2D Electron transport in a microwave field

Dima Shepelyansky
www.quantware.ups-tlse.fr/dima/

- **arxiv:0707.2694[cond-mat]**
  Synchronization, zero-resistance states and rotating Wigner crystal
  A.D.Chepelianskii (ENS, rue d'Ulm, Paris)
  A.S.Pikovsky (Univ. of Potsdam)
  and DS
- **Discussions:**
  A.Bykov (Inst. of Physics of Semiconductors, Novosibirsk)
  F. von Oppen (Freie Univ. Berlin)
  J.Schmidt and F.Spahn (Univ. of Potsdam)
- **Support:**
  ANR PNANO project MICONANO of French government

Hero of the talk

- Christian Huygens
  (1629 - 1695)

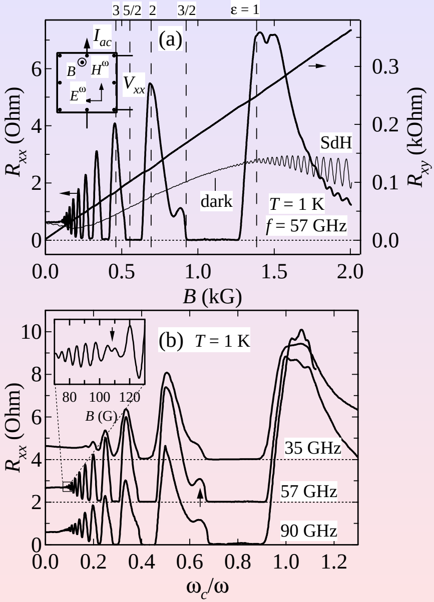
Experiments on Zero-Resistance States (ZRS)

- High mobility 2DEG in a microwave field

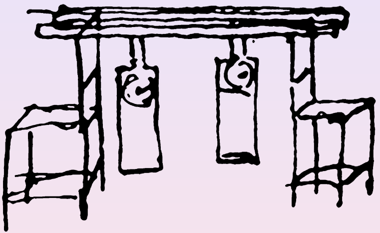
  *M.A.Zudov, R.R.Du, J.A.Simmons, J.I.Reno PRB **64**, 201311 (2001)

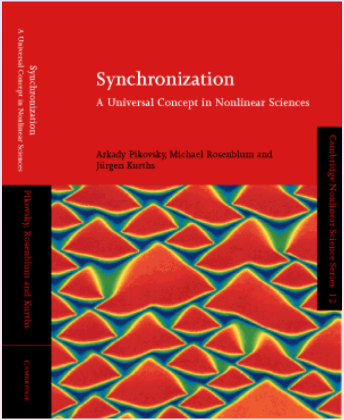
  *R.G.Mani, J.H.Smet, K. von Klitzing, V.Narayanamurti, W.B.Johnson, V.Umansky Nature **420**, 646 (2002)

  *M.A.Zudov, R.R.Du, I.N.Pfeiffer, K.W.West PRL **90**, 046807 (2003) (Fig. image)



Synchronization

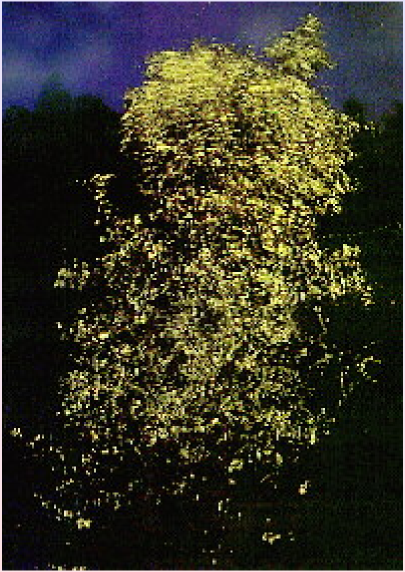




Ch.Huygens (1665) “sympathy of two clocks”

A. Pikovsky, M. Rosenblum, and J. Kurths, Cambridge University Press (2001)

Fireflies in Siam



Numerical studies

Classical 2D electron dynamics with short range and Coulomb interactions.
Short range interactions: the Nosè-Hoover thermostat (Hoover (1999)) combined with interactions treated in the frame of the mesoscopic multi-particle collision model (Kapral (2004)):

$$\dot{\mathbf{q}}_i = \mathbf{p}_i/m\ ,\ \dot{\mathbf{p}}_i = \mathbf{F}_i + \mathbf{f}_{Li} + \mathbf{f}_{ac} - \gamma \mathbf{p}\ , \tag{1}$$

$$\dot{\gamma} = [\langle \mathbf{p}^2 \rangle/(2mT) - 1]/\tau^2 \tag{2}$$

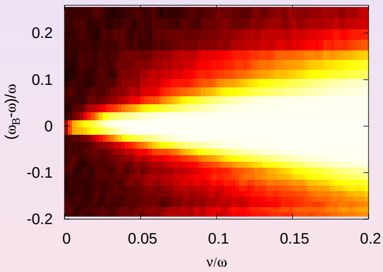
where $\mathbf{q}_i, \mathbf{p}_i$ are the coordinate and the momentum of particle i, $\mathbf{f}_{Li} = e[\mathbf{p}_i \times \mathbf{B}]/mc$ is the Lorentz force, $\mathbf{F}_i$ is an effective force produced by particles collisions, τ is the relaxation time in the thermostat and $\langle \mathbf{p}^2 \rangle$ means average over all N particles.

Another choice, used for short range and Coulomb interactions, is to equilibrate the heating induced by the microwave field we introduce in Eq. (1) an energy-dependent dissipation with $\gamma = \gamma_0(E - E_F)/E_F$ for $E = p^2/2m > E_F$ and $\gamma = 0$ for $E < E_F$. In such a way the dynamics remains Hamiltonian for $E < E_F$ while above E_F the dissipative processes are switched on as it is usually the case for 2DEG; thus E_F plays a role of Fermi energy.

In experiment electrons are at a Landau level $n_L \approx 100$.

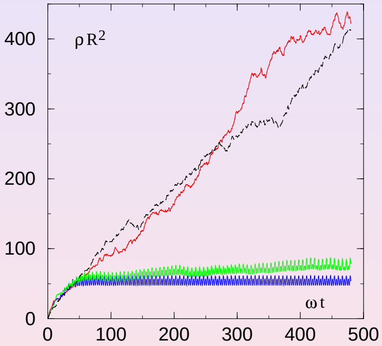
Kapral collision model

Diffusion rate density D/D_0 as a function of frequency detuning $(\omega_B - \omega)/\omega$ and rescaled microwave field strength ν/ω with $\nu = f/(mv_T)$ where $v_T = \sqrt{2T/m}$ is the thermal velocity and D_0 is the diffusion rate in absence of microwave at $\omega_B = \omega$. The particle dynamics is described by the NH thermostat at temperature T with short range interactions treated in the MMPCM formalism (see Eqs. (1,2)). The system parameters described in the text are: $N = 1000$ $N_c = 4 \times 10^4$, $\omega\Delta t = 0.2$, $\omega\tau = 10$, $\omega t = 500$, $L/r_B = 10$, $D_0/D_c = 0.12$ (with $D_c = v_T^2/\omega$, $\rho = N/L^2$ and r_B taken at $\omega_B = \omega$, thus a number of particles inside a Larmor circle is $N_B = \pi r_B^2 \rho = \pi \rho v_T^2/\omega^2 = 10\pi$, ω is kept constant). Color is proportional to D/D_0 (black maximum $D/D_0 \approx 1.2$; white minimum $D/D_0 = 0$).



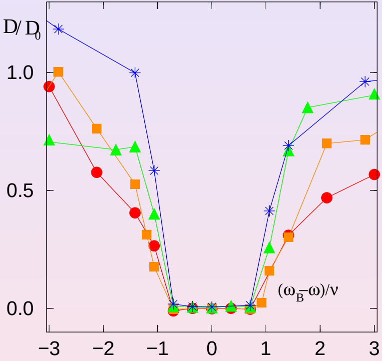
Coulomb interactions

Dependence of electron square displacement R^2, rescaled by electron density ρ, on the rescaled time ωt. Here the Larmor frequency is $\omega_B = \omega$ at microwave field strength $f = 0$ (red top curve); $f/(mv_F\omega) = 0.059$ ($fa/E_F = 0.02$) for $\omega_B = \omega$ (blue bottom curve), $\omega_B = 0.875\omega$ (second from top black dashed curve), and $\omega_B = \omega$ with impurity scattering mean free path $l_i = 96r_B$ (second from bottom green curve). Total number of electrons is $N = 100$ and $N_B = \pi\rho v_F^2/\omega^2 = 34.7$. The linear fit gives the diffusion rates $D/D_c = 0.089, 0.068, 0.0040, 9 \times 10^{-6}$ with $D_c = v_F^2/\omega$ (respectively for curves from top to bottom ordered at $\omega t = 400$).



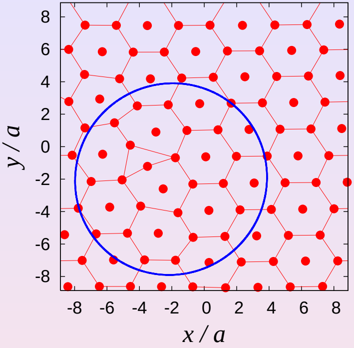
Coulomb interactions: diffusion rate

Dependence of rescaled diffusion rate D/D_0 on the rescaled frequency difference $(\omega_B - \omega)/\nu$. Here $\nu = f/mv_F$, D_0 is diffusion rate in absence of microwave at $\omega_B = \omega$, $fa/E_F = 0.02$ and number of electrons in a Larmor circle is $N_B = 2$ (stars), 8 (triangles), 34.7 (squares), 138.8 (points) with $D_0/D_c = 0.054, 0.089, 0.12, 0.14$ and $D_0/v_F a = 0.20, 0.35, 0.53, 0.64$ respectively. Total number of electrons is $N = 100$, $L = \sqrt{N/\rho} \approx 17.72a$.



Rotating Wigner crystal

Instant image of the rotating Wigner crystal formed by $N = 100$ electrons (points) in a periodic cell with $L = \sqrt{N/\rho} \approx 17.72a$, $\omega t = 480$ $\omega_B = \omega$, $fa/E_F = 0.02$ and $N_B = 34.7$ (bottom curve in one of previous Fig.); the circle shows an orbit of one electron for $240 \leq \omega t \leq 480$; lines are drawn to adapt an eye showing a hexagonal crystal with a defect.



Synchronization domain of ZRS phase:

$$|\omega_B - \omega| \leq 0.8f/mv_F \ , \qquad (3)$$

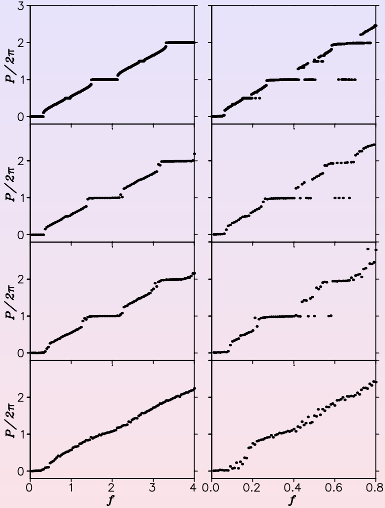
For experiment conditions the relative size of ZRS plateau is $\Delta\omega/\omega \approx 2\nu/\omega \approx fv_F/\omega E_F$ and with $E_F \sim 100K^o$, $v_F \sim 3 \times 10^7 cm/s$ and $\omega/2\pi = 35GHz$ this gives $\Delta\omega/\omega \approx 0.1$ if the field strength acting on an electron is $f/e \approx 5V/cm$. The coherent rotation of electrons in the crystal creates a rotating current in 2D plane which in its turn generates a magnetic field $B_W \sim \mu_0 ev_F\rho \sim 1G$ parallel to 2DEG and rotating in the plane with a frequency close to ω.

Quantum synchronization

A model of kicked rotator with friction

$$\bar{p} = (1-\gamma)p + (1-\gamma)K \sin x + f\gamma/g^2 \, ,$$
$$\bar{x} = x + \gamma p/g^2 + (\gamma K/g^2) \sin x + f(g^2-\gamma)/g^4 \, , \quad (4)$$

Fig: Dependence of the average momentum P on static force f at $K = 0.8$ for $\gamma = 0.25$ (left column) and $\gamma = 0.05$ (right column). From top to bottom: classical case at $\hbar = 0$, $\hbar = 0.012$, $\hbar = 0.05$, $\hbar = 0.5$.

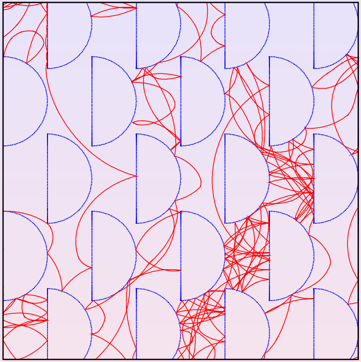


O.V.Zhirov, DS, Eur. Phys. J. D **38**, 375 (2006)

Quantum synchronization is possible at small values of $\hbar$.

=> possible tests in experiments with cold atoms in kicked optical lattices “à la Raizen”

Aside note: Ratchets in asymmetric nanostructures



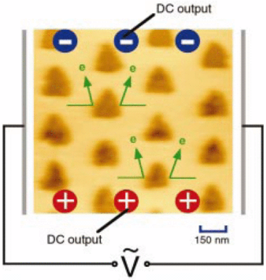


FIG. 1. (Color) An atomic force microscope image of the nanomaterial is shown. Each etched triangular hole (obstacle) scatters electrons in a predetermined upward direction independent of the direction of the applied field, as illustrated by the arrows. Therefore, dc signals are generated even when ac or random fields are applied.

A most rapid ratchet in the world: $v_r/v_F \approx 0.1(r_d f/E_F)^2$, $v_r \sim 10^5 cm/s$.

Lund experiment at room temperature:

A.M.Song, P.Omling, L.Samuelson et al. Appl. Phys. Lett. **79**, 1357 (2001)

Theory of deterministic ratchets:

A.D.Chepelianskii, M.V.Entin, L.Magarill, DS, Eur. Phys. J. B **56**, 323 (2007)

Coulomb crystals in rf-traps

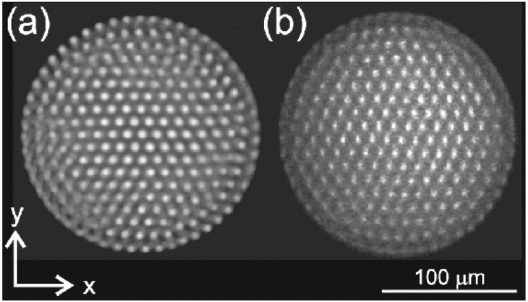
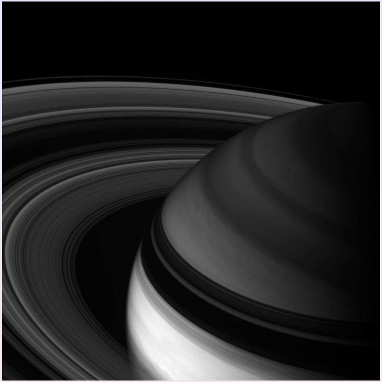


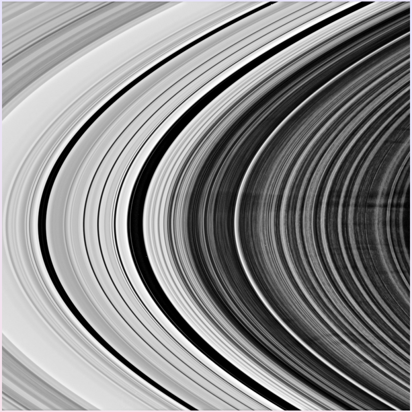
FIG. 3. Images of Coulomb crystals. (a) Time averaged image based on data from MD simulations of Coulomb clusters with 2685 ions at $\Gamma \sim 400$ (temperature: $\sim$5 mK). The averaging time is 10 ms. (b) Image from experiments with clusters containing $\sim$2700 ions.

Mainz experiments: G.Werth et al. Phys. Rev. A. **56**, 4023 (1997); Eur. Phys. J. D **18**, 295 (2002)

Aarhus experiments: A.Mortensen, E.Nielsen, T.Matthey, and M.Drewsen, Phys. Rev. Lett. **96**, 103001 (2006) [Fig. image]

Synchrony Conjecture for Planetary Rigns





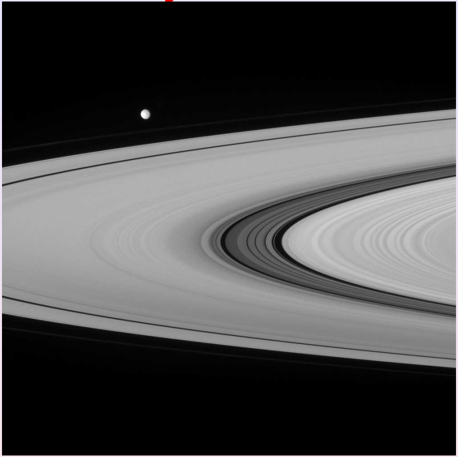
NASA Cassini images of Saturn Ring (2007)

Features to explain: enormously long life time (10^{12} rotations)
and very sharp edges ($10m$)

Ch.Huygens coded in 1655:

"It (Saturn) is girdled by a thin flat ring, nowhere touching, inclined to the ecliptic"

Summary



Mimas and the Great Division
(NASA Cassini Mission
image of Sept 7, 2007;
http://photojournal.jpl.nasa.gov/
catalog/PIA09750)

A generic mechanism of
SYNCHRONIZATION INDUCED SELF-ASSEMBLY is proposed