TWO ELECTRON VIEW ON METAL-INSULATOR TRANSITION IN TWO DIMENSIONS

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The model of two electrons with Coulomb interaction on a two-dimensional (2D) disordered lattice is considered. It is shown that the interaction can give a sharp transition to delocalized states in a way similar to the Anderson transition in 3D. The localized phase appears when the ratio of the Coulomb energy to the Fermi energy becomes larger than some critical value dependent on the disorder. The relation to the experiments on metal-insulator transition in 2D is also discussed.

1 Introduction

According to D. Tsui [1], "the important thing is the interplay between disorder and electron-electron interactions. The FQHE (fractional quantum Hall effect) is, in some sense, the clean limit. But there's another limit, where both interaction and disorder are important ... there's always some disorder." Indeed, the recent experimental discovery of metal-insulation transition in two dimensions (2D) by Kravchenko *et al.* [2] attracted a great interest to this problem. This transition is especially surprising since according to the well established theoretical result [3] all states of non-interacting electrons in 2D disordered potential are exponentially localized. However, in reality the electron-electron interaction is present and the original result [2], as well as the new results of different groups in experiments with different materials [4,5,6,7,8,9,10], show that the interaction can induce metallic behavior. Indeed, the majority of experiments are done in the situation where the parameter $r_s = 1/\sqrt{\pi n_s} a_B^* \simeq E_{ee}/E_F \gg 1$. Here, E_{ee} is the energy of Coulomb interaction, E_F is the Fermi energy determined by the charge density n_s and a_B^* is the effective Bohr radius. In some experiments the r_s value was as large as 10 - 30. In this situation the electrons are located far from each other and in a first approximation it is natural to consider the problem of only two electrons with Coulomb interaction. The first consideration of two particles with strong attraction was done by Dorokhov [11] but it was ignored by the community. The studies of two interacting particles with short range interaction showed that repulsive/attractive interaction can lead to a strong increase of localization length or even to delocalize pairs of particles in dimension $d > 2$ [12,13,14,15,16]. According to [12,13] in 2D the pairs of particles remain localized and their localization length l_c grows smoothly with the increase of disorder strength U or one-particle localization length l_1: $\ln(l_c/l_1) \sim \kappa >$ with $\kappa \sim \Gamma_2 \rho_2$, where $\Gamma_2 \sim U^2/V l_1^2$ is the interaction induced transition rate, $\rho_2 \sim l_1^4/V$ is two-particle density of states in the middle of the band and V is the hopping strength proportional to the energy band size B ($B = 4V$ for weak disorder). The case of the long range Coulomb interaction requires separate analysis. Generally, one can expect that the delocalization effect will be even stronger in this case since the particles are always interacting, in a difference from a short range interaction case.

2 Analytical estimates

The first estimates for two electrons with Coulomb interaction on a 2D disordered Anderson lattice were presented in [17]. The lattice is characterized by the nearby hopping V and the diagonal disorder in the interval $[-W/2, W/2]$, while the interaction is $U/|r_1 - r_2|$. Then the parameter $r_s = U/(2V\sqrt{\pi n_s})$ where n_s is the filling factor. If the distance R between the electrons is much larger than the one-particle localization length l_1 ($\ln l_1 \sim (V/W)^2$) then the

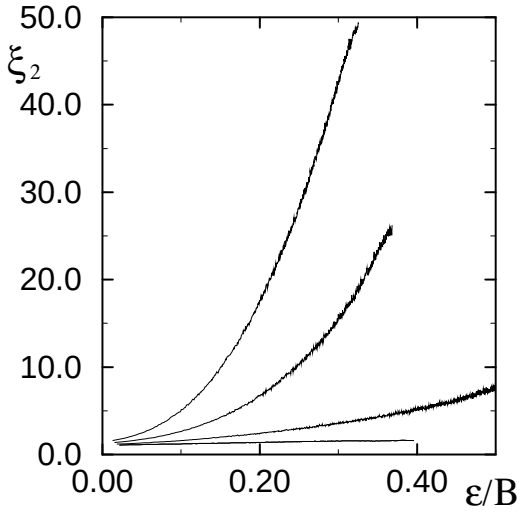

Figure 1: Dependence of the inverse participation ratio ξ_2 on rescaled one-particle energy ϵ/B for $U/V = 2$, $L = 16$ ($r_s = 6.38, B = 4V$) and $W/V = 5, 7, 10, 15$ (from up to down).

two-body coupling appears only in the dipole-dipole interaction term. This gives the typical matrix element $U_s \sim U/R^3$ [17] and the transition rate $\Gamma_2 \sim U_s^2 \rho_2$, where still ρ_2 is determined by the estimate given above since the electrons can have a jump only on a distance l_1 from the initial position (otherwise wave function overlap drops exponentially). As the result the two electron levels become mixed by interaction when $\kappa_e = \chi_e^2 \sim \Gamma_2 \rho_2 \sim (r_L^{4/3}/r_s)^2 > 1$, where r_L is the value of r_s at the density $n_s = 1/l_1^2$ (one electron in a box of l_1 size). For $\chi_e > 1$ the Coulomb interaction leads to a delocalization of two electrons in a way similar to 3D Anderson transition[17]. Indeed, in this case the hopping goes effectively in 3D: the center of mass moves in 2D and in addition the electrons slowly rotate around it that gives 3 dimensions. The rotation goes on a ring of width l_1 and of radius $R \sim l_1^{4/3} \gg l_1$ (for $U \sim V$); the size of the ring is fixed by the energy conservation $\epsilon \sim U/R$. Due to that the length l_c changes sharply from $l_c \sim l_1$ to $l_c \sim l_1 \exp(\pi l_1^{1/3} \kappa_e)$ when κ_e crosses the critical value $\kappa_e \sim 1$ [17]. It is interesting to note that, as in the experiments (see Refs. 2,4-10), the localized phase corresponds to the large values of r_s: physically the two-body interaction becomes weaker at low density. The diffusion rate in the metallic phase can be estimated as $D_e \sim l_1^2 \Gamma_2 \sim V \kappa_e / l_1^2$. Near the critical point $\kappa_e \sim 1$ the diffusion rate (conductivity) drops with the decrease of disorder (increase of l_1). These estimates are done for the excited states in the middle of the band.

3 Numerical results

The above problem of two electrons in the 2D Anderson model in the triplet state is studied numerically. The maximal lattice size is $L = 24$. The numerical diagonalization is done in the following way: the Hamiltonian is rewritten in the basis of noninteracting eigenstates, from which only first M low energy one-particle states (orbitals) are selected and after that the Hamiltonian is diagonalized exactly. The special check is done to ensure that the low energy states are not effected by the above cutoff (e.g. by changing M in few times). Usually ND=4000 disorder realizations are used to average the fluctuations. The fact that the effect of interaction strongly depends on l_1 (or W) is demonstrated in Fig. 1. Indeed, here the number of noninteracting eigenstates ξ_2 contributing in an eigenfunction at fixed interaction $U/V = 2$ is increased in about 50 times only by the change of the disorder W. This confirms the analytical result

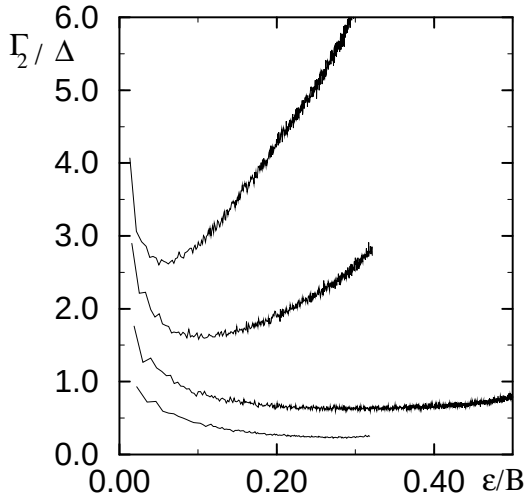

Figure 2: Dependence of the rescaled transition rate Γ_2 (defined via the relation $\Gamma_2 = (\xi_2 - 1)/\rho_2(\epsilon)$ with $\rho_2(\epsilon)$ being the two-electron density of states) on ϵ/B for the parameters of Fig.1 and the same order of curves, $\Delta = B/L^2$.

according to which the effect of interaction becomes stronger for larger l_1 since $\xi_2 \sim \Gamma_2\rho_2(\epsilon)$. The last relation allows to determine numerically the dependence of the transition rate Γ_2 on the excitation energy $E = 2\epsilon$, counted from the ground state. This dependence is presented in Fig. 2 and for a moderate disorder shows a very flat dependence on ϵ and even a certain increase of Γ_2 very close to the ground state. If to assume that the matrix element U_s is independent of ϵ then Γ_2 should drop linearly with ϵ since $\rho_2(\epsilon) \approx l_1^4\epsilon/V^2$ [13]. However, for a short range interaction is has been shown that for localized states Γ_2 can be independent of ϵ due to enhanced return probability near the Fermi level [18]. For the case of long range interaction similar effects can be responsible for the flat variation of Γ_2 with ϵ in Fig. 2. More detailed studies are required to understand the properties of Γ_2 near the ground state.

Another part of numerical studies is devoted to the investigation of the level spacing statistics in the above model. Indeed, it is known that the localized phase is characterized by the Poisson distribution $P_P(s)$, the metallic phase has the Wigner-Dyson statistics P_{WD} while the critical transition point has an intermediate statistics independent of the system size [19]. It is convenient to study the transition between two limits with the help of the parameter $\eta = \int_0^{s_0}(P(s) - P_{WD}(s))ds/\int_0^{s_0}(P_P(s) - P_{WD}(s))ds$, where $s_0 = 0.4729...$ is the intersection point of $P_P(s)$ and $P_{WD}(s)$. In this way $\eta = 1$ corresponds to $P_P(s)$, and η=0 to $P_{WD}(s)$. The dependence of η on ϵ is determined in the following way. For each disorder realization the spacing between nearby energy levels E_i is determined and then is averaged over ND disorder realizations for each i giving the $P(s)$ statistics and η as a function of averaged excitation energy $\epsilon = E/2$. At higher energies the values of η are in addition averaged in a fixed energy interval. In this way the total statistics obtained for $P(s)$ and η varies from $NS = 12000$ for low energy states up to $NS = 10^6$ at high energy with high density of levels. Un example of η variation with energy for different system size L is shown in Fig. 3 (see also Fig. 1 in [17] for a stronger disorder). For large L the statistics becomes close to $P_P(s)$ at low energy and to $P_{WD}(s)$ at ϵ larger than a critical energy ϵ_c dependent on the disorder and independent of L. The transition in the spectral statistics can be qualitatively understand on the basis of the estimates given in the previous section. Indeed, since the interaction energy is $U/R \sim \epsilon$ the high energy states allow to have particles closer to each other ($R \sim U/\epsilon$) that increases their interaction and finally leads

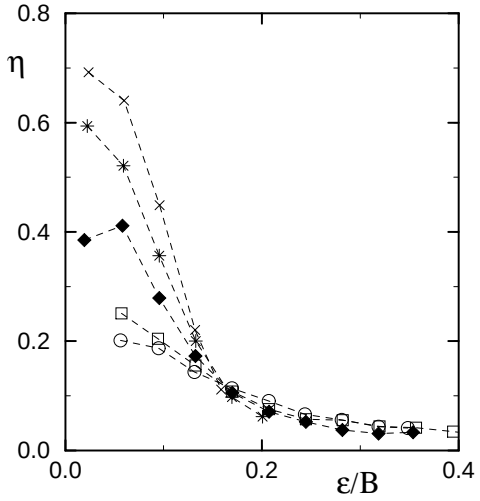

Figure 3: Dependence of η on ϵ/B for $U/V = 2, W/V = 5$ and different L: 10 (o); 12 (□); 16 (full diamond); 20 (*); 24 (x); so that $2.39 \leq r_s \leq 9.57$.

to delocalization for $R > l_1^{4/3}(U \sim V)$. In agreement with this picture the critical energy ϵ_c decreases with the increase of l_1 (decrease of W/V) as it can be seen from Fig. 1 in [17] and Fig. 3. The fact that an interaction increases the localization length for two particles in 2D has been also seen in the other numerical simulations [20,21]. However, the claim made there that the short range interaction gives a transition from localized to delocalized states is in a sharp contradiction with the theoretical arguments [12,13,16] and probably should be attributed to small sizes used in [20,21]. The numerical data for the Coulomb case presented in [21] are somewhat similar to the data presented here and in [17], even if any theoretical arguments in the favor of transition were presented in [21].

The variation of $P(s)$ with the interaction strength is shown in Fig. 4. At small U the statistics approaches to the Poisson distribution while with the increase of U it tends to the Wigner-Dyson case. In the vicinity of the critical point ϵ_c the statistics is close to the critical statistics in the 3D Anderson model with periodic boundary conditions [19,22,23] (see Fig. 4). This gives one more support for the physical picture developed in the previous section according to which two electrons in 2D are delocalized in a way similar to the 3D Anderson transition.

4 Conclusion

The analytical and numerical results obtained show that the Coulomb interaction leads to delocalization of two electron states in a way similar to the Anderson transition in 3D for $r_L < r_s < r_L^{4/3}$. The model is restricted only by two interacting electrons and has delocalization only for excited states that represents its weak point. However, it gives a picture qualitatively similar to experimentally observed metal-insulator transition in 2D [2] and therefore it can be useful for a future complete theory.

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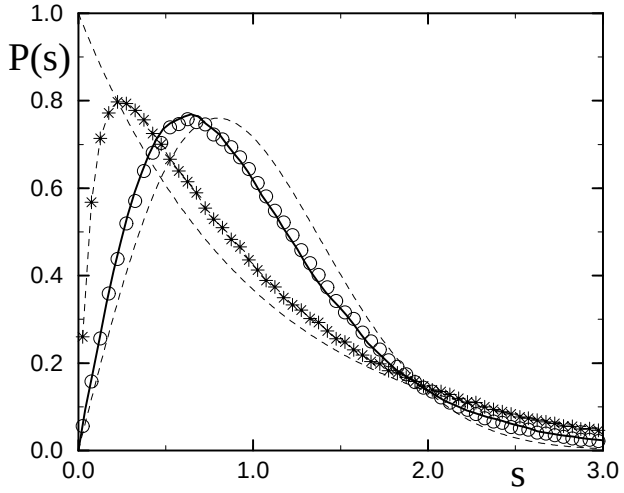

Figure 4: Level statistics $P(s)$ for two 2D electrons at $W/V = 7, L = 16$ in the energy interval $0.25 \leq \epsilon/B \leq 0.3$: (o) $U/V = 2$, vicinity of the critical point (see Fig. 1c in Ref. 17); (*) $U/V = 0.2$; total statistics is $NS = 5 \times 10^5$. The full line shows the critical $P(s)$ in 3D Anderson model ($W/V = 16.5, L = 14$, taken from Ref. 23); the dashed lines give Poisson statistics and Wigner surmise.

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