

## How Many “Arrows of Time” Do We Really Need to Comprehend Statistical Laws?

In a recent paper [1] Schulman claims to exhibit “a model in which two weakly coupled systems maintain opposite running thermodynamic arrows of time.” The main purpose of this Comment is to point out that such interpretation of the well known and well understood dynamics of this elementary model [1] is neither necessary nor acceptable. In fact, the process shown in Fig. 1 is simply a big fluctuation out of the equilibrium steady state, quite rare and relatively short, but perfectly regular within the stationary small fluctuations. To see this it is sufficient to slightly extend the time evolution of the model in both directions of time beyond the artificial restriction imposed by the author. In doing this, one immediately sees a quick relaxation to the steady state in both directions of time in agreement with the well developed theory of dynamical systems (see, e.g., Ref. [2]). Notice that in the interpretation [1] such a longer run would imply a U-turn of both “arrows of time.”

Moreover, if one extends the dynamical evolution to much longer times, fluctuations of the above type, as well as any others, would spontaneously arise and decay as many times as one can run the system. The above qualitative picture does not depend on the coupling or other details of the model, nor even on any external noise. What is even more important, the fluctuations themselves do not depend on the very specific initial (or initial and final) conditions imposed in [1] by an external to the system observer—the Supervisor who is supposed to freely create whatever He likes, e.g., the system, the initial conditions, etc. Needless to say, no observer, communication, signaling, and the like can be included in the model [1].

It is perhaps appropriate to remark that the presence of a big and regular fluctuation as described by one of the two parts (*A* or *B*) in [1], was conjectured already by Boltzmann as the basis of his fluctuation hypothesis for the Universe. Again, as is well known by now, such a hypothesis is incompatible with the present structure of

the Universe, as it would immediately imply the notorious “heat death” (see, e.g., Ref. [3]). The origin of the latter is the existence of a stable steady state (statistical equilibrium) which is the case in the model [1] but not in our Universe.

Our final remark concerns the relation of the “two-arrows of time scenario,” originally speculated upon by Boltzmann (and much later by Wiener also), with the recent “strange attraction” to an old misconception on the notorious (single) arrow of time (see, e.g., “Round Table on Irreversibility” in [4]). To us, the “two-arrow conception,” or “2-time formulation” in [1] does not help at all to clear out the too-long-standing confusion around (ir)reversibility of statistical laws. On the contrary, it further obscures this issue which, on the other hand, is quite clear and simple. In actual running trajectories of his 2-time model, the author [1] does prefer the standard dynamics with a single time, and the initial conditions at  $t = 0$ .

In the very conclusion, our answer to the question in the title of this Comment is certainly none.

G. Casati

Universita' degli Studi dell' Insubria  
Via Castelnuovo, 7  
22100 Como, Italy

B. V. Chirikov and O. V. Zhirov

Budker Institute of Nuclear Physics  
630090 Novosibirsk, Russia

Received 18 February 2000

PACS numbers: 05.70.Ln, 05.20.-y, 95.35.+d

- [1] L. Schulman, *Phys. Rev. Lett.* **83**, 5419 (1999).
- [2] I.P. Kornfeld, S.V. Fomin, and Ya.G. Sinai, *Ergodic Theory* (Springer, New York, 1982).
- [3] L.D. Landau and E.M. Lifshitz, *Statistical Physics* (Pergamon Press, New York, 1980).
- [4] *Proceedings of the 20th IUPAP International Conference on Statistical Physics, Paris, 1998* [*Physica* (Amsterdam) **263A**, 516 (1999)].