation, the limiting case. When you said the way around, you meant that everything can be described by Newtonian classical equations? There are such theories: Bohm's theory, and some versions of that. I don't believe at all in this possibility, for me it is not a question. But there is another question: do you have two separate mechanics? I don't know. Quantum mechanics for micro-world is not necessarily strictly related to the classical mechanics of the macro-world. This essentially is the difficulty of the statement that the correspondence principle fails. Some people say this. But they should understand the implications of the statement on two different mechanics. You can take such point of view but then there is a difficulty (I never thought about it very much because my preference is different): where is the borderline and how to divide the world between these two mechanics. If they are separate we need to divide. Instead, you may try to take the point of view that only quantum mechanics is fundamental. That does not mean you cannot use the classical mechanics, it is a perfect approximation in the macro-world but not in fundamental problems. Everything should have quantum explanation principally. If you cannot find one or the quantum theory gives you a different result as, for example, it may seem in the dynamical chaos, a very unusual and relatively new phenomenon, then there is a problem. Now, my statement, not opinion but statement, is: so far we have no contradiction with the idea that there is only quantum mechanics. So far I don't know any contradiction. We may find one later on but so far no contradiction exists, and in this sense I said that the correspondence principle was confirmed.

Schurz: Did I understand you right that already in classical mechanics you have two possibilities of representation, via trajectories and via phase density. And you said the representation via a phase density is always linear.

Chririkov: The equation is linear.

Schurz: Yes. This happens also in classical mechanics. So, the question of getting chaos from non-chaos could be studied already in classical mechanics because if you use a linear phase density equation in classical mechanics it should not be chaotic. If you describe it by trajectories you have a system which behaves chaotic. But if you describe it via phase density the equation is linear and it does not exhibit chaotic behavior.

Chirikov: It does!

Schurz: But it is linear. You said a necessary condition for chaos is non-linearity.

Chirikov: No, for trajectory equation.

Schurz: How does such a linear phase density equation produce chaos in classical mechanics?

Chirikov: The question is how to imagine this because, first of all, this is a rigorous result, both types of description are completely equivalent. The situation

is the following. Sometimes people say: quantum equations are linear, hence, the quantum chaos is not classical. This is a completely wrong statement because we can refer to the Liouville equation. So, the question is not whether the equation is linear or nonlinear but what kind? There are different kinds of linear equations with qualitatively different properties. Now about the relation of nonlinearity to chaos. There are different definitions of chaos we discussed yesterday in relation to the motion stability. The most common definition is related to the exponential instability. Now, the exponential instability is a property of the linearized equations of motion, by the way. Generally, you don't need nonlinear equations. Nevertheless, we do need some nonlinearity even in terms of the linear equations for Lyapunov exponent or in terms of linear Liouville equation. What is the role of nonlinearity then? To make the motion bounded. Because if it is unbounded then unstable motion is not necessarily chaotic. If you have simple exponential instability, you would never call it chaotic. Why? It is just an explosion of trajectories. So, from this point of view nonlinearity simply restricts the motion to a finite phase volume, makes it bounded in phase space. Then, in combination with instability, you obtain the mixing of trajectories. This is a graphical view of the chaos mechanism. Now, another interesting question: what means exponential instability in terms of wave or Liouville equation? It depends in which space you consider the instability. If you consider the space of density itself, slightly change the distribution function or ψ-function, then the difference is described by the same equation and you have no instability at all. Nevertheless, in Schrödinger equation, in classically chaotic case, there is a relatively short time interval when the quantum motion is exponentially unstable if you do not use the Hilbert space but the phase space of classical mechanics. Consider, for example, two narrow wave packets or two distribution functions very close to each other. Because of the instability each of them is spreading, and also they diverge from each other very quickly. But why is it different? Because in terms of linear equation it is not a small change of the wave function: you have one wave packet and then another one. Let me show briefly a picture how it looks in a quantum system, on a simple model. This is phase space and this is the initial wave packet, the so-called coherent state, which is the most narrow wave packet. You see the wave packet is spreading very quickly as under the classical exponential instability. And then, after three steps of the map, everything is destroyed, and there is no more relation to the classical picture. Instead, you see wild fluctuations of the wave function but nevertheless during a much longer time the diffusion and relaxation still remain classical even though the ψ-function itself has almost no resemblance to the classical distribution.

Noyes: Just a quick comment on this question of correspondence principle. This is a question which is being investigated empirically by Tony Leggett at the University of Illinois. He asks whether a system containing 10^{15} atoms still behaves as a quantum system, or if for such a large number quantum coherence has to disappear. The system he uses is two superconducting flux loops ("SQUID's") can be shown to contain either zero or one flux quantum. By coupling them through a Josephson junction and insuring that the system contains a single flux quantum in one loop and none in the other, one can then ask whether the "tunneling" of this quantum state from one macroscopic loop to the other fits the quantum mechanical prediction or not. If it does not, he will have discovered a complexity parameter which could be interpreted as showing where the transition from quantum to classical occurs. Then there would have to be a new theory which might show that there actually is a classical regime. Despite the success of the usual assumption that quantum mechanics rather than classical physics *has* to be the fundamental theory, it's an open question from the point of view of experiment.

Chirikov: I agree. Such a question you would never answer completely. We have a common solution and you must be ready to change this solution and not necessarily to follow your preferences. I would like to mention that the phenomenon you spoke about is a particular case in the very intensively studied field. Now it is called mesoscopics. Maybe you heard the word: mesoscopic is something intermediate. But what people have in mind is that you may be very far in the quasi-classical region, with quantum numbers arbitrarily large, but nevertheless, under some additional conditions, the behavior may be essentially quantum. This is called mesoscopic phenomena. Of course, the extreme case of this is well known since long ago: superfluidity and superconductivity. Mesoscopic phenomena are called also the intermediate asymptotics. It means that the quantum numbers may be arbitrarily large but still it is not the final answer for the correspondence principle, you must go further and you will reach the quantum chaos, it is a theorem. But the example you mentioned is more complicated and more interesting.

Suppes: I'd like to use the privilege of the chair to ask one quick question. In your example a minute ago of the two wave functions that separate exponentially, if you take the expectations of the wave functions then you get a path trajectory. Are those two paths classically scar-paths - in the language of quantum optics, are those expected paths chaotic in the classical sense?

Chirikov: You mean some average in the spirit of the Ehrenfest theorem or something like this? No. Unfortunately, I had no time. But I mentioned that there are characteristic time scales of quantum motion. The most important one I mentioned is the scale on which classical diffusion and relaxation proceed. But there is another one, very short, proportional only to logarithm of quantum number or of Planck's constant on which initially narrow wave packet remains relatively narrow and simply follows the classical trajectory. So, if this trajectory is random, then the motion of the packet on this time scale is equally random. But it terminates because of the spreading of the wave packet and of its eventual destruction I showed. Then, you can no longer follow the wave packet as you have instead a very complicated structure of ψ-function but, nevertheless, the classical diffusion still persists.