

Boris Chirikov - Legislator of Chaos

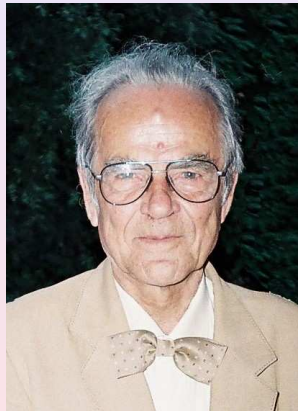


Dima Shepelyansky (CNRS, Toulouse)

www.quantware.ups-tlse.fr/dima

It is not so much important to be rigorous as to be right

A.N.Kolmogorov (from Chirikov Phys Rep (1979))



1928 Orel – 2008 Akademgorodok; MFTI-1952, arrived to Novosibirsk in 1958 with Budker IYaF ~ 1972 (left), at his 70th Toulouse (right)

Loschmidt - Boltzmann dispute on time reversibility (1876-1877)

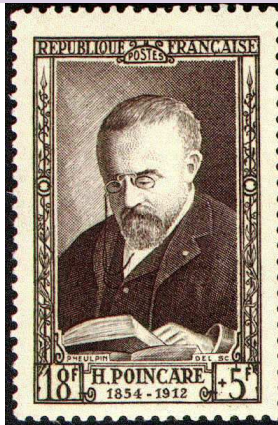
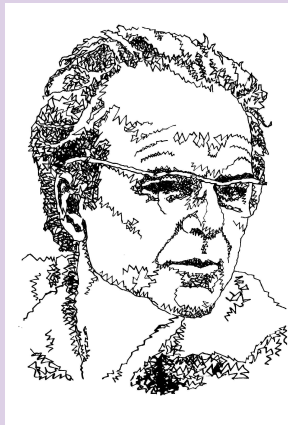
- * irreversible statistical laws from reversible dynamical equations



Boltzmann (1872), Loschmidt (1876), Boltzmann (1877)

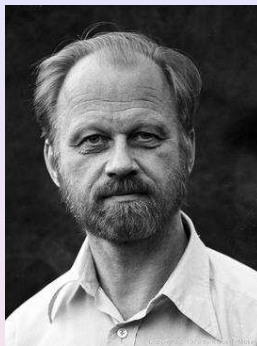
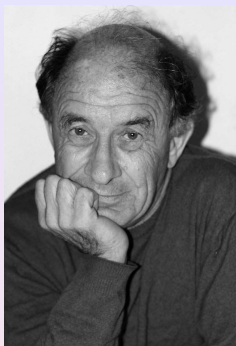
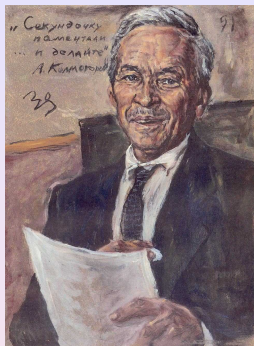
- * and quantum Planck constant $\hbar$ (1900) ?

Deterministic dynamical chaos



- * Poincaré (1893) - 3-body problem; Einstein (1917) - how to quantize ?
- * Chirikov resonance overlap criterion:
explained plasma trap experiments of Rodionov at Kurchatov Inst. (1959)
- * visit to Kolmogorov 1959

Kolmogorov-Arnold-Moser theory (1954-1963)



Integrability and Chaos:

Chirikov criterion (1959) (e.g. for Chirikov standard map $K < K_c$)
term “KAM theory” coined by Chirikov (INP preprint 1969)

- * Kolmogorov complexity (1963) - almost all numbers on interval $(0,1)$ are uncomputable; exponential instability of dynamics extracts information of high digits of initial conditions leading to chaos inspite of Laplace determinism
- * Sinai billiard (1963) - square with an elastic disk

Chirikov talk (celebrating his 70th at Toulouse)

CHAOS in SIBERIA

A story for history

Short (40') account of a long (40 yrs) work

Boris Chirikov & many coworkers in

Budker Institute of Nuclear Physics

& elsewhere

My (accidental) start

($\approx$ 1958, Kurchatov Institute, Moscow)

a simple-looking but turned-out rich

Budker's problem: single particle confinement in

Budker's (adiabatic) magnetic trap

(for the great END – controlled nuclear fusion !)

Toulouse

16 July 1998

BACKGROUND (retrospectively)

Revival of intensive studies

into nonlinear dynamics,

surprising rediscovery of chaos (stochasticity) after

Boltzmann ... Poincare ...

- new applications:

strong focusing accelerators,

controlled nuclear fusion

- computers, NUMERICAL EXPERIMENTS !!

– Lehmer, 1951

pseudorandom number generators after

Galton Board (= Lorentz gas in external field !)

– Goward and Hine (CERN), 1953, accelerators

→ Fermi, Pasta and Ulam, 1955

foundations of statistical mechanics

– Symon and Sessler, 1956, accelerators

→ Kolmogorov, 1954, KAM-integrability
(in spite of Poincare theorem !)

Chirikov talk (celebrating his 70th at Toulouse)

instructive
examples

SIMPLE CHAOS:
most complex dynamics
BUT simple statistics

• MICROTRON, Veksler, 1944

$$E_{n+1} = E_n + V_0 \sin \phi_n$$

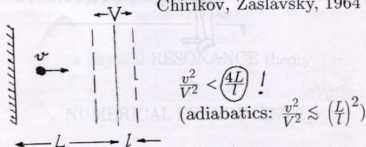
$$\phi_{n+1} = \phi_n + E_{n+1}/E_0, \quad F = 2 \text{ freedoms}$$

$$K = V_0/E_0 \gg 1$$

$$E_0 = \omega_0/2\pi\Omega, \quad e=m=c=1$$

• FERMI ACCELERATION, Ulam, 1961

Chirikov, Zaslavsky, 1964



• COMET HALLEY

diffusion backwards in time

$$t_L \sim 10 \text{ Myrs} \quad ?$$

Chirikov, Vecheslavov, 1989

Fermi's problem (1923)

ergodicity of nonlinear oscillation

$$H = \sum_{i=1}^N \frac{\dot{x}_i^2}{2} + \frac{(x_i - x_{i-1})^2}{2} + (x_i - x_{i-1})^p, \quad p=3;4, \quad N \gg 1$$

Fermi, Pasta, Ulam, 1955

nonergodic (generally) $\rightarrow$ solitons Kruskal, Miura, Zabusky, 1966
 $\rightarrow$ dynamical chaos Chirikov, Izrailev, 1966

De Luca, Lichtenberg, Lieberman, 1995
 Shepelyansky, 1997

Fermi-Pasta-Ulam problem (1955)

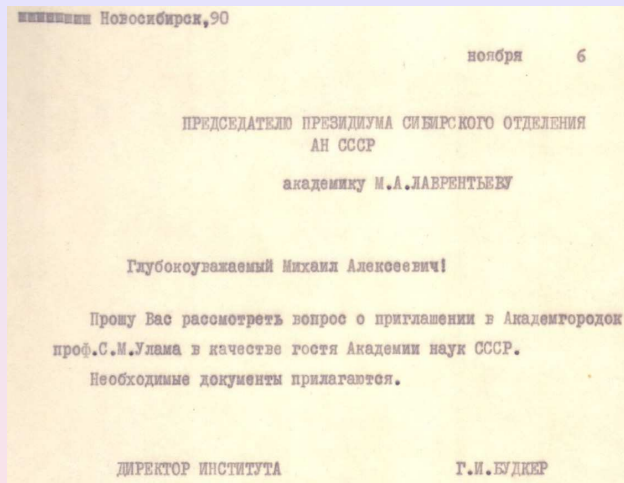


"In January 1951, Ulam and Teller came up with the Teller-Ulam design, which is the basis for all thermonuclear weapons. ...

"After the H-bomb was made," Bethe recalled, "reporters started to call Teller the father of the H-bomb. For the sake of history, I think it is more precise to say that Ulam is the father, because he provided the seed, and Teller is the mother, because he remained with the child. As for me, I guess I am the midwife." "

Wikipedia

Virtual visit of Ulam to Akademgorodok (1967)



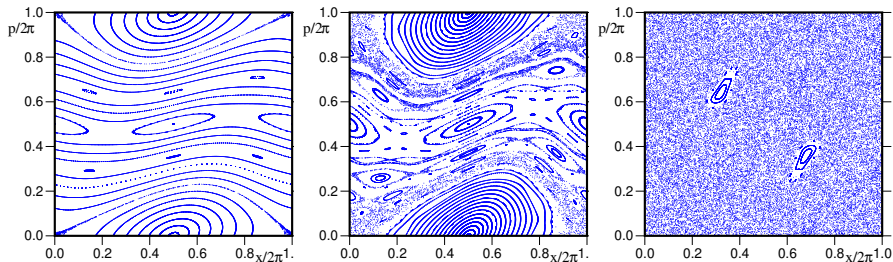
invitation Ulam never got

Meeting of Chirikov and Ulam at Mathematical Congress, Moscow 1966

Invitation of Ulam to Novosibirsk for summer 1967 (letter of Budker to Lavrentiev Nov 1966 (initiated by Chirikov)) from Archive of Chirikov

<http://www.quantware.ups-tlse.fr/chirikov/archive/ulamprog.pdf>

Chirikov standard map (1969, 1979)



$$\bar{p} = p + K \sin x, \quad \bar{x} = x + \bar{p}; \quad K = 0.5; 0.971635; 5$$

related systems: Frenkel-Kontorova model (1937); Veksler microtron (1944); Ulam map for Fermi acceleration (1961), Kepler map for microwave ionization of Rydberg atoms, Halley comet, dark matter dynamics in the Solar system ...

* Exponential divergence of nearby trajectories, positive Kolmogorov-Sinai entropy $h \approx \ln(K/2) > 0$ (Lyapunov exponent)

* Diffusion in momentum and energy $\langle p^2 \rangle = Dt \approx K^2 t/2$ for $K > 0.971635...$ ($E = p^2/2$)

Chirikov Phys Rep (1979) - mostly cited article of Russian scientists till 2008 (per author, from www.scientific.ru)

Chirikov, DS Scholarpedia (2008-2020) - various examples

Quantization: Ehrenfest time and chaos

*Bohr correspondence principle (1920); Ehrenfest theorem (1927)



Quantum evolution for the Chirikov standard map (kicked rotator):

$$\bar{\psi} = \exp(-ik \cos x) \exp(-iTn^2/2)\psi, \quad H = p^2/2 + K \cos x \sum_m \delta(t - m);$$
$$k = K/\hbar, \quad T = \hbar, \quad K = kT; \quad n = -i\partial/\partial x; \quad p = -i\hbar\partial/\partial x;$$

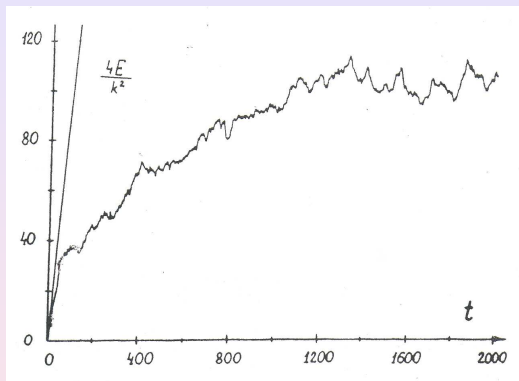
exponentially fast spreading of wave packet:

logarithmically short Ehrenfest time $t_E \sim |\ln \hbar|/h$

Chirikov, Izrailev, DS (1981); DS (1981)

Quantum localization of chaos

Quantum Chirikov standard map (kicked rotator)



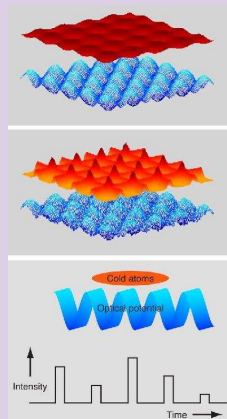
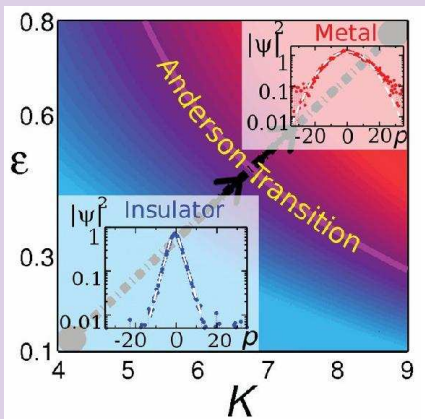
Chirikov, Izrailev, DS (1981) → Raizen cold atom experiment (1995):

$\bar{\psi} = \exp(-ik \cos x) \exp(-iTn^2/2)\psi$; $k = K/\hbar$, $T = \hbar$, $n = -i\partial/\partial x$;

diffusive time scale t_D , localization length: $t_D \sim \ell \approx \Delta n \approx D \sim K^2/\hbar^2 \gg t_E$

Analogy with Anderson localization (AL) in disordered solids (1958; Nobel prize 1977) shown by Fishman, Grempel, Prange (1982)

EXP: 3d-delocalization of cold atomic waves



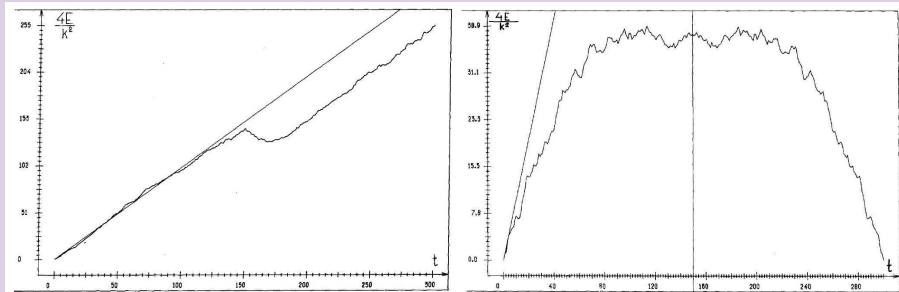
quantum chaos in kicked rotator => Chirikov localization in momentum space
=> dynamical analog of 3d Anderson transition

$$H = p^2/2 + K \cos x [1 + \epsilon \cos(\omega_2 t) \cos(\omega_3 t)] \sum_m \delta(t - m), \quad \hbar_{\text{eff}} = 2.89$$

J.C.Garreau *et al.* PRL **101**, 255702 (2008); theory prediction DS (1983-89-96)

Quantum version of Boltzmann-Loschmidt dispute

* Time reversal for the Chirikov standard map



BESM-6 computation, rescaled energy or squared momentum vs. time t :

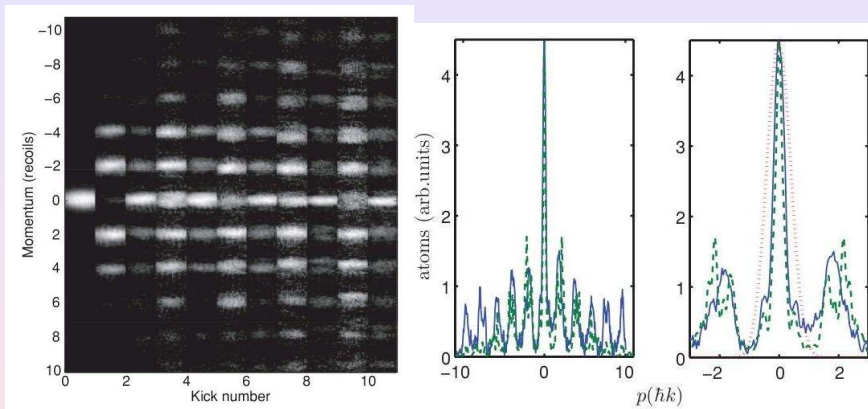
$K = 5$, $\hbar = 0$ (left), $\hbar = 1/4$ (right), DS (1983)

proposal for BEC in kicked optical lattice (Martin, Georgeot, DS (2008))

$k = K/\hbar$, $\hbar = T = 4\pi + \epsilon$ (forward), $4\pi - \epsilon$ (back);

$\cos x \rightarrow -\cos x$ by a π shift in x

Time reversal of atomic matter waves by Hoogerland group (2011)

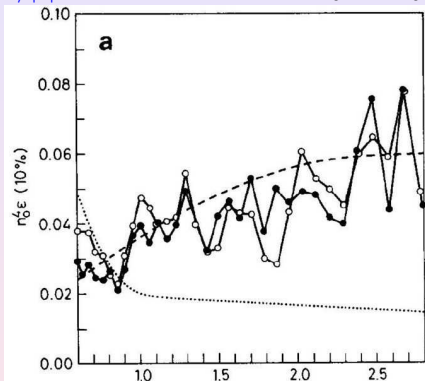
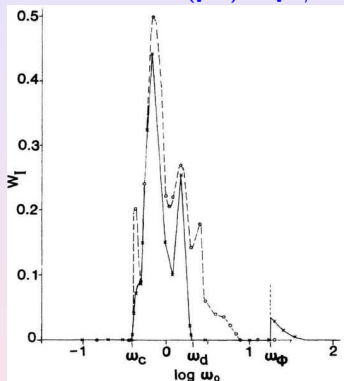


- $k = 2 - 3$, $T = 4\pi \pm \epsilon$ (Ullah, Hoogerland (2011))
- Ultracold ^{67}Rb atoms BEC: 10^4 atoms, $T_{\text{cool}} = 50\text{ nK}$,
 $\lambda = 2\pi/k_L = 760\text{ nm}$, $\epsilon = 1$; 5 + 5 kicks; right panel shows zoom near
initial distribution shown by red dotted curve (initial/final width is
0.43/0.21 recoils; full/dashed curve for experiment/numerics).

Microwave ionization of hydrogen atoms

Bayfield, Koch experiment (1974), Koch et al. (1988): $n_0 \sim 66$, $\omega \sim 10\text{GHz}$

Hamiltonian: $H(\mathbf{p}, \mathbf{r}) = \mathbf{p}^2/2 - 1/|\mathbf{r}| - \epsilon \mathbf{r} \cos \omega t$; $\omega_0 = \omega n_0^3$



Kepler map: $\bar{N} = N + k \sin \phi$, $\bar{\phi} = x + 2\pi\omega(-2\omega\bar{N})^{-3/2}$

$N = E/\omega$, number of photons for ionization $N_I \approx n_0/(2\omega_0) \sim 60$, $\omega_0 = \omega n_0^3$,
 $\epsilon_0 = \epsilon n_0^4 \sim 0.05$ (static field border 0.13); $k = 2.58\epsilon/\omega^{5/3}$ (atomic units)

photonic localization length $\ell_\phi \approx k^2/2 < N_I$ or delocalization $> N_I$

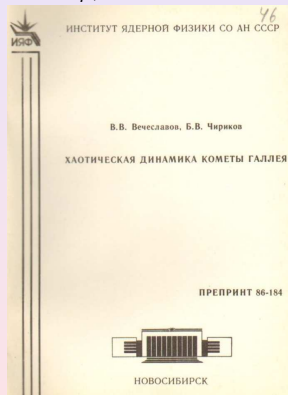
Chirikov et al. (1984-90), DS Scholarpedia (2012)

Halley comet chaos

Chirikov, Vecheslavov (1986 BINP, A&A 1989) from 46 appearances (27 historic from 240 BC); Halley - same comet 1705; period around 75 yrs

Kepler map: $\bar{w} = w + J \sin \phi = w + F(\phi/2\pi)$, $\bar{\phi} = \phi + 2\pi/\bar{w}^{3/2}$

$J \sim m_p/M$, comet image on 8 March 1986

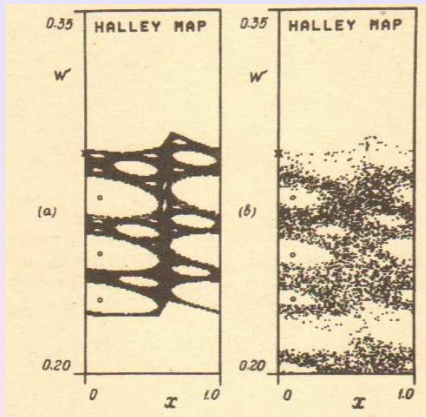
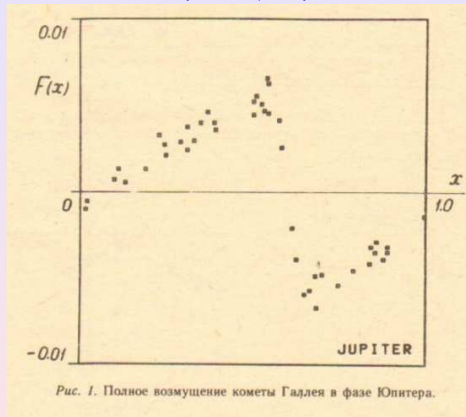


Yeomans, Kiang (1981) - 46 events computed; Petrosky (1986) - sin-map

Halley comet chaos

Chirikov, Vecheslavov (1986 BINP, A&A 1989)

kick function $F(x = \phi/2\pi)$ (left), phase-space (right)



comet dynamics is chaotic

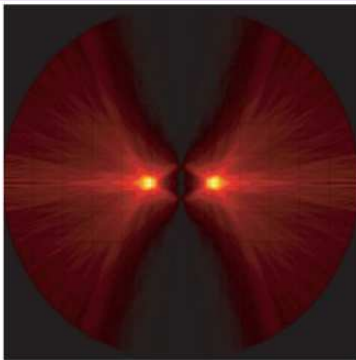
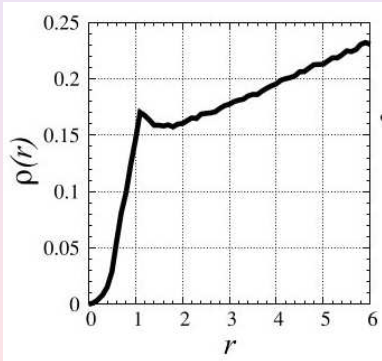
life time in the Solar system is around 10 millions yrs

($t_D \sim 1/D \sim 2/J^2 \sim (M/m_p)^2 \sim 10^6 \text{ periods} \sim 10^7 \text{ yrs}$)

Further advancements and applications:

Dark matter chaos in the Solar System

Capture of dark matter particles (DMP) in SS: perihelion distance $q \sim r_p$, energy of one kick $v_{cap} \sim v_p \sqrt{m_p/M} \sim 1 \text{ km/s}$ for Jupiter, enormous capture cross section $\sigma \sim r_p^2 (v_p/v)^2 \sim r_p^2 M/m_p$, dynamics of 10^{14} DMP for 4.5 milliads yrs with Kepler map approach, captured DMP mass (inside Neptune orbits) $10^{15} g$



Khriplovich, DS (2009), Lages, DS (2011), Lages, Shevchenko, DS (2018)

Wigner crystal in a periodic potential

Frenkel-Kontorova model (ZhETF 1938); Chirikov standard map (1969-1979); Aubry transition $K > K_c = 0.9716$ (1983)

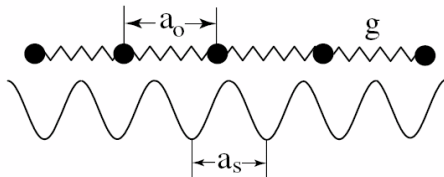


Fig. 1.1. Schematic presentation of the Frenkel-Kontorova model: A chain of particles interacting via harmonic springs with elastic coupling g is subjected to the action of an external periodic potential with period a_s .

Introducing the dimensionless variables, we re-write the Hamiltonian (1.1)–(1.5) in the conventional form ($H = 2\mathcal{H}/\varepsilon_s$)

$$H = \sum_n \left\{ \frac{1}{2} \left(\frac{dx_n}{dt} \right)^2 + (1 - \cos x_n) + \frac{g}{2} (x_{n+1} - x_n - a_0)^2 \right\}, \quad (1.8)$$

fixed particle density $\nu = 0.618... \rightarrow$ golden KAM curve

O.M.Braun and Yu.S.Kivshar, *The Frenkel-Kontorova model ...* Springer (2004)

Wigner crystal in a periodic potential

The dimensionless Hamiltonian:

$$H = \sum_{i=1}^N (P_i^2/2 - K \cos x_i + m\omega^2 x_i^2/2) + \sum_{i>j} 1/|x_i - x_j|$$

here P_i, x_i are ion momentum and position, K gives the strength of optical lattice potential and all N ions are placed in a harmonic potential; lattice constant d in $K \cos(x_i/d)$ is unity, energy $E = H$ is given in units of ion charge energy e^2/d . In the quantum case $P_i = -i\hbar\partial/\partial x_i$ with dimensionless $\hbar$ measured in units $\hbar \rightarrow \hbar/(e\sqrt{md})$, m is unit charge mass.

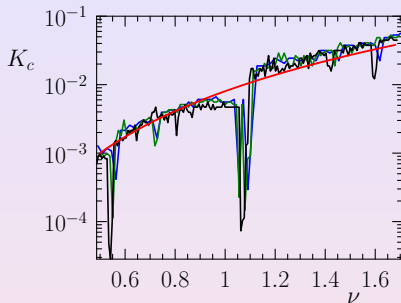
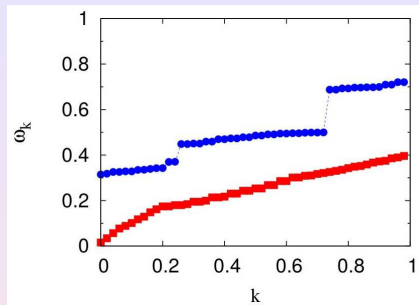
Related map: $p_{i+1} = p_i + Kg(x_i)$, $x_{i+1} = x_i + 1/\sqrt{p_{i+1}}$, where the effective momentum conjugated to x_i is $p_i = 1/(x_i - x_{i-1})^2$ and the kick function is $Kg(x) = -K \sin x$.

For the [Frenkel-Kontorova model](#) the equilibrium positions are described by the [Chirikov standard map \(1969-1979\)](#): $p_{i+1} = p_i + K \sin x_i$, $x_{i+1} = x_i + p_{i+1}$ with $K_c = 0.971635...$ for the golden mean density $\nu = (\sqrt{5} + 1)/2$.

Garcia-Mata, Zhirov, DS (2007) $\rightarrow$ Wigner crystal $K_c = 0.046$ vs. 0.034

first experimental realisations Vuletic et al. (MIT 2016)

Density dependence of Aubry transition



* Left: spectrum of excitations $N = 150$, $\nu \approx 1.618\dots$, $K = 0.03/0.2$ red/blue

* Right: Linearized approximation by the Chirikov standard map gives $K_c = 0.034(\nu/1.618)^3$; number of wells $N_w = 55, 89, 144$ and varied number of ions N_{ions} with $\nu = N_{ions}/N_w$; red curve is from the Chirikov standard map approximation; high thermoelectric figure of merit up to $ZT \approx 8$; (Zhiron, Lages, DS (2019))

* $K > K_c$ gap for excitations $\rightarrow$ protects quantum computer gates (DS (2019)) in contrast to Cirac-Zoller proposal with zero gap at many ions

Quantum computing and chaotic maps

Universal quantum computing: single qubit rotations + two-qubit gates (e.g. CNOT) (Nielsen, Chuang (2000)); At present: about 20 cold-ion qubits; 64 superconducting qubits; restricted by number of gates due to decoherence and imperfections (Deutsch PRX (2020)); here Arnold cat map at 26 qubits (Georgot, DS (2002)); polynomial number of gates for quantum st.map

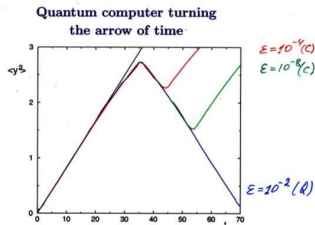
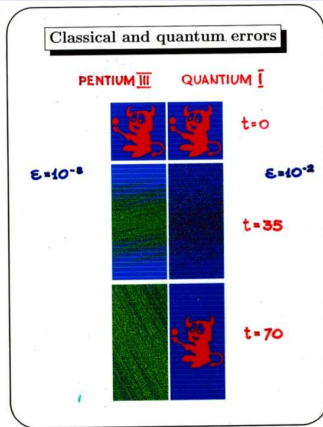


Figure 1: Diffusive growth of the second moment $\langle y^2 \rangle$ of the distribution $w(y, t)$ generated by the Arnold cat map with $L = 8$, simulated on a classical (Pentium III) and quantum ("Quantum I") computers. At $t = t_r = 35$ Maxwell's demon inverts all velocities. For Pentium III inversion is done with precision $\epsilon = 10^{-4}$ (red line) and $\epsilon = 10^{-8}$ (green line); 10^8 orbits are simulated, initially distributed inside initial distribution. For Quantum I, the computation is done with 26 qubits ($n_q = 7, n_r = 10$) (blue line); each quantum gate operates with imperfections of amplitude $\epsilon = 0.01$ (unitary rotation on a random angle of this amplitude). The black straight line shows the theoretical macroscopic diffusion with $D = 1/12$.

$$\bar{y} = y + x \pmod{L}, \quad \bar{x} = x + \bar{y} \pmod{L}$$



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Quantum hardware melting induced by quantum chaos

The quantum computer hardware is modeled as a (one)two-dimensional lattice of qubits (spin halves) with static fluctuations/imperfections in the individual qubit energies and residual short-range inter-qubit couplings. The model is described by the many-body Hamiltonian

$$H_S = \sum_i (\Delta_0 + \delta_i) \sigma_i^z + \sum_{i < j} J_{ij} \sigma_i^x \sigma_j^x,$$

where the σ_i are the Pauli matrices for the qubit i , and Δ_0 is the average level spacing for one qubit. The second sum runs over nearest-neighbor qubit pairs, and δ_i , J_{ij} are randomly and uniformly distributed in the intervals $[-\delta/2, \delta/2]$ and $[-J, J]$, respectively.

Quantum chaos border for quantum hardware:

$$J > J_c \approx \Delta_c \approx 3\delta/n_q \gg \Delta_n \sim \delta 2^{-n_q}$$

Emergency rate of quantum chaos:

$$\Gamma \sim J^2/\Delta_c.$$

(B.Georgeot, DS (2000))

Åberg criterion for many-body quantum chaos

Onset of quantum chaos in systems with two-body interactions:

spacing between adjacent energy levels drops exponentially with number of particles L : $\Delta_L \propto \exp(-L)$; interaction induced coupling, two-body matrix element between directly coupled states $U_c = U$.

Spacing between directly coupled states: $\Delta_c \gg \Delta_L$.

Åberg criterion for onset of quantum chaos: $U_c \approx \Delta_c \gg \Delta_L$ ($A = U_c/\Delta_c > 1$)

Åberg (PRL1990,1992) (random signs of interaction, argument refers on Weidenmuller works even if in (1998) there is no theory for that);

DS, Sushkov (1997) (argument of 3 interacting particles);

Jacquod, DS (1997) $\rightarrow$ dynamical thermalization near Fermi energy in TBRIM (quantum dot with one-particle level spacing Δ and two-body random interactions U):

$$\delta E > \delta E_{ch} \approx g^{2/3} \Delta \gg \Delta;$$

$\rightarrow$ dynamical thermalization border/conjecture (DTC); (in metallic quantum dots $U/\Delta = J/V \approx 1/g \ll 1$, $g = E_{Thouless}/\Delta \gg 1$ dot conductance)

Analytical confirmation of DTC for TBRIM:

Gornyi, Mirlin, Polyakov *et al.* (2016,2017)

Sinai billiard $\rightarrow$ Sinai oscillator for BEC

Sinai billiard (1963; 1970) (Abel prize 2014)

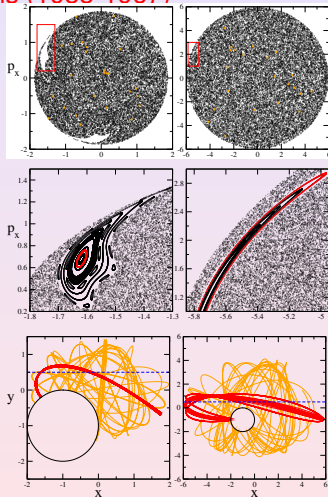
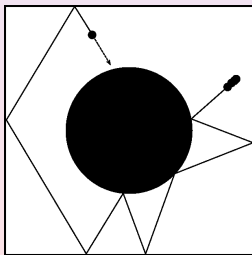
Wigner Random Matrix Theory for nuclei, atoms (1955-1967)

Bohigas-Giannoni-Schmidt conjecture (1984)

$$H = (p_x^2 + p_y^2)/2 + x^2/2 + y^2$$

$$r_d = 1, x_d = y_d = -1; E = 2; 18 \rightarrow$$

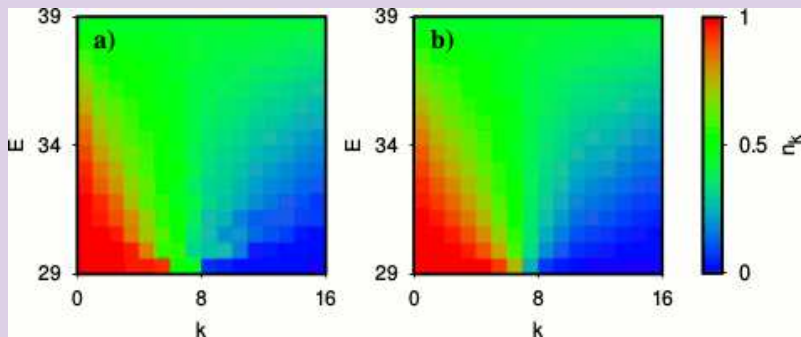
Ketterle BEC experiment (1995) in 3D



dynamics in Sinai oscillator

Ermann, Vergini, DS (2015-2017)

Dynamical thermalization in Sinai-oscillator trap



Orbital occupation numbers n_k for interacting fermionic atoms in a Sinai-oscillator trap at $\tilde{A}$ parameter $A = 3.5$; numerical result from many-body eigenstates (left), theoretical Fermi-Dirac distribution (right); $M = 16$ orbitals and $L = 7$ fermions, $N = 11440$.

Interaction between fermionic atoms $v(r)$ has a form of disk of certain small radius $r_c = 0.2$ and amplitude U .

Frahm, Ermann, DS (2019),
links to SYK black hole model at $\Delta \ll U_2$ (1993 - 2015)

Ulam method (1960) and Ulam networks (2010)

Small-world networks (6 degrees of separation Milgram (1967)):

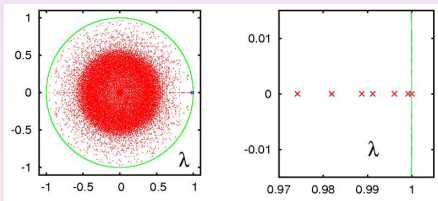
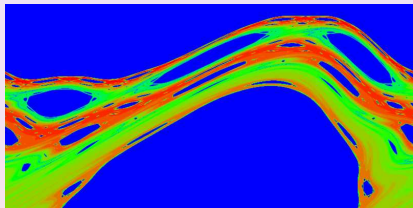
Ulam method for the Chirikov standard map $\rightarrow$ Markov chains (1906),

Google matrix, World Wide Web, PageRank algorithm $\rightarrow$ Brin, Page (1998);

Ulam networks $\rightarrow$ DS, Zhirov; Ermann, DS (2010);

Chirikov standard map $\rightarrow$ Frahm, DS (2010)

TBRIM, SYK models $\rightarrow$ quantum small world Frahm, DS (2018))



Google matrix analysis of directed networks: WWW, world trade, Wikipedia, Linux Kernel ..., PageRank, CheiRank

Chepelianski (2010);

Ermann, Frahm, DS Rev Mod Phys (2015)

Google matrix of fibrosis protein-protein interactions (bioRxiv 2021)

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Edward Lorenz, father of chaos theory and butterfly effect, dies at 90

April 16, 2008

Edward Lorenz, an MIT meteorologist who tried to explain why it is so hard to make good weather forecasts and wound up unleashing a scientific revolution called chaos theory, died April 16 of cancer at his home in Cambridge. He was 90.

A professor at MIT, Lorenz was the first to recognize what is now called chaotic behavior in the mathematical modeling of weather systems. In the early 1960s, Lorenz realized that small differences in a dynamic system such as the atmosphere—or a model of the atmosphere—could trigger vast and often unsuspected results.

These observations ultimately led him to formulate what became known as the butterfly effect—a term that grew out of an academic paper he presented in 1972 entitled: "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?"

Lorenz's early insights marked the beginning of a new field of study that impacted not just the fields of mathematics but virtually every branch of science—biological, physical and social. In meteorology, it led to the conclusion that it may be fundamentally impossible to predict weather beyond two or three weeks with a reasonable degree of accuracy.

Some scientists have since asserted that the 20th century will be remembered for three scientific revolutions—relativity, quantum mechanics and chaos.

"By showing that certain deterministic systems have formal predictability limits, Ed put the last nail in the coffin of the Cartesian universe and foreshadowed what some have called the third scientific revolution of the 20th century, following on the heels of relativity and quantum physics," said Kerry Emanuel, professor of atmospheric science at MIT. "He was also a perfect gentleman, and through his intelligence, integrity and humility set a very high standard for his and succeeding generations."

Memorialist service for
Edward Lorenz
Sunday, April 20, 2008
3 p.m.
Swendenborg Chapel, 60
Quincy St., Cambridge



Edward Lorenz

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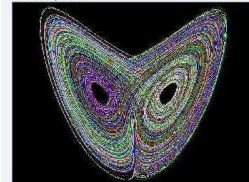
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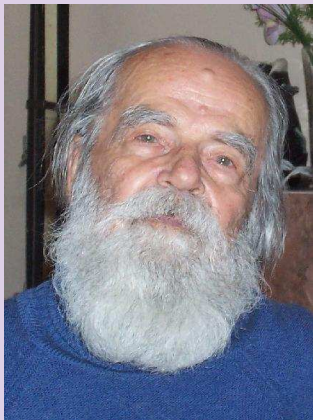
Аттрактор Лоренца.
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КОММЕНТАРИИ

Аттрактор и бабочка Памяти Эдварда Нортона Лоренца

В 1961 году метеоролог и математик Эдвард Лоренц, скончавшийся 16 апреля 2008 года, ввел в созданную им компьютерную модель погоды данные, округлив их не до шестого, а до третьего знака после запятой. В результате был сформулирован эффект бабочки, открыт один из странных аттракторов, обнаружена непредсказуемость поведения многих детерминированных систем и, в конечном итоге, создана теория хаоса.

1959 - Hamiltonian chaos - Chirikov, 1963 - dissipative chaos - Lorenz, ...



Johnson Phil D
 Dick Matis
 Tami Gage
 and Mgrs
 V. L. L. L.
 V. F. Anderson
 F. F. F. F.
 T. W. T. W.
 S. W. S. W.
 M. W. M. W.
 1998, Toulouse

I

$$K - S^2 = \left(\frac{\Delta \omega r}{\Omega_d} \right)^2 > 1$$

(1959)

II	
$\beta E_0 > N/k$	/FPU/
III	(1966/73)

$$\bar{I} = I + K \sin \theta, \quad \bar{\theta} = \theta + \bar{I}$$

$$K > K_c = \frac{\pi^2}{4} S^2 - 1; \quad h = \ln(K/2) > 0$$

/KAM/ /KS/ (1969/79)

$$\begin{aligned} \text{IV} \\ \omega_s = 4\pi\epsilon\lambda^3 \exp(-\pi\lambda/2), \quad \lambda = \Omega/\omega_0 \\ D_A = D_0 \exp(-\pi\lambda) \sim \omega_s^2 \\ (1969/79) \end{aligned}$$

$$\bar{\psi} = \exp(-i\frac{T\hbar^2}{2}) \exp(-ik \cos \hat{\theta}) \psi$$

$$(\Delta n)^2 \sim k^2 t^* < \infty / KR /$$

(1979)

VI
 $h > \Lambda_m \approx 0.38H^{1/4} > 0$
 /YM/ (1981)

VII
 $P(\tau) \sim 1/\tau^p$, $p \approx 1.5$ (1981)
 $p = 3$ ($\tau \rightarrow \infty$) (1998)

VIII
 $t^* \sim \Delta n \sim D \sim k^2 \gg t_f \sim \ln k / h$
 (1981)

$$\varepsilon_q \approx 0.5 \omega_0^{7/6} / \sqrt{n_0} \cdot H / (1984)$$

$\bar{\omega} = \omega + F(x)$, $\bar{x} = x + \bar{\omega}^{-3/2}$
 $h > 0$, $t_0 = 4 \times 10^6 \text{ yrs}$
 /CH/ (1989)