To the problem of Poincaré recurrences in generic Hamiltonian systems

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We discuss the problem of Poincaré recurrences in area-preserving maps and the universality of their decay at long times. The work is related to to the results presented in Refs. [1,2].

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The authors of Comment [1] question the existence of universal power-law decay of Poincaré recurrences discussed in our Letter [2] for generic Hamiltonian systems. There are two aspects in their quarry:

The first one states that the asymptotic decay $P(\tau) \propto 1/\tau^3$, which is expected from the scaling theory of universal phase-space structure in the vicinity of critical golden curve (see Refs. 1-2, 12-18 in [2]), is not valid. Such a conclusion is based on Fig. 1 in [1] obtained from numerical simulations of the standard map with the total number of map iterations N_{tot} being only by a factor 10 larger than for the similar case of Fig. 2 in [2] where $N_{tot} = 10^{12}$. New data for $8 < \log \tau \leq 9$ indeed demonstrate noticeable deviations from the theoretical estimates for $P(\tau)$ obtained from the exit times τ_n from golden resonance scales r_n (Fig. 1 in [2]). These deviations are rather intriguing. Indeed, the local properties of critical golden curve are known to be self-similar and universal (e.g. phase-space structure, size of resonances and local diffusion rate, see e.g. Refs. 2, 15 in [2]). Moreover, the exit times τ_n, found in [2] for a first time, give an example of a non-local characteristic which follows the universal scaling law. The arguments based on this scaling lead to the asymptotic decay $P(\tau) \propto 1/\tau^3$ which however can start after extremely long times $\tau > \tau_a$. In [2] we have shown that at least $\tau_a > \tau_g \approx 2 \times 10^5$ and it is not excluded that this time scale is still much longer [3], and is not yet reached even in the simulations presented in [1].

If to assume this then there is an interesting possibility to see if $P(\tau)$ would have some universal properties on the presently available (intermediate) time scales ($\tau \leq 10^9$). This is the second aspect of the quarry on which the authors of the Comment tend to give a negative answer. To this end we show in Fig. 1 an example of another map with critical golden curve which power-law decay $P(\tau) \propto 1/\tau^p$ has the exponent $p \approx 1.5$. This average value of exponent p in the range $10^2 < \tau < 10^8$ agrees, indeed, with that of the decay in the standard map in the range $10^5 < \tau < 10^9$ shown in [1]. There is also a clear similarity for the decay of $P(\tau)$ in two maps (see Fig. 1). For other values of parameter λ the decay $P(\tau)$ in the separatrix map is no longer a simple power law but rather some irregular oscillations around that which represent a peculiar phenomenon of the so-called renormalization chaos (Ref. 12 in [2]). However, averaging over several λ values smoothes away the signs of renormalization chaos and reveals the underlying picture of the power-law distribution with the same exponent $p \approx 1.5$ (see Ref. 9 in [2] and [4]). Certainly, further studies are required for the understanding of the origin of this intermediate asymptotics and for the estimates of the time scale τ_a after which a transition to $p = 3$ is expected.

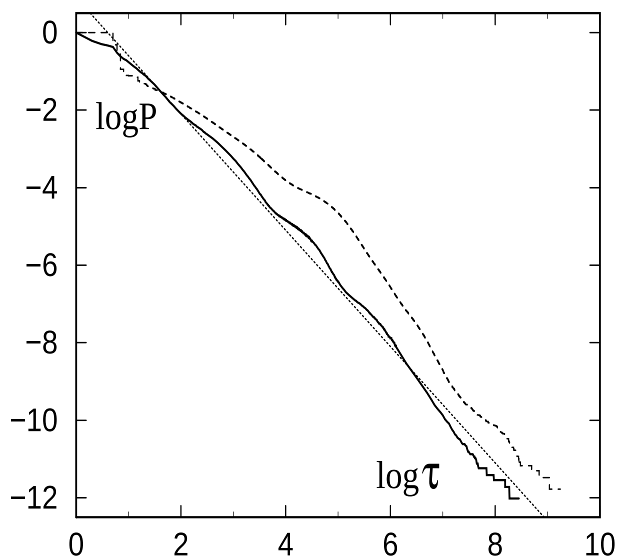

FIG. 1. Poincaré recurrences $P(\tau)$ in the separatrix map ($\bar{y} = y + \sin x, \bar{x} = x - \lambda \ln |\bar{y}|$) with critical golden boundary curve at $\lambda = 3.1819316$ (return line $y = 0$, average return time $< \tau > \approx 10$). Data are obtained from 10 orbits computed for 10^{12} map iterations (solid curve). Dashed curve shows $P(\tau)$ for the standard map at $K = K_g$ (data of the lower curve of Fig. 1 in [1]). The dotted straight line shows the power-law decay $P(\tau) \propto 1/\tau^p$ with $p = 1.5$; logarithms are decimal.

[1] M. Weiss, L. Hufnagel, and R. Ketzmerick, Comment submitted to Phys. Rev. Lett., see also nlin.CD/0106021.
[2] B. V.Chirikov, and D. L.Shepelyansky, Phys. Rev. Lett **82** 528 (1999) (cond-mat/9807365).

[3] The measure of a sticking region is of the order of $\mu(\tau) \sim \tau P(\tau)/ < \tau > \sim 1/q^2$ and therefore even for $\tau \sim 10^9$ (Fig. 1 in [1]) only resonances with $q \sim 100$ are reachable for an orbit.
[4] B. V.Chirikov, and D. L.Shepelyansky, Phys. Rev. Lett **61** 1039 (1988).