**Chirikov and Shepelyansky Reply:** The authors of the Comment [1] question the existence of universal power-law decay of Poincaré recurrences discussed in our Letter [2] for generic Hamiltonian systems. There are two aspects in their query:

The first one states that the asymptotic decay $P(\tau) \propto 1/\tau^3$, which is expected from the scaling theory of universal phase-space structure in the vicinity of critical golden curve (see Refs. [1,2,12–18] in [2]), is not valid. Such a conclusion is based on Fig. 1 in [1] obtained from numerical simulations of the standard map with the total number of map iterations $N_{\rm tot}$ being only by a factor 10 larger than for the similar case of Fig. 2 in [2] where $N_{\rm tot} = 10^{12}$. New data for $8 < \log\tau \leq 9$ indeed demonstrate noticeable deviations from the theoretical estimates for $P(\tau)$ obtained from the exit times $\tau_n$ from golden resonance scales $r_n$ (Fig. 1 in [2]). These deviations are rather intriguing. Indeed, the local properties of critical golden curve are known to be self-similar and universal (e.g., phase-space structure, size of resonances, and local diffusion rate; see, e.g., Refs. [2,15] in [2]). Moreover, the exit times $\tau_n$ found in [2] for a first time, give an example of a nonlocal characteristic which follows the universal scaling law. The arguments based on this scaling lead to the asymptotic decay $P(\tau) \propto 1/\tau^3$ which, however, can start after extremely long times $\tau > \tau_a$. In [2] we have shown that at least $\tau_a > \tau_g \approx 2 \times 10^5$ and it is not excluded that this time scale is still much longer [3], and is not yet reached even in the simulations presented in [1].

If to assume this then there is an interesting possibility to see if $P(\tau)$ would have some universal properties on the presently available (intermediate) time scales ($\tau \leq 10^9$). This is the second aspect of the query on which the authors of the Comment tend to give a negative answer. To this end we show in Fig. 1 an example of another map with critical golden curve which power-law decay $P(\tau) \propto 1/\tau^p$ has the exponent $p \approx 1.5$. This average value of exponent $p$ in the range $10^2 < \tau < 10^8$ agrees, indeed, with that of the decay in the standard map in the range $10^5 < \tau < 10^9$ shown in [1]. There is also a clear similarity for the decay of $P(\tau)$ in two maps (see Fig. 1). For other values of parameter $\lambda$ the decay $P(\tau)$ in the separatrix map is no longer a simple power law but rather some irregular oscillations around that which represent a peculiar phenomenon of the so-called renormalization chaos (Ref. [12] in [2]). However, averaging over several $\lambda$ values smooths away the signs of renormalization chaos and reveals the underlying picture of the power-law distribution with the same exponent $p \approx 1.5$ (see Ref. [4] and Ref. [9] in [2]). Certainly, further studies are required for the understanding of the origin of this intermediate asymptotics and for the estimates of the time scale $\tau_a$ after which a transition to $p = 3$ is expected.

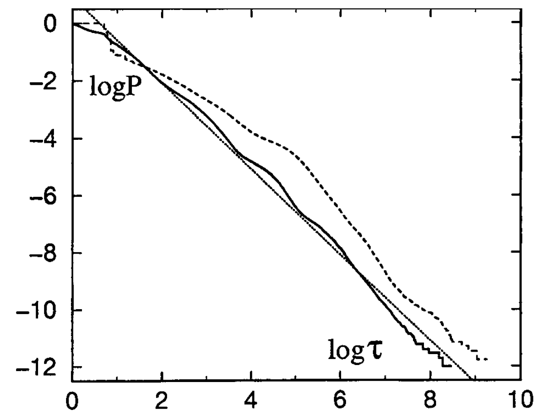


FIG. 1. Poincaré recurrences $P(\tau)$ in the separatrix map ($\bar{y} = y + \sin x$, $\bar{x} = x - \lambda \ln|\bar{y}|$) with critical golden boundary curve at $\lambda = 3.1819316$ (return line $y = 0$, average return time $\langle\tau\rangle \approx 10$). Data are obtained from 10 orbits computed for $10^{12}$ map iterations (solid curve). Dashed curve shows $P(\tau)$ for the standard map at $K = K_g$ (data of the lower curve of Fig. 1 in [1]). The dotted straight line shows the power-law decay $P(\tau) \propto 1/\tau^p$ with $p = 1.5$; logarithms are decimal.

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Received 30 July 2002; published 14 November 2002
DOI: 10.1103/PhysRevLett.89.239402
PACS numbers: 05.45.Mt

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