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CORRELATED FERMIONS AND TRANSPORT IN MESOSCOPIC SYSTEMS

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INTERACTIONS AND LOCALIZATION: TWO INTERACTING PARTICLES APPROACH

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It is shown that two repulsing / attracting particles in a random potential can propagate coherently on a distance l_c much larger than one-particle localization length l_1 without interaction. In dimension $d > 2$ this leads to delocalization of pairs formed by two repulsing / attracting particles. The results of numerical simulations allow to understand some specific features of this effect.

I. INTRODUCTION

The problem of interacting particles in a random potential attracts more and more interest during last years (see [1] and refs. therein). The situation at finite particle density is rather complicated both for analytical and numerical analysis and therefore it is desirable to have some relatively simple models which could be solved and would lead to a better understanding of effects of interaction in the presence of disorder and localization.

One of the ways to analyse the effects of interaction is to treat them in a self-consistent way. The simplified model of this approach can be described by nonlinear Schroedinger equation on a discrete lattice with disorder

$$i\frac{\partial \psi_n}{\partial t} = E_n\psi_n - \beta V|\psi_n|^2\psi_n + V(\psi_{n+1} + \psi_{n-1}) \qquad (1)$$

where V is the intersites hopping matrix element, E_n are randomly distributed in the interval $(-W, W)$ and β represents the dimensionless strength of self-consistent interaction. The model (1) and other models of a similar type with nonlinearity and disorder had been studied in [2,3]. It had been found that below critical strength of nonlinearity $|\beta| < \beta_c \sim 1$ Anderson localization persists while above β_c the localization is destroyed and anomalous diffusive spearing of probability along the lattice takes place [2,3]. Such type of transition is independent of the sign of β and therefore it looks to be quite different from the results [4,5] according to which repulsive interaction between particles ($\beta > 1$) in solid state systems leads only to a stronger localization near the ground state.

The main physical reason for delocalization transition in (1) is due to destruction of interference phase by nonlinear term representing interactions in some kind of mean field approximation. A more realistic way to study the effect of decoherence due to interaction is to consider only two interacting particles (TIP) on a disordered lattice. For a short range interaction a mean field approach leads to a nonlinear equation of type (1). However, the investigation of

the TIP quantum problem gives somewhat different result [6]. According to [6] two repulsing/attracting particles, being on a distance of one-particle localization length l_1 from each other, propagate together on a much larger length

$$l_c \sim {l_1}^2 M \frac{U^2}{32V^2} \tag{2}$$

where U is the strength of on site interaction between two particles, V is the one-particle hopping element between nearest sites which determines the size of one-particle energy band. The parameter M notes the number of transverse channels for the case when both particles are moving in a thick wire, where by itself $l_1 \propto M$. Also in (2) it is assumed that the intersite constant $a = 1$ and the total TIP energy is somewhere close to the center of the band so that wavevector $k_F \sim 1/a = 1$. The same expression for l_c was also derived in another way by Imry [7] whose approach is based on the Thouless block picture and scaling of conductance. The reason for the difference between the model (1)and TIP result (2) is not completely clear. Probably, the mean field approximation is not exact enough.

The derivation of the result (2) is based on some assumptions which look quite natural but still at the moment have not been rigorously justified. The main of them are the estimate of interaction induced transition matrix elements which are assumed to have the same order of magnitude and a partial neglect of correlations between TIP moving in the same random potential. Therefore it was quite important to check the existence of the TIP effect (2). First numerical simulations indicating the existence of enhancement for TIP localization length had been presented in [6]. Much more advanced numerical investigations for TIP in one-dimensional Anderson model had been done by Pichard and coworkers [8] and von Oppen and coworkers [9]. Their results show that the exponent γ of power growth $l_c \propto {l_1}^\gamma$ is close to the theoretical value $\gamma = 2$. The value of $\gamma \approx 2$ was also found by the transfer matrix technique for a bag model [6,8] in which particles do not interect if the distance between them is less than the bag size $B > l_1$. A model with strong attraction between particles in a well of size $B \ll l_1$ had been first studied analytically by Dorokhov [10] who had found that the propagation length of such strongly coupled particles could be enhanced. By an extrapolation to $B \sim l_1$ he argued that in this case the localization length is proportional to ${l_1}^2$. The case with a short range repulsive/attractive interaction is much less evident and more detailed numerical simulations are still required especially since the results [9] give first power of U for l_c instead of U^2 in (2).

The TIP effect in higher dimensions d have been studied in [7,11–13] where it has been shown that in $d > 2$ TIP pairs can be delocalized below Anderson transition when all one-particle states are exponentially localized. At the moment only few results have been obtained for a number of particles larger than two [14,15] and for a finite density of particles [7,16] where the existence of the Fermi level plays an important role.

II. TIP LOCALIZATION

The derivation of expression (2) can be done in the following way. It is convinient to rewrite the Sroedinger equation for TIP in the basis of noninteracting eigenstates. In this basis the diagonal part is given by the sum of one-particle eigenenergies $\epsilon_{m_1} + \epsilon_{m_2}$. The transitions between noninteracting eigenstates are only due to interaction and their matrix elements are given by

$$U_s = U \sum_{n_1,\tilde{n}_1,n_2,\tilde{n}_2} \tilde{R}^+_{n_1,\tilde{n}_1,m_1,\tilde{m}_1} \tilde{R}^+_{n_2,\tilde{n}_2,m_2,\tilde{m}_2} \tilde{R}_{n_1,\tilde{n}_1,m_1',\tilde{m}_1'} \tilde{R}_{n_2,\tilde{n}_2,m_2',\tilde{m}_2'} \delta_{n_1,n_2} \delta_{\tilde{n}_1,\tilde{n}_2} \tag{3}$$

where indices $n_{1,2}, \bar{n}_{1,2}$ mark correspondingly the positions of first and second particles along and transverse a strip with M channels, m-s are the indices of eigenstates without interaction which mark the maximum of a state along the strip. The matrix R gives the transformation between the lattice basis and one-particle eigenstates so that $\bar{R}_{n,\bar{n},m,\bar{m}} \approx \exp(-\mid n-m \mid/l_1 - i\theta_{n,\bar{n},m,\bar{m}})/\sqrt{Ml_1}$ where θ randomly changes with indices.

Due to exponential decay of R only about $(Ml_1)^{1/2}$ terms with random signs contribute in sum (3) so that the typical value of U_s for $|m_1 - m_2| < l_1$ is $U_s \approx U/(Ml_1)^{3/2}$ [2,6]. For $|m_1 - m_2| > l_1$ the matrix elements decay exponentially fast and at first approximation they can be neglacted. Thefore, the total number of coupled states is $b \sim (Ml_1)^2$. All these states are inside the energy band $4V$ so that the density of coupled states is $\rho_c \sim (Ml_1)^2/V$. Using the Fermi golden rule we can now determine the interaction induced transition rate

$$\Gamma \sim {U_s}^2 \rho_c \sim \frac{U^2}{VMl_1} \tag{4}$$

The typical size of such transitions is of order l_1 so that they give the TIP pair diffusion rate along the strip

$$D_p \sim {l_1}^2 \Gamma \sim (U/V)^2 V l_1/M \sim (U/V)^2 D_1 \tag{5}$$

where D_1 is one-particle diffusion rate on a short time scale. In all these estimates it was assumed $U < V$ and therefore we see that the diffusion rate due to interaction is not enhanced ($D_p \leq D_1$) that is in agreement with the numerical results [6,12]. The diffusion of TIP pair arises as the result of interparticle collisions which destroy quantum interference phase and give coherent TIP propagation.

Knowing the diffusion rate D_p it is possible to find the localization length l_c for a pair in a way similar to that one used for dynamical localization in the kicked rotator [17,18]. Indeed, due to diffusion the total number of excited noninteracting eigenlevels grows as $\Delta N \sim \Delta m_1 (M^2 l_1) \delta E/V$ where $M\Delta m_1 \sim M(D_p t)^{1/2}$ gives the number of excited sites for the first particle and the additional factor Ml_1 takes into account that the distance between two particles is approximately l_1. Also generally not all coupled nointeracting eigenlevels are excited but only a fraction of levels in some energy interval δE. Usually, $\delta E \sim \Gamma$ [19] but similar to the case of photonic localization [18] the actual value of δE does not enter in the final answer for localization length. All ΔN levels are homogeneously distributed in the energy interval δE and therefore the level spacing between them is $\Delta\nu \sim \delta E/\Delta N$. Due to uncertainty relation between frequency and time after the time t^* defined from the relation $\Delta\nu \sim 1/t^*$ the discrete nature of the lines in the spectrum is resolved and the diffusion, which should have a continuous spectrum, is stopped. This gives the localization time and the localization length

$$t^* \sim M^4 {l_1}^2 D_p/V^2, \quad \Delta m_{1,2} \approx \Delta n_{1,2} \approx (D_p t^*)^{1/2} \approx l_c \sim M^2 l_1 D_p/V \sim l_1 \Gamma \rho_c \tag{6}$$

in agreement with (2). The last relation $l_c/l_1 \sim \Gamma\rho_c$ established in [6] is the same as for photonic localization in a complex molecular spectrum [18] with the only difference that here the size of transition is not the photon frequency but one-particle localization length. This relation shows that the length l_c is determined by two-particle spread width Γ which can be extracted from the Breit-Wigner distribution of TIP eigenstates over eigenbasis of noninteracting particles (see [19] and section V).

It is interesting to note that the final answer for l_c/l_1 looks in such a way as interaction is enhanced by the squareroot from the number of components $N_1 \sim l_1$ in one-particle eigenfuction ($U_{eff} \sim U\sqrt{l_1}$). A similar effect had been intensively studied for enhancement of weak interactions in nulcei [20]. However, there even being enhanced the effect was small and did not give large physical changes.

III. TIP DELOCALIZATION

Similar approach based on the uncertainety relation between frequency and time can be used also in higher dimensions d. For that we should take into account that $U_s \sim U/N_1^{3/2}$ with $N_1 \sim l_1^d$. Therefore $\Gamma \sim U^2/(VN_1)$ and the TIP diffusion rate $D_p \sim l_1^2\Gamma \sim V(U/V)^2 l_1^{2-d}$. It is interesting to note that for $d = 2$ the diffusion D_p is independent on l_1 while for $d = 3$ it decreases with l_1 which in its own turn increases when approaching the one-particle Anderson transition. Due to diffusion the number of excited levels grows as $\Delta N \sim (D_p t)^{d/2} l_1^d \delta E/V$. The level spacing is $\Delta\nu \sim \delta E/\Delta N$ and should be compared with frequency resolution $1/t$. For $d = 2$ the ratio $1/(\Delta\nu t) \sim (U/V)^2 l_1^2$ is independent on time and as usual in $d = 2$ the localization length is proportional to the exponent of this ratio:

$$\ln(l_c/l_1) \sim (Ul_1/V)^2 > 1 \tag{7}$$

Since in $d = 2$ the localization length l_1 grows exponentially with decrease of disorder ($\ln l_1 \sim (V/W)^2$) the enhancement (7) is enormous.

In $d = 3$ the spacing $\Delta\nu$ decreases faster than $1/t$ and therefore the TIP pair will be delocalized if at the moment $t' \sim l_1^6/V$, which is determined by two particle level spacing for a block of size l_1 and during which diffusion is always going on, the value of $\Delta\nu$ is less than $1/t'$. This gives the condition of TIP pair delocalization in $d = 3$ while one-particle states are well localized (see also [7,12]):

$$(U/V)^2 l_1^3 > 1 \tag{8}$$

While the results (7), (8) are qualitatively correct they however don't take into account the effect of possible pair size growth with time which for a first time was discussed in [12]. Indeed, the above derivation of Γ and D_p is local and it assumes that the TIP pair size is always of the order of l_1. This would be correct for a bag model in which particles are confined in a bag of size l_1 with infinite walls. But for our short range interaction the separation of particles is not excluded. Indeed, there are always matrix elements U_- which give an increase of the pair size $n_- = |n_1 - n_2|$. Due to exponential decrease of the operlapping probability these transitions decay exponentially with n_- as $U_- \sim U\exp(-n_-/l_1)/l_1^{3d/2}$ that gave the reason to neglact them in the above consideration. However, the existence of such transitions should definitely produce a slow diffusive pair size growth [12] $n_-^2/t \sim D_- \sim D_p \exp(-2n_-/l_1)$. This gives the logarithmic growth $n_- \sim l_1 \ln t/2$ which should also change the diffusion rate D_p of pair propagation. Qualitatively the modification of $D_p \to \tilde{D}_p$ can be understood in the following way. Since the pair size becomes in $\ln t$ larger the probability of particles collisions which from ergodicity is inversely proportional to the pair volume becomes in $(\ln t)^d$ times smaller. The probability of collisions is proportional to Γ so that finally $\tilde{D}_p \sim D_p/(\ln t)^d$ where we assumed that still the typical size of transition is l_1. Therefore, the average square of displacement of the center of mass of the pair $\sigma_+ =< (n_1 + n_2)^2 > /4$ grows in a subdiffusive way $\sigma_+ \sim D_p t/(\ln t)^d$. Here the power of $\ln t$ is the same as in [13] where it was obtained on the basis of supersymmetry approach. However, the stickings in the regions with $n_- \gg l_1$ can lead to quite large fluctuations and therefore a more rigorous analysis of logarithmic corrections is still desirable.

The numerical simulations for TIP in 3d are very heavy and at the moment there are only numerical results obtained in [12] for the model of two interacting kicked rotators in effective 2-3 dimensions. The evolution operator of the model is

$$\begin{aligned}\hat{S}_2 = &\exp\{-i[H_0(\hat{n}) + H_0(\hat{n}') + U\delta_{n,n'}]\} \\ &\times \exp\{-i[V(\theta, t) + V(\theta', t)]\}\end{aligned} \tag{9}$$

with $\hat{n}^{(\prime)} = -i\partial/\partial\theta^{(\prime)}$. Here $H_0(n)$ is a random function of n in the interval $[0, 2\pi]$ and it describes the unperturbed spectrum of rotational phases. The perturbation V gives the coupling between the unperturbed levels and has the form $V(\theta, t) = k(1 + \epsilon\cos\theta_1\cos\theta_2\cos\theta_3)\cos\theta$ with $\theta_{1,2,3} = \omega_{1,2,3}\, t$. For incommensurate frequencies one can go to the extended phase space by replacing $H_0(n) \to H_0(n)+\omega_1 n_1+\omega_2 n_2+\omega_3 n_3$ where new actions $n_{1,2,3}$ are conjugated to phases $\theta_{1,2,3}$. We used $\omega_1 = 2\pi\lambda^{-1}$, $\omega_2 = 2\pi\lambda^{-2}$ with $\lambda = 1.3247...$ the real root of the cubic equation $x^3-x-1 = 0$ and $\omega_3 = 2\pi/\sqrt{2}$. Then without interaction the effective dimension for one rotator is $d = 4$ and at fixed $\epsilon > 0$ the one-particle delocalization takes place for $k > k_{cr}$. With switched on interaction U the total dimension of the extended phase space is five for two rotators so that it is possible to say that each rotator is moving in effective dimension $d_{eff} = 5/2$. According to the above picture of TIP delocalization the pair in (9) can be delocalized due to interaction for $k < k_{cr}$. An example of such delocalization for second moments $\sigma_+ =< (n + n')^2/4 >$ and $\sigma_- =< (n - n')^2 >$ is shown in Fig.1.

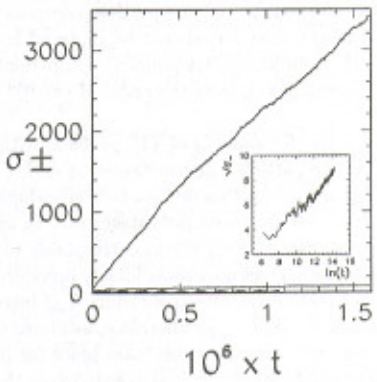

FIG. 1. Dependence of second moments on time in model (9) with $k = 0.7$, $\epsilon = 0.9$ ($k_{cr} \approx 1.15$); upper curve is σ_+ ($U = 2$), middle is σ_- ($U = 2$), lower is σ_+ ($U = 0$). At $t = 0$ both particles are at $n = n' = 0$, basis is $-250 \le n, n' \le 250$. Inset shows the dependence of $\sigma_-^{1/2}$ on $\ln(t)$ (after [12] b).

In agreement with the above theoretical arguments the pair size grows logarithmically with time as $n_- \approx \sqrt{\sigma_-} \sim \ln t$. The behaviour of σ_+ is consistent with $\sigma_+ \propto t/\ln t$. In the model (9) the interaction is only along one direction and therefore the probability of collision

decreases as $1/n_- \sim 1/\ln t$ (and not as $\ln^{-3} t$ for real $d = 3$) that explains the dependence $\sigma_+/t \sim 1/n_- \sim 1/\ln t$ (see also [12] b). The numerical results for two interacting kicked rotators with incommensurate frequencies clearly demonstrate the effect of pair delocalization below one-particle delocalization border.

IV. OTHER MODELS

Let us now discuss other different models of TIP in a random potential. A different type of situation corresponds to the case with short but finite radius of interaction $R < l_1$. In d dimensions the sum similar to (3) should be taken over R^d nearby sites. All these terms have random signs and therefore the effective value of U_s becomes $R^{d/2}$ times larger $U_s \sim UR^{d/2}/{N_1}^{3/2}$ where $N_1 \sim {l_1}^d$ is the number of components in one-particle eigenfunction. As the result, the enhancement parameter is $\eta \sim [(U/V)^2 R^d]{l_1}^d$ that is R^d times larger than for on site interaction. As before the enhancement parameter determines the ratio $l_c/l_1 \sim \eta$ in $d = 1$, $\ln l_c/l_1 \sim \eta$ in $d = 2$ and the TIP pair delocalization border $\eta > 1$ in $d = 3$. However, it should be taken into account that similar to the case $R = 1$ the maximal value of η cannot be larger than N_1. Indeed, in this case the interaction is too strong and it starts to deform the noninteracting density of state. Also it is clear that interaction creates some effective wire along the diagonal on the lattice of two-particle index (n_1, n_2) and the number of channels in this wire cannot be larger than the number of one-particle components N_1. Therefore, if the parameter $(U/V)^2 R^d$ becomes larger than 1 it should be replaced by 1. For the case of small energies when $k_F \ll 1/a = 1$ the number of independent components is proportional to $k_F l_1$ and the number of terms in the sum for U_s is of the order of $(k_F R)^d$ so that the enhancement is $\eta \sim k_F l_1 (U/V)^2 (k_F R)^d$.

The above result can be used also for analysis of TIP problem with a long range interaction. For concreteness let us consider two particles on a distance r_{12} with Coulomb interaction α/r_{12} in $d = 3$ random potential. Without interaction one-particle eigenfunctions are spreaded over a size of localization length l_1. Interaction two-particle states mixing appears only in the second order of expansion over small parameter l_1/r_{12} which corresponds to dipole-dipole interaction $U_{dd} \sim \alpha {l_1}^2/{r_{12}}^3$. Indeed, the first term gives only some locally homogeneous field which does not destroy localization. Due to localization the effective radius R of interaction which determines the number of terms in the sum for U_s is $R \sim l_1$. Therefore, as above the enhancement factor in $d = 3$ is $\eta \sim [(U_{dd}/V)^2 {l_1}^3]{l_1}^3$ and TIP delocalization takes place for $\eta > 1$. As before the term in the square brackets is supposed to be less than 1. It is easy to see that TIP can be delocalized even when two particles are very far from each other $r_{12} \gg l_1$. Due to the homogeneous local field between particles they will diffusively approach to each other in the case of attraction ($\alpha < 1$) or separate up to distance r_{12} where $\eta \sim 1$ in the case of repulsion ($\alpha > 1$).

We also can consider other type of TIP problem in which two particles are moving in parallel strips with independent disorder in each strip. The number of channels in the strips is M_1 and M_2 while the localization length for each particle without interaction is l_1 and l_2 correspondingly. We will assume that $M_2 \geq M_1$ and $l_2 \geq l_1$ and that interaction is local with $U\delta_{n_1,n_2}\delta_{\tilde{n}_1,\tilde{n}_2}$ where $n_{1,2}$ are the indices along the strips while $\tilde{n}_{1,2}$ mark the transverse direction. To estimate the interaction induced transition matrix elements it is necessary to take into account that the number of terms contributing to the sum for U_s similar to (3) is of the order of $l_1 M_1$ and therefore $U_s \sim U/(l_2 M_2 \sqrt{l_1 M_1})$. The density of coupled states is $\rho_c \sim l_1 l_2 M_1 M_2/V$ and the transition rate $\Gamma \sim U^2/(V l_2 M_2)$. The diffusion rate of the first particle is $D_1 \sim {l_1}^2 \Gamma \sim U^2 {l_1}^2/(V l_2 M_2)$. In a way similar to the one used above we obtain the localization length l_{c1} for the first particle

$$l_{c1}/l_1 \sim \Gamma\rho_c \sim (U/V)^2 l_1 M_1 \tag{10}$$

Surprisingly, l_{c1} does not depend on the characteristics of the second particle. The localization length for the second particle is $l_{c2} \approx l_2$ if $l_2 \gg l_{c1}$ and $l_{c2} \approx l_{c1}$ if $l_2 \ll l_{c1}$.

It is possible to modify slightly the model taking interaction independent on the transverse direction $U\delta_{n_1,n_2}$ and putting $M_1 = 1$ (see also [15]). Then the number of terms in the sum of type (3) is in M_2 times larger. Therefore, we have the transition rate $\Gamma \sim U^2/(Vl_2)$, the diffusion rate $D_1 \sim U^2 l_1^2/(Vl_2)$ and $l_{c1}/l_1 \sim (U/V)^2 l_1 M_2$. Such kind of situation in higher dimension corresponds to the model (9) where interaction depends only on one direction and where $l_1 \sim l_2$. Therefore the TIP diffusion rate in (9) is $D_1 \sim V(U/V)^2 l_1$ and it grows when approaching one-particle delocalization border that is in agreement with numerical data (see [12]b, Fig.13). It is interesting to note that a similar type of model effectively describes the case of three interacting particles in a 1d chain where $M_2 \sim (U/V)^2 l_1$ and three-particle localization length is $l_{c1}/l_1 \sim (U/V)^4 l_1^2 > 1$ [15].

V. SUPERIMPOSED BAND RANDOM MATRICES

Under some approximations the TIP problem can be reduced to some kind of band random martix (BRM) model. Indeed, if to write the Hamiltonian in the noninteracting eigenbasis then it will be represented by a matrix with a strong diagonal ($\epsilon_{m_1} + \epsilon_{m_2} \sim V$) and weak ($U_s \sim U/l_1^{3/2} \ll V$) but broad BRM with approximately b diagonals where $b \sim l_1^2$ is the number of noninteracting eigenstates coupled by direct transitions and for concreteness we discuss 1d case. Normalizing the nondiagonal elements in a usual way (amplitude $\pm 1/\sqrt{2b+1}$) and ordering the levels which are in the strip of size $\sim l_1$ along levels with $m_1 \approx m_2$ the Hamiltonian matrix will be reduced to a superimposed BRM (SBRM) with diagonal fluctuations in the interval $\pm W_b$ with $W_b \sim V\sqrt{l_1}/U$ [6]. By transfer matrix technique it is easy to investigate the dependence of localization length l_{sb} in SBRM on different parameters. It was shown [6] that for $W_b < \sqrt{b}$ the length scales approximately as $l_{sb} \approx 0.5(b/W_b)^2$ while in the perturbative regime $W_b > \sqrt{b}$ it is $l_{sb} \sim b/\ln(W_b^2/b)$. This result can be understood in a way similar to (6) [6]. Indeed, the density of coupled states is $\rho_c \sim b/W_b$ and then the transition rate $\Gamma \sim (1/\sqrt{b})^2 b/W_b \sim 1/W_b$. As the result, the number of transitions is $l_{sb}/b \sim \Gamma\rho_c \sim b/W_b^2 > 1$. Taking into account that for TIP $b \sim l_1^2, W_b \sim U\sqrt{l_1}/V$ and $l_c \sim l_{sb}/l_1$ we can see that the result for SBRM leads to the same expression (2) for l_c.

If the transition rate $\Gamma \sim 1/W_b$ is larger than the level spacing $1/\rho_c$ then an eigenstate contains many unperturbed sites with diagonal energies E_n being in the interval of size Γ near the eigenvalue E_λ. In other words the local density of states $\rho_W(E-E_n) = \sum_\lambda |\psi_\lambda(n)|^2 \delta(E-E_\lambda)$ has the spread width Γ and is described by the well-known Breit-Wigner distribution [19]:

$$\rho_{BW}(E-E_n) = \frac{\Gamma}{2\pi((E-E_n)^2 + \Gamma^2/4)};\ \Gamma = \frac{\pi}{3W_b} \tag{11}$$

The numerical results [19] confirm that the local density of states ρ_W is well fitted by (11). This result is correct both for infinite matrix and for matrix of finite size $N < l_{sb}$. The Breit-Wigner distribution leads to a peaked structure of eigenfunctions since only levels within $|E_n - E_\lambda| < \Gamma$ are populated. The number of peaks determines the inverse participation ratio (IPR) $\xi \sim \Gamma\rho$. For delocalized case $N \ll l_{sb}$ the level density is $\rho \sim N/W_b$ so that $\xi \sim N/W_b^2 \ll N$ is much smaller than the system size. In the localized case $N \gg l_{sb}$ the value of N should be replaced by l_{sb} and then the IPR $\xi \sim l_{sb}/W_b^2$ is much less than the localization length l_{sb} [19]. These results have been also ontained on a more rigorous basis by supersymmetry

approach in [22,21]. For original TIP problem this means that the IPR in the noninteracting eigenbasis $\xi_c \sim (U/V)^4 {l_1}^2 > 1$ is much less than the number of lattice sites $l_1 l_c$ contributing in an eigenfunction [19]. Similarly, if one-particle eigenfunction is ergodic in a d-dimensional box of size L then still the IPR value in noninteracting eigenbasis is much smaller than the total Hilbert space: $\xi_c \sim \Gamma\rho \sim L^d (U/V)^2 \ll L^{2d}$ [19]. The existence of Breit - Wigner distribution leads to a deviation of the number variance $\Sigma^2(E)$ from the random matrix behavior for energies $E > \Gamma$ where the rigidity of levels disappeares [23].

Above we analysed SBRM with bounded fluctuations of matrix elements. It is interesting to look what will happen if the nondiagonal matrix elements $H_{nn'}$ have a Cauchy distribution: $H_{nn'} = \tan(\phi_{n,n'})/\sqrt{2b+1}$ where $\phi_{n,n'}$ is a random phase in the interval $[0, \pi]$ while the fluctuations on the diagonal are still bounded $-W_b < H_{nn} < W_b$. One of the reasons to be interested in such kind of fluctuations is due to numerical results [8,24] which have shown that the distribution of interaction induced matrix elements U_s for TIP problem on 1d lattice has very long power tails. There are some numerical and analytical indications [24] that U_s can be discribed by a Cauchy distribution with a typical width $U/{l_1}^2$ and a cutoff at $U_s > U/l_1$.

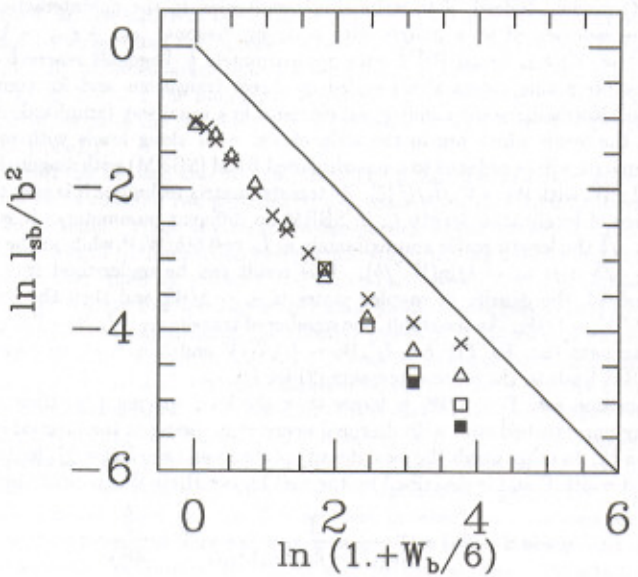

FIG. 2. Dependence of localization length l_{sb} in SBRM with Cauchy fluctuations on the strength of diagonal fluctuations W_b for different band widths with $2b+1 = 41(X)$; 81 (triangle); 161 (open square); 321 (full square). Full line shows the slope -1.

The transfer matrix numerical investigations for SBRM with off diagonal Cauchy fluctuations defined above show that localization length in this case scales as $l_{sb}/b \approx 3b/W_b > 1$ (see Fig.2).

The $1/W_b$ behaviour is similar to the case of Lloyd model however its analytical derivation still should be done. If to map this result on the TIP case assuming that there U_s has the Cauchy distribution with width $U/{l_1}^2$ then it would give $l_c/l_1 \sim Ul_1$ [24] in agreement with numerical result [9] for the center of the band. However, for a serious application to the TIP problem more rigorous investigations are required.

VI. CONCLUSIONS

Above we discussed the effect of interaction induced enhancement of localization length or delocalization mainly for only two or few particles. In the real physical situation the density of particles is finite and the situation is much more complicated. However, it is possible to think that an effect similar to TIP effect can take place for quasi-particles. The first estimates for such a case have been done by Imry [7]. They indicate that in 3d case a mobility edge for pairs of two quasi-particles near Fermi energy is lower than one-particle mobility edge. According to Imry such difference in one- and two-particle edges can be responsable for anomalous dependence of conductance on temperature observed in the experiments [25]. However, more detailed investigations in this direction are required to clearify the situation. An interesting approximate numerical approach has been developped [16] for finite density case and applied for 1d chain. There the enhancement of l_c can take place only sufficiently far from Fermi energy in agreement with [7]. However, the 3d case still remains open for investigations. The TIP effect can be also important for photo-conductance when an excited pair of quasi-particles is sufficiently far from Fermi edge and suppression of interaction due to small phase volume disappeares.

Another interesting question is if the TIP effect can take place in Luttinger liquid. On the first glance it seems to be not the case since the dynamics for Luttinger liquid is in some sense completely integrable while for TIP a quite important element was associated with the ergodic structure of eigenfunctions and nonintegrability. However, a more detailed analysis is required to answer this question.

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INTERACTIONS ET LOCALISATION : APPROCHE PAR UN MODÈLE DE DEUX PARTICULES EN INTERACTION

Nous montrons que deux particules en interaction répulsive ou attractive placées dans un potentiel aléatoire peuvent se propager de façon cohérente sur une distance beaucoup plus grande que la longueur de localisation d'une particule sans interaction. En dimension $d > 2$, ceci conduit à une délocalisation des paires formées par deux particules en interaction répulsive ou attractive. Les résultats des simulations numériques permettent de comprendre certaines caractéristiques spécifiques de cet effet.