Chirikov standard map

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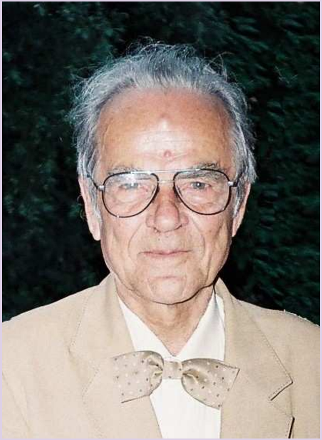
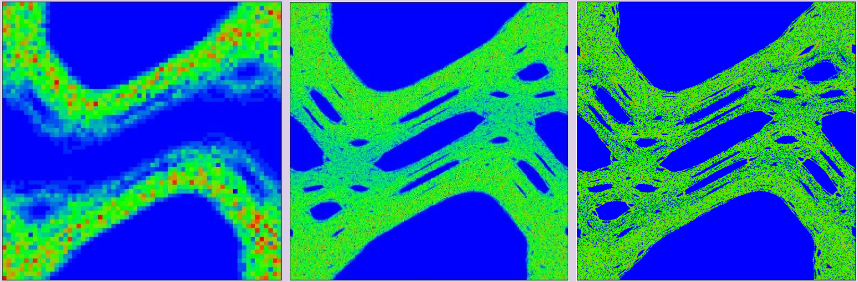
www.quantware.ups-tlse.fr/chirikov



- (1959) Chirikov criterion
- (1969) Classical map:
  $\bar{p} = p + K \sin x$
  $\bar{x} = x + \bar{p}$
- (1979) Quantum map (kicked rotator):
  $\bar{\psi} = e^{-i\hat{p}^2/2\hbar} e^{-iK/\hbar \cos \hat{x}} \psi$
- (1959-2008) Hamiltonian classical/quantum chaos:
  $H(\hat{p}, \hat{x}) = \hat{p}^2/2 + K \cos \hat{x} \sum_m \delta(t - m)$
  $[\hat{p}, \hat{x}] = -i\hbar$
- (2001) Quantum computations
- (2008) Ongoing experiments with cold atoms and Bose-Einstein condensates



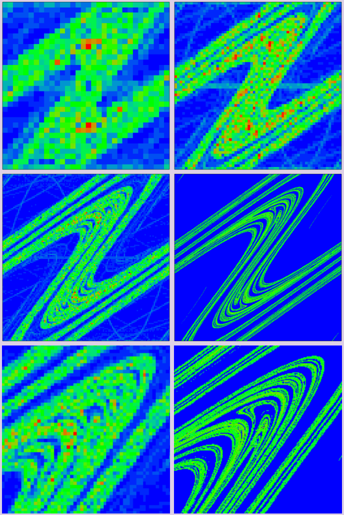
Examples of quantum/classical Poincaré sections: initial coherent state at $p = x = \pi/5$,

$K = 1.1$, $t = 2 \times 10^4$, $\hbar = 2\pi/N$, $N = 2^{n_q}$, $n_q = 12$ (left), $n_q = 16$ (middle), classical (right)

- A quantum computer with about n_q qubits performs one map iteration in $O(n_q^3)$ quantum gates for a vector of $N = 2^{n_q}$ states while a usual computer needs $O(2^{n_q})$ operations (B.Georgeot, DS, PRL **86**, 2890 (2001))
- Random matrix theory for quantum errors (K.Frahm, R.R.Fleckinger, DS, EPJD **29**, 139 (2004))
- Quantum map implementation on a 3-qubit NMR-based quantum computer at MIT with $\cos x \to x^2$ (M.K.Henry, J.Emerson, R.Martinez, D.G.Cory, PRA **74**, 062317 (2006))

Various faces of the Chirikov standard map

- Frenkel-Kontorova model (1938) - atomic chain in a periodic potential
- Veksler (1944) - particle dynamics in a microtron
- Chirikov (1969 - 1979) - properties of chaos, universality, applications
- Casati, Chirikov, Ford, Izrailev (1979) - quantum map (kicked rotator)
- Koch *et al.* (1988) - hydrogen atoms in a microwave field
- Chirikov, Vecheslavov (1989) - comet Halley
- Raizen *et al.* (1995) - dynamical localization with cold atoms
- Phillips *et al.* (2006) - Bose-Einstein condensates in kicked optical lattices
- ***
- fractal Weyl law
- Chirikov typical map
- Boltzmann - Loschmidt dispute on time reversibility,
  time reversal of Bose-Einstein condensates (BEC)
- ***
- Other faces & links: B.Chirikov, DS, Scholarpedia, 3(3):3550 (2008)



- Gamow states in the kicked rotator with absorption:

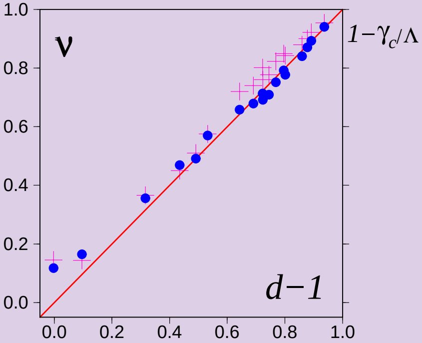
  $\bar{\psi} = \hat{P} e^{-i\hat{p}^2/4\hbar} e^{-iK/\hbar \cos\hat{x}} e^{-i\hat{p}^2/4\hbar} \psi = e^{-i\lambda - \gamma/2}\psi$

  projection on $-N/2 < n = p/\hbar < N/2$, $\hbar N/K = a = 2$, $K = 7$

  (Fig: Husimi function of quantum fractal eigenstates with minimal escape rate γ;

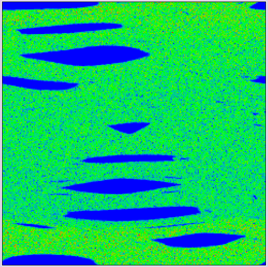
  $N = 1025, 4097, 16349$, classical; fractal dimension of strange repeller $d = 1.723$)

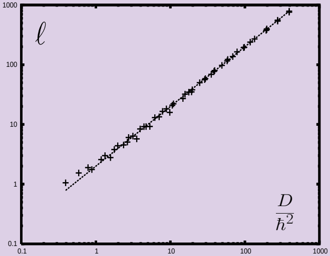
- $N_\gamma \propto N^\nu \propto \hbar^{-(d-1)}$: number of states with $0 < \gamma < \gamma_b = 8/a^2$



DS, PRE **77**, 015202(R) (2008)

Chirikov typical map (1969)





- Standard map with random, periodically repeated phases ϕ_m:
  $\bar{p} = p + K\sin(x + \phi_m), \quad \bar{x} = x + \bar{p},$
  $\phi_{m+T} = \phi_m$
  chaos border: $T^{-3/2} < K \ll 1$
  Kolmogorov-Sinai entropy: $h \sim K^{2/3} \ll 1$,
  diffusion rate per period T: $D = K^2 T/2$,
  => continuous time flow
  (Fig: Husimi function at $K = 0.1$, $T = 10$, $t = 2 \times 10^4$, $\hbar = 2\pi/N$, $N = 2^{16}$, initial coherent state at $p = 0$, $x = \pi$)

- $\ell \approx 2D/\hbar^2$: dynamical localization
  (Fig: $0.1 \le K \le 1$, $10 \le T \le 100$, $\hbar = 2\pi/17.618$)

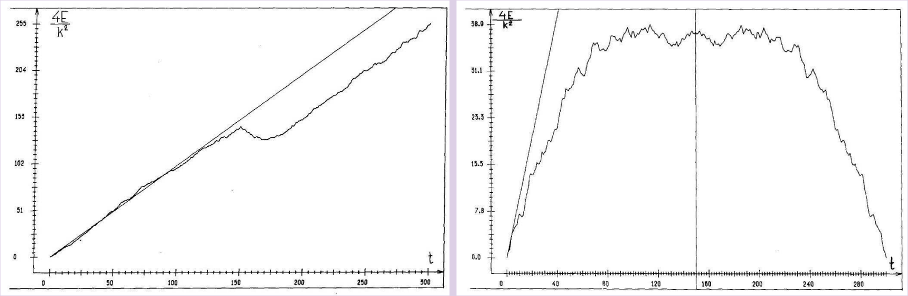
K.Frahm, DS, in preparation (2008)

Boltzmann - Loschmidt dispute on time reversibility (1876)

* irreversible kinetic theory from reversible equations

Sitzungsberichte der Akademie der Wissebschaften, Wien,
II **73**, 128 (1876); **75**, 67 (1877)

Time reversal for the Chirikov standard map



BESM-6 computation, rescaled energy or squared momentum vs. time t:

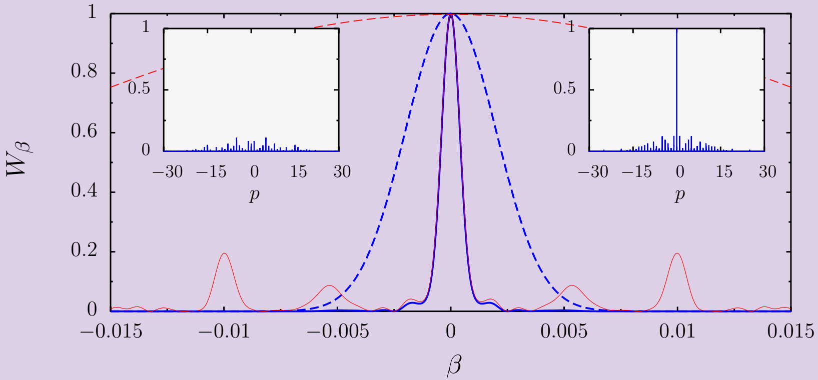
$K = 5$, $\hbar = 0$ (left), $\hbar = 1/4$ (right)

DS, Physica D **8**, 208 (1983)

* Experimental realization of time reversal:

spin echo (E.L.Hahn (1950)); acoustic waves (M.Fink (1995)); electromagnetic waves (M.Fink (2004))

* Loschmidt cooling by time reversal of atomic matter waves



proposal of time reversal in kicked optical lattices:

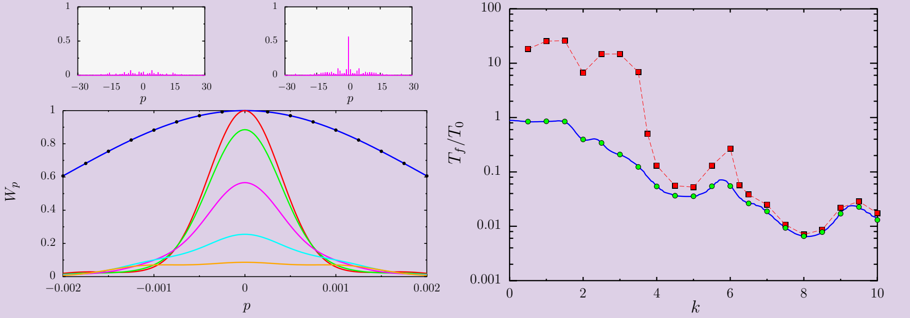
$k = K/\hbar$, $\hbar = 4\pi + \epsilon$ (forward), $\hbar = 4\pi - \epsilon$ (back) and $k \to -k$;

Fig: $k = 4.5, \epsilon = 2, t_r = 10$, $k_B T_o/E_r = 2 \times 10^{-4}$ (red), $k_B T_o/E_r = 2 \times 10^{-6}$ (blue);

momentum β and energy E_r are give in recoil units

J.Martin, B.Georgeot, DS, PRL **100**, 044106 (2008)

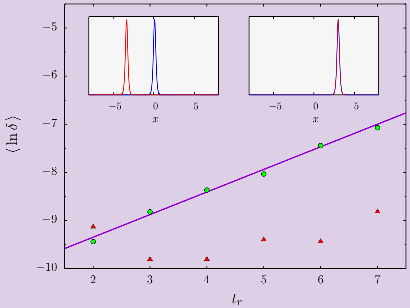
* Time reversal of Bose-Einstein condensates

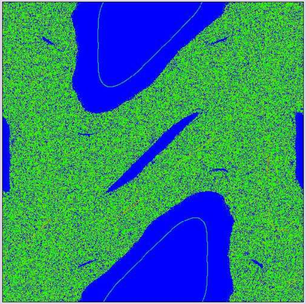


The Gross-Pitaevskii equation with kicks:

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - g|\psi|^2 + k\cos x\ \delta_T(t)\right)\psi$$

Left: same as in previous Fig. for $g = 0, 5, 10$(*insets*), 15, 20 (top to bottom), ($t = 0$);
Right: cooling ratio T_f/T_0 for $g = 0$ (blue curve), $g = 0.5$ (green), $g = 10$ (red)
J.Martin, B.Georgeot, DS, arXiv:0804.3514[cond-mat] (2008)





Soliton initial condition (Zakharov, Shabat (1973)):

$$\psi(x,t) = \frac{\sqrt{g}}{2} \frac{\exp\left(ip_0(x-x_0-p_0t/2)+ig^2t/8\right)}{\cosh\left(\frac{g}{2}(x-x_0-p_0t)\right)}$$

Left: time resersal of soliton at $g = 10, k = 1, T = \hbar = 2, K = kT = 2, t_r = 40$ inside chaotic (left inset) and regular (right inset) domains; line shows divergence given by the Kolmogorov-Sinai entropy $h = 0.45$. Right: Poincaré section at $K = 2$

But the real BEC is quantum and should return back since the Ehrenfest time $t_E \sim |\ln \hbar_{eff}|/h \sim \ln N/2h \sim 13$ for BEC with $N = 10^5$ atoms

J.Martin, B.Georgeot, DS, arXiv:0804.3514[cond-mat] (2008) massachusetts institute of technology

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Edward Lorenz, father of chaos theory and butterfly effect, dies at 90

April 16, 2008

Edward Lorenz, an MIT meteorologist who tried to explain why it is so hard to make good weather forecasts and wound up unleashing a scientific revolution called chaos theory, died April 16 of cancer at his home in Cambridge. He was 90.

A professor at MIT, Lorenz was the first to recognize what is now called chaotic behavior in the mathematical modeling of weather systems. In the early 1960s, Lorenz realized that small differences in a dynamic system such as the atmosphere--or a model of the atmosphere--could trigger vast and often unsuspected results.

These observations ultimately led him to formulate what became known as the butterfly effect--a term that grew out of an academic paper he presented in 1972 entitled: "Predictability: Does the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?"

Lorenz's early insights marked the beginning of a new field of study that impacted not just the field of mathematics but virtually every branch of science--biological, physical and social. In meteorology, it led to the conclusion that it may be fundamentally impossible to predict weather beyond two or three weeks with a reasonable degree of accuracy.

Some scientists have since asserted that the 20th century will be remembered for three scientific revolutions--relativity, quantum mechanics and chaos.

"By showing that certain deterministic systems have formal predictability limits, Ed put the last nail in the coffin of the Cartesian universe and fomented what some have called the third scientific revolution of the 20th century, following on the heels of relativity and quantum physics," said Kerry Emanuel professor of atmospheric science at MIT. "He was also a perfect gentleman, and through his intelligence, integrity and humility set a very high standard for his and succeeding generations."

Memorial service for Edward Lorenz
Sunday, April 20, 2008
3 p.m.
Swedenborg Chapel, 50 Quincy St., Cambridge

Edward Lorenz

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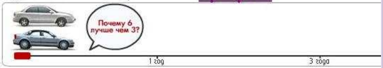

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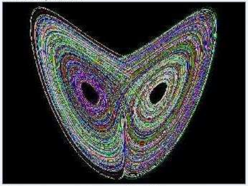

Аттрактор Лоренца. Изображение получено с помощью программы Fractint for Windows.

КОММЕНТАРИЙ

Аттрактор и бабочка

Памяти Эдварда Нортона Лоренца

В 1961 году метеоролог и математик Эдвард Лоренц, скончавшийся 16 апреля 2008 года, ввел в созданную им компьютерную модель погоды данные, округлив их не до шестого, а до третьего знака после запятой. В результате был сформулирован эффект бабочки, открыт один из странных аттракторов, обнаружена непредсказуемость поведения многих детерминированных систем и, в конечном итоге, создана теория хаоса.

1959 - Hamiltonian chaos - Chirikov, 1963 - dissipative chaos - Lorenz, ...

Boris Chirikov - Sputnik of Chaos