Density matrix properties of quantum strange attractor

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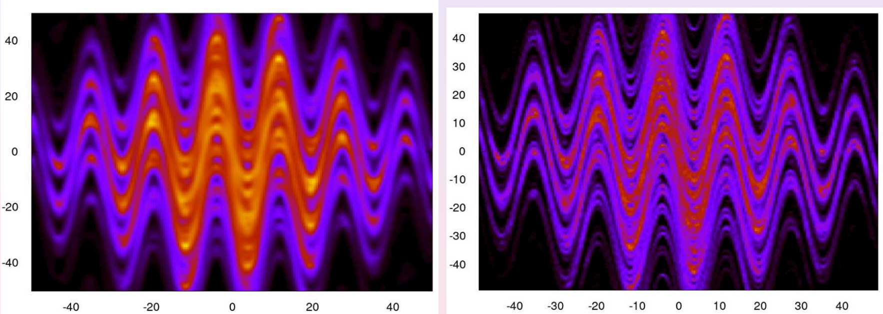
work in progress



Lindblad eq, 4 millions components for dissipative kicked ocillator

OCTAVES pubs: K.Frahm, DS: Eur. Phys. J. D (2021) [chaotic EPR pairs]; J. Phys. A (2022) [Loschmidt echo]; AC+DS: Entropy (2024) [qubits+resonator]

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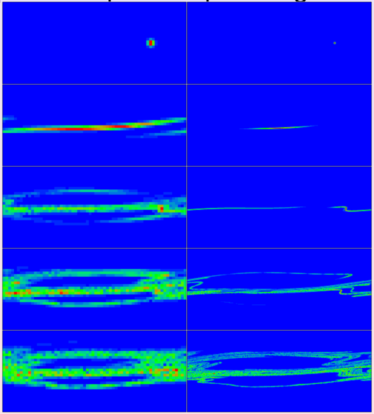
Ehrenfest time and chaos

Ehrenfest theorem(1927) $\rightarrow$ coherent wave packet follows its quantum trajectory during Ehrenfest time t_E: $t_E \propto 1/\hbar_{eff}$ for integrable dynamics; $t_E \propto |\ln \hbar_{eff}|/h$ for chaotic dynamics

where h is Kolmogorov-Sinai entropy proportional to Lyapunov exponent, due to exponentially rapid wave packet spreading Chirikov, Izrailev, DS (1981)



Frahm, DS PRE (2009) Husimi (smoothed Wigner) function of chaotic map $\hbar_{eff} = 2\pi/N$, $N = 2^{12}$ (left), 2^{16} (right), $t = 0, 20, 60, 100, 150$

Dissipation and quantization of chaos

Quantum trrajectories description for Chirikov standard map with dissipation:
$p_{t+1} = (1-\gamma)p_t + K \sin x_t, \;\; x_{t+1} = x_t + p_{t+1}$

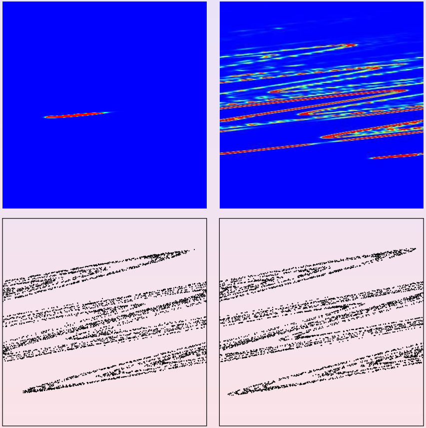


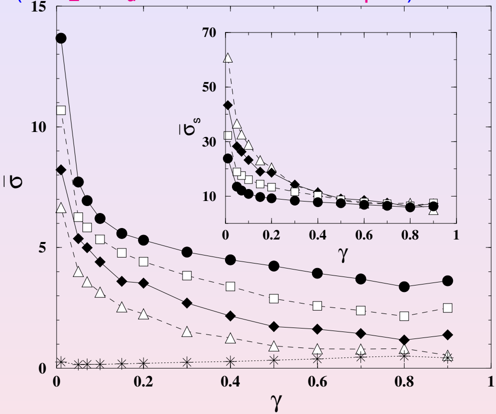
FIG. Top: Husimi functions in phase space for a single quantum trajectory taken after $t = 300$ kicks, at $K = 7$, $\hbar = 0.012$, $\gamma = 0.5$ (left), and $\gamma = 0.01$ (right). Here x (horizontal axis) and p (vertical axis) vary in the intervals: $0 \le x < 2\pi$, $-25 \le p \le 25$ (left), and $-100 \le p \le 50$ (right); the width of the p interval is the same in both cases for comparison purposes. The initial Gaussian wave packet is located at $(\langle x \rangle, \langle p \rangle) = (5\pi/4, 0)$. The color is proportional to density: blue for zero and red for maximum. Bottom: quantum Poincaré section (left), obtained from average quantum x, p values for the case of top left panel and its classical counterpart (right); here $0 \le x < 2\pi$ and $-15 \le p \le 15$.

Carlo, Benenti, DS PRL (2005)

Ehrenfest collapse and explosion

Explosion: Ehrenfest time t_E becomes smaller than dissipation time t_d

$t_E \approx |\ln \hbar|/h < t_d = 1/\gamma$ (for $t_E > t_d$ there is wave collapse)



Average dispersion $\overline{\sigma}$ as a function of γ, for $K = 7$, $\hbar = 0.33$ (circles), 0.11 (squares), 0.036 (diamonds) and 0.012 (triangles). Stars show the same quantity for the integrable case $K = 0.7$, at $\hbar = 0.012$. Inset: scaled dispersion $\overline{\sigma}_s = \overline{\sigma}/\sqrt{\hbar}$ versus γ (same symbols).

Carlo, Benenti, DS PRL (2005)

Zaslavsky web map

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Minimal chaos and stochastic webs

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Particles within a stochastic—or chaotic—web in phase space can be accelerated to high energies even by weak magnetic

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Quantum Chaos in an Ion Trap: The Delta-Kicked Harmonic Oscillator

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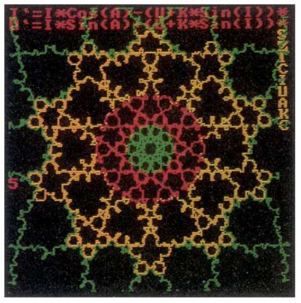
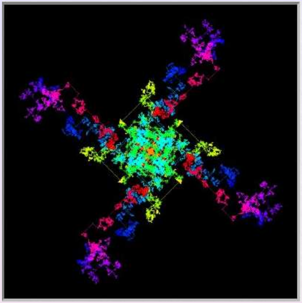
(Received 15 August 1997)

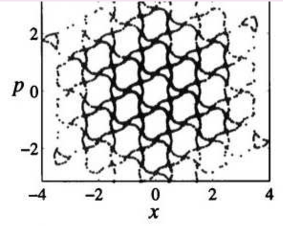
We propose an experimental configuration, within an ion trap, by which a quantum mechanical delta-kicked harmonic oscillator could be realized, and investigated. We show how to directly measure the

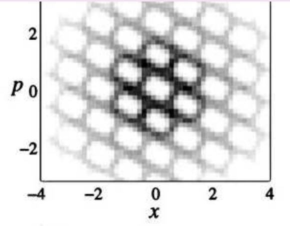
Hamiltonian $H(p,x,t) = (p^2 + \omega^2 x^2)/2 + K\cos(qx)\sum_m \delta(t - mT)$
$\rightarrow$ kick + free rotation;
$\omega T/2\pi = 1/M$; $M = 3,4,6$ or $5,8$. We consider $M = 4$ + dissipation γ;
mass=1, $\omega = \hbar = 1$, between kicks $dp/dt = -2\gamma p - \omega^2 x$; $dx/dt = p$

Zaslavsky web map









$M = 5; \mathbf{4}; 6(\textit{classical}/\textit{quantum})$

Lindblad equation for density matrix

It is useful to consider a model with oscillator basis (kicked oscillator), then for the Lindblad equation we have (kick + free rotation):

$$\int_{-\infty}^{\infty} \cos qx \ \psi_n(x)\psi_{n+m}(x)dx = \frac{1+(-1)^m}{2} 2^{-m/2}\sqrt{\frac{n!}{(m+n)!}} q^m e^{-q^2/4} L_n^m(q^2/2)$$

where H_n are Hermite polynomals and L_n^m are Laguerre polynomials.
The kick unitary operator is given by:

$$\hat{U} = \exp\left(iK\cos q\hat{x}\right)$$

The lindblad equation is:

$$\partial_t \hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + 2\gamma\left(\hat{a}\hat{\rho}\hat{a}^+ - \frac{1}{2}\hat{a}^+\hat{a}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{a}^+\hat{a}\right)$$

in the oscillator eigenbasis is:

$$\partial_t \rho_{nm} = i\omega(m-n)\rho_{nm} + 2\gamma\left(\sqrt{(n+1)(m+1)}\rho_{n+1,m+1} - \frac{(n+m)}{2}\rho_{n,m}\right)$$

introducing interaction picture variables:

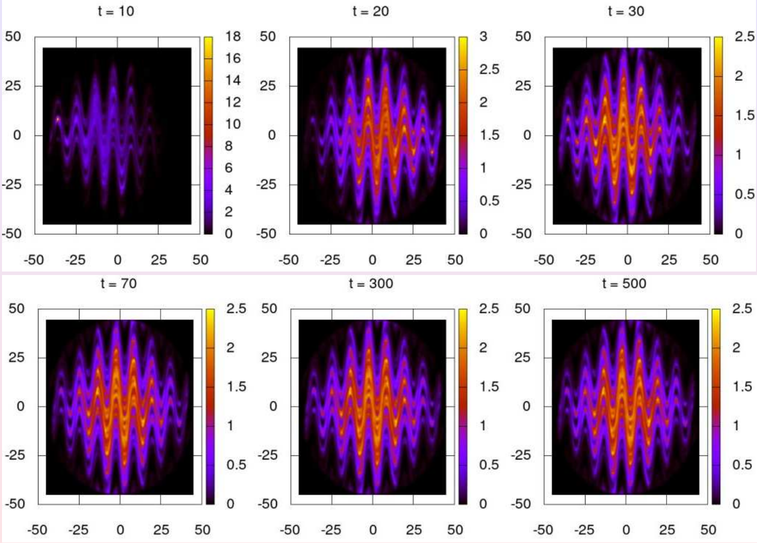
$$\rho(t) = \tilde{\rho}(t)e^{i\omega(m-n)t}$$
$$\partial_t \tilde{\rho}_{nm} = 2\gamma\left(\sqrt{(n+1)(m+1)}\tilde{\rho}_{n+1,m+1} - \frac{(n+m)}{2}\tilde{\rho}_{n,m}\right)$$

we integrate the propagators explicitly in each independent k indexed sub-block $\tilde{\rho}_{n,n+k}$ (n an integer in the basis
During the kick the denisty matrix is changed to:

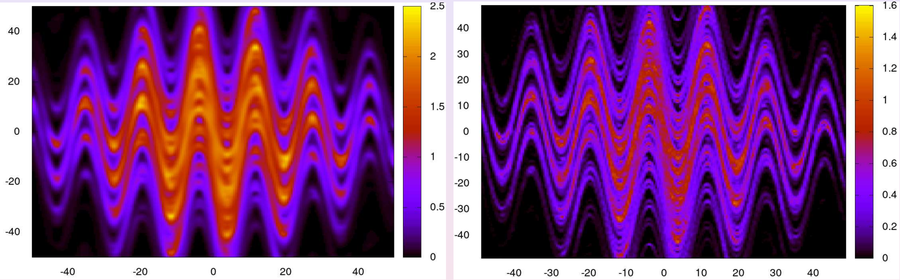
$$\hat{\rho} \to \exp\left(iK\cos q\hat{x}\right)\hat{\rho}\exp\left(-iK\cos q\hat{x}\right)$$

Lindblad dissipative evolution

Husimi after t kicks, color $\times 10^3$, $N = 2000$, $K = 40$, $q = 0.4$, $\gamma = 0.05$



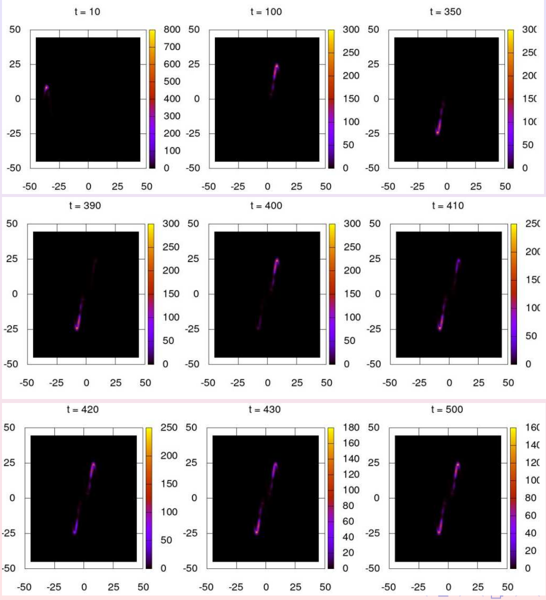
Quantum/classical strange attractor



Quantum/classical steady-state,
color $\times 10^3$, $N = 2000$, $K = 40$, $q = 0.4$, $\gamma = 0.05$, $\hbar = \omega = 1$

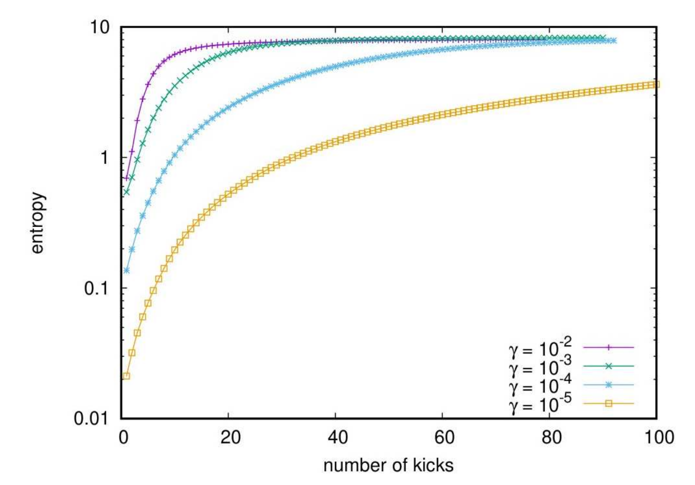
Collapse of density matrix eigenstate at max λ_i

Eigenstate Husimi at t, color $\times 10^3$, $N = 2000$, $K = 40$, $q = 0.4$, $\gamma = 0.05$



Entropy of entanglement S_E of density matrix

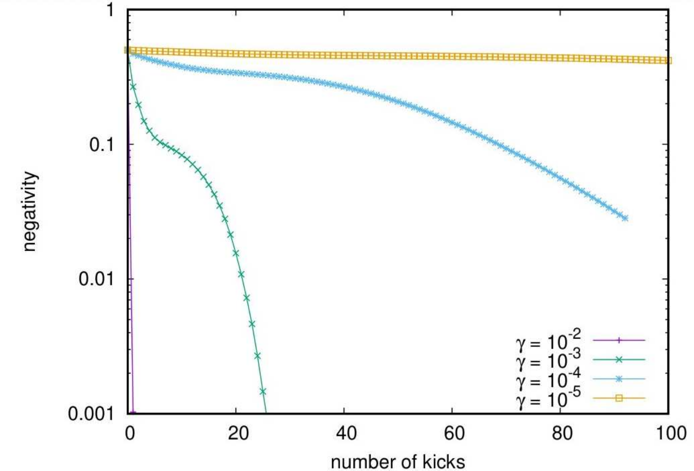
$S_E = -Tr[\rho \ln \rho] = -\sum_i \alpha^2{}_i \ln(\alpha^2{}_i)$; Schmidt decomposition
$|\psi> = \sum_i \alpha_i |u_i> \bigotimes |v_i>$



Entropy of entanglement S_E vs time t, here $K = 8$, $q = 1$, $N = 2000$ thus 4×10^6 components of ρ

Quantum negativity ℵ of density matrix

entanglement measure between effective two spins-$1/2$,
Zoller et al PRL (1997)



Here $K = 8$, $q = 1$, $N = 2000$

Discussion

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Ehrenfest collapse and explosion for quantum strange attractor in the frame of Lindblad evolution, classical quantum correspondence

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Localisation of main eigenstate of density matrix in the phase of collapse

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Delocalisation of main eigenstate of density matrix in the phase of explosion ?

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Rapid decay of quantum negativity with time

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Experimental detection of localised eigenstate of density matrix ?