Synchnonization, zero-resistance states and rotating Wigner crystal

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We show that rotational angles of electrons moving in two dimensions (2D) in a perpendicular magnetic field can be synchronized by an external microwave field which frequency is close to the Larmor frequency. The synchronization eliminates collisions between electrons and thus creates a regime with zero diffusion corresponding to the zero-resistance states observed in experiments with high mobility 2D electron gas (2DEG). For long range Coulomb interactions electrons form a rotating hexagonal Wigner crystal. Possible relevance of this effect for planetary rings is discussed.

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The discovery of microwave-induced resistance oscillations (MIRO) [1] and of striking zero-resistance states (ZRS) of a 2DEG in a magnetic field [2, 3] attracted a great interest of the community. A variety of theoretical explanations has been pushed forward to explain the appearance of ZRS (see Refs. in [4]). These approaches provide certain MIRO which at large microwave power even produce a current inversion. However, these theories do not give zero resistance, and it is usually argued that ZRS are created as a result of some additional instabilities which mysteriously compensate currents to zero. Hence, a physical origin of ZRS still remains a puzzling problem.

In this work we suggest a generic physical mechanism which leads to a suppression of electron-electron collisions and creates ZRS. Its main element is the synchronization phenomenon which has abundant manifestations in science, nature, engineering and social life [5, 6]. A simple picture of the effect is the following: a microwave field excites electrons above the Fermi level and switches on dissipation processes in energy, which compensate microwave-induced energy growth, thus creating a nonequilibrium steady-state distribution. Due to this dissipation, when the microwave frequency ω is close to a resonance with the Larmor frequency ω_B, the synchronization of the phases of Larmor rotations of electrons with the phase of microwave field is established. In this way all electrons start to oscillate in phase - like male fireflies blink in phase on trees in Siam, emitting rhythmic light pulses in order to attract females [5, 6]. But compared to fireflies stationary sitting on trees, the synchronization of moving electrons brings a new element not presented in the common synchronization science: due to synchrony the collisions between electrons extinct what leads to a drastic drop of the collision-induced diffusion constant D and to creation of ZRS (we note that D is proportional to experimentally measured resistance R_{xx} since $R_{xy} \gg R_{xx}$ [2, 3]). A simple image of such synchronized electrons is given by an ensemble of particles randomly distributed on a 2D plane, which rotates as a whole on a Larmor circle of radius $r_B = v_F/\omega_B$ with frequency ω_B. Indeed, in such a rotating ensemble (plane) particles never collide, and we demonstrate below that this can happen with 2D electrons synchronized with a microwave field phase in a magnetic field B. The synchronization origin of ZRS allows one also to understand qualitatively why ZRS exist only in high mobility samples. Indeed, it is well known that synchronization remains robust to a weak noise but disappears at strong one [5], hence a weak impurity scattering will not destroy ZRS. It is also important to note that the above picture is based on classical dynamics that has its grounds since in the experiments [2, 3] the Landau quantum level is large $n_L \sim 100$. Thus, we start our analysis with a classical mechanics treatment and will turn to a discussion of quantum effects later.

To justify the synchronization picture of ZRS described above we perform extensive numerical simulations using two main models of classical electrons (particles) with short range and Coulomb interactions. In the simplest setup, we model particle dynamics with short range interactions in magnetic and microwave fields with the Nosè-Hoover (NH) thermostat (see e.g. [7, 8]) combined with interactions treated in the frame of the mesoscopic multiparticle collision model (MMPCM) [9]. The NH thermostat produces an effective friction γ which keeps the average kinetic energy $\langle \mathbf{p}^2/2m \rangle$ equal to a given thermostat temperature T and equilibrates heating induced by a microwave field $\mathbf{f}_{ac} = \mathbf{f}\cos\omega t$. At the same time the MMPCM drives system toward ergodic state with the equilibrium Maxwell distribution at a given temperature. In this way the particle dynamics is described by the equations:

$$\dot{\mathbf{q}}_i = \mathbf{p}_i/m \; , \; \dot{\mathbf{p}}_i = \mathbf{F}_i + \mathbf{f}_{Li} + \mathbf{f}_{ac} - \gamma \mathbf{p} \; , \tag{1}$$

$$\dot{\gamma} = [\langle \mathbf{p}^2 \rangle/(2mT) - 1]/\tau^2 \tag{2}$$

where $\mathbf{q}_i, \mathbf{p}_i$ are the coordinate and the momentum of particle i, $\mathbf{f}_{Li} = e[\mathbf{p}_i \times \mathbf{B}]/mc$ is the Lorentz force, $\mathbf{F}_i$ is an effective force produced by particles collisions, τ is the relaxation time in the NH thermostat and $\langle \mathbf{p}^2 \rangle$ means average over all N particles. We usually consider the case of a linearly polarized microwave field $\mathbf{f}_{ac}$ since numerical data give no significant dependence on polarization. In numerical simulations N particles are placed randomly on a square cell $L \times L$ which is periodically continued all over the plane. The collisions are treated in the MM-PCM formalism, namely the main cell is divided into N_c small collision cells in which after a time step Δt the velocities of particles are reshuffled randomly but keeping conserved the momentum and energy of particles in the collision cell [9]. In absence of microwave radiation the system evolves to a usual thermal equilibrium with the Maxwell distribution. The average rate D of particles diffusion in space is computed via their displacements after a large time interval t. In presence of the microwave field the diffusion rate D is drastically changed in the vicinity of the resonance $\omega_B \approx \omega$ as it is shown in Fig. 1 for typical values of parameters.

Fig. 1 clearly shows the existence of a synchronization Arnold tongue inside which the diffusion drops to zero (here as well as in numerical simulations below, its residual numerical value $D/D_0 \lesssim 10^{-8}$ is essentially determined by roundoff errors and fit accuracy and is non distinguishable from zero). According to Fig. 1 the synchronization regime and the ZRS exist inside the detuning range

$$|\omega_B - \omega| \leq sf/(mv_T) \ , \qquad (3)$$

where $v_T = \sqrt{2T/m}$ and a numerical constant $s \approx 0.7$. We note that s is not sensitive to the relaxation time τ which has been varied by an order of magnitude. In fact the domain of ZRS given by (3) is very similar to a usual synchronization domain for one particle [5] which is also not sensitive to the dissipation rate. The origin of this similarity is rather clear: the synchronization with the microwave field phase eliminates collisions between particles, so that they move independently and hence the Arnold tongue becomes the same as for one-particle synchronization. The fact that in the ZRS the collisions are eliminated, is confirmed by direct counting of the number of collisions in the numerical code and by computation of the synchronization parameter $S = \sum_{i<j} (\mathbf{v}_i - \mathbf{v}_j)^2/(N^2 v_T^2/2)$ which in the ZRS drops down to $S \sim 10^{-10}$ being determined by roundoff errors. This means that all particles have the Larmor phase synchronized with the microwave field phase while their positions in the coordinate space are disordered. Outside of the ZRS particles continue to diffuse with a rate D which is comparable with the unperturbed rate D_0. At small values of N_c and Δt when the collision rate becomes rather large and $D_0 \sim D_c = v_T^2/\omega \approx v_T r_B$, the ZRS regime is destroyed. Another model of collisions, in which the velocities of colliding particles are changed randomly in a bounded relatively small scattering angle, gives essentially the same result (3) for the ZRS.

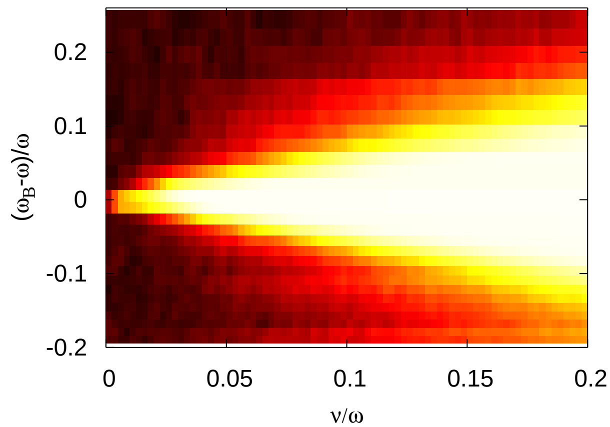

FIG. 1: (color online) Gray-scale plot of the normalized diffusion rate D/D_0 as a function of frequency detuning $(\omega_B - \omega)/\omega$ and rescaled microwave field strength ν/ω with $\nu = f/mv_T$ where $v_T = \sqrt{2T/m}$ is the thermal velocity and D_0 is the diffusion rate in absence of microwave at $\omega_B = \omega$. The system parameters are: $N = 1000$ $N_c = 4 \times 10^4$, $\omega \Delta t = 0.2$, $\omega\tau = 10$, $\omega t = 500$, $L/r_B = 10$, $D_0/D_c = 0.12$ (with $D_c = v_T^2/\omega$, $\rho = N/L^2$ and r_B taken at $\omega_B = \omega$, thus a number of particles inside a Larmor circle is $N_B = \pi r_B^2 \rho = \pi\rho v_T^2/\omega^2 = 10\pi$, $\omega = const$). Color intensity is proportional to D/D_0 (black for maximum $D/D_0 \approx 1.2$ and white for minimum $D/D_0 = 0$).

To check the existence of the ZRS in the case of long range Coulomb interactions we use the molecular dynamics (MD) simulations of a classical two-dimensional electron liquid as described in [10]. The results obtained in [10] show that such an approach correctly describes plasmon modes in presence of a magnetic field even when the Coulomb energy $E_C = e^2/a$ is large compared to classical temperature T. Here, $a = 1/\sqrt{\pi\rho}$ is an average distance between electrons determined by the electron density ρ. We ensured that our numerical code with the Ewald resummation technique reproduces correctly the results presented in [10]. To equilibrate the heating induced by the microwave field we introduce in Eq. (1) an energy-dependent dissipation with $\gamma = \gamma_0(E - E_F)/E_F$ for $E = p^2/2m > E_F$ and $\gamma = 0$ for $E < E_F$. In such a way the dynamics remains Hamiltonian for $E < E_F$ while above E_F the dissipative processes are switched on as it is usually the case for 2DEG; thus E_F plays a role of Fermi energy [11]. Usually we use $E_F/T \approx 2$ but the obtained results are not sensitive to this ratio. The main part of simulations is done at an intermediate interaction strength $r_s = E_C/E_F = 0.3$ but we ensured that an increase(decrease) of r_s by a factor 7(3) does not change qualitatively the results (samples studied in [2, 3] have $r_s \approx 2$). Also a variation of the dissipation rate γ_0 by an order of magnitude does not affect significantly the results and we present data at $\gamma_0 \approx 0.7 v_F/a$. The same is true for the total number of electrons varied from 20 to 200 at $\rho = const$, thus we present data at $N = 100$.

A typical example of the dependence of average elec-

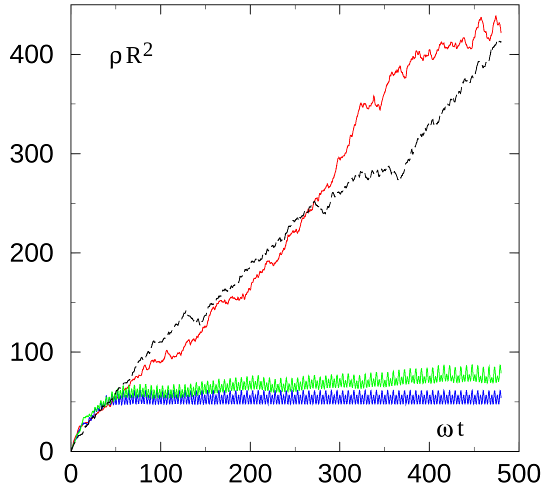

FIG. 2: (color online) Dependence of electron square displacement R^2, rescaled by electron density ρ, on the rescaled time ωt. Here the Larmor frequency is $\omega_B = \omega$ at microwave field strength $f = 0$ (red top curve); $f/(mv_F\omega) = 0.059$ ($fa/E_F = 0.02$) for $\omega_B = \omega$ (blue bottom curve), $\omega_B = 0.875\omega$ (second from top black dashed curve), and $\omega_B = \omega$ with impurity scattering mean free path $l_i = 96r_B$ (second from bottom green curve). Total number of electrons is $N = 100$ and $N_B = \pi\rho v_F^2/\omega^2 = 34.7$. The linear fit gives the diffusion rates $D/D_c = 0.089, 0.068, 0.0040, 9\times10^{-6}$ with $D_c = v_F^2/\omega$ (respectively for curves from top to bottom ordered at $\omega t = 400$).

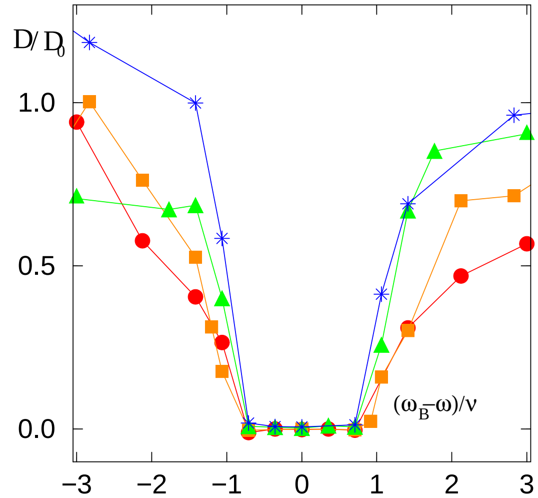

FIG. 3: (color online) Dependence of rescaled diffusion rate D/D_0 on the rescaled frequency difference $(\omega_B - \omega)/\nu$. Here $\nu = f/mv_F$, D_0 is diffusion rate in absence of microwave at $\omega_B = \omega$, $fa/E_F = 0.02$ and number of electrons in a Larmor circle is $N_B = 2$ (stars), 8 (triangles), 34.7 (squares), 138.8 (points) with $D_0/D_c = 0.054, 0.089, 0.12, 0.14$ and $D_0/v_F a = 0.20, 0.35, 0.53, 0.64$ respectively. Total number of electrons is $N = 100$, $L = \sqrt{N/\rho} \approx 17.72a$.

tron square displacement R^2 on time is shown in Fig. 2. The introduction of microwave field leads to the synchronization of electron Larmor phases and to a drastic drop of diffusion rate at $\omega_B = \omega$, formally by 4 orders of magnitude; the synchronization parameter S drops down to $S \approx 10^{-11}$ in this case that means that collisions are completely switched off. A shift in the Larmor frequency $\omega_B = 0.875\omega$ destroys synchronization and diffusion D is restored being close to its unperturbed value D_0 at $f = 0$. An introduction of a weak noise linked to impurity scattering with a scattering time t_i and mean free path $l_i = v_F\tau_i$ leads to a finite diffusion rate D which is however much smaller than D_0 until $l_i \gg r_B$ (see Fig. 2). A decrease of the mean free path down to $l_i \approx 10r_B$ destroys synchronization and restores a diffusion with rate $D \approx D_0$. We note that $l_i \approx 100r_B$ approximately corresponds to experimental conditions in [2, 3].

The dependence of D on the frequency detuning is shown in Fig. 3. The numerical data for Coulomb interactions between electrons show that the ZRS exist inside the synchronization window near the resonance $\omega_B \approx \omega$ with the width given by Eq. (3) where v_T should be replaced by v_F and $s \approx 0.8$. Inside the ZRS the diffusion drops practically to zero as discussed above. The validity of the relation (3) shows that the effect is not very sensitive to the type of interactions between particles.

However, the long range nature of Coulomb interactions significantly modifies the structure of the ZRS configuration: for short range interactions particles are distributed over the plane in a disordered way, while for the Coulomb interactions electrons form a hexagonal Wigner crystal as it is shown in Fig. 4. The whole crystal (as well as each electron) is rotating in the plane with the frequency $\omega \approx \omega_B$ and rotation radius $r_B = v_F/\omega_B$. A remarkable property of the rotating Wigner crystal is that formally it is formed at a rather small parameter $r_s \approx 0.3$ while the usual Wigner crystal requires r_s values by more than two orders of magnitude larger [12]. We attribute this to synchronization of electron Larmor phases with the microwave field phase, what eliminates collisions between electrons and suppresses fluctuations, thus yielding an effectively large r_s in the rotating frame. In the crystal all Coulomb forces acting on an electron are compensated, thus the size of synchronization domain in frequency range given by Eq. (3) is essentially the same as for one-particle synchronization and is practically independent of dissipation rate γ_0 [5].

In conclusion, we have suggested a generic mechanism which for 2D particle rotational dynamics (e.g the Larmor rotation, but it may be also relevant for other 2D systems like planetary rings) produces synchronization of rotational angles of all particles with the phase of external driving periodic field. As a result a rotating Wigner crystal is created and a collisional diffusion is suppressed by several orders of magnitude. The collective crystal structure also suppresses the diffusion due to impurities. We propose that this effect explains the appearance of ZRS in 2DEG observed in [2, 3]. According to Eq. (3) the relative size of ZRS plateau is $\Delta\omega/\omega \approx 2\nu/\omega \approx fv_F/\omega E_F$ that for

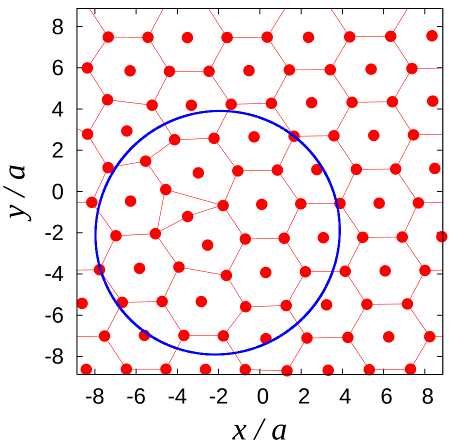

FIG. 4: (color online) Instant image of the rotating Wigner crystal formed by $N = 100$ electrons (points) in a periodic cell with $L = \sqrt{N/\rho} \approx 17.72a$, $\omega t = 480$, $\omega_B = \omega$, $fa/E_F = 0.02$ and $N_B = 34.7$ (as in Fig. 2, bottom curve); the circle shows an orbit of one electron for $240 \leq \omega t \leq 480$; lines are drawn to adapt an eye showing a hexagonal crystal with a defect.

experiments [2, 3] with $E_F \sim 100K^o$, $v_F \sim 3 \times 10^7 cm/s$ and $\omega/2\pi = 35GHz$ gives $\Delta\omega/\omega \approx 0.1$ if the field strength acting on an electron is $f/e \approx 5V/cm$. This relative width is in a reasonable agreement with the experimental results [2, 3, 13] where unfortunately an exact value of f is not known. The synchronization energy scale $E_S \sim fr_B \sim 10K$ and the crystal Coulomb energy $E_C \sim 200K$ might be the origin of large energy scale $E_A \sim 10K$ in the ZRS activated transport [2, 3]. An important discrepancy from the experiments is that our theory gives synchronization only near the main resonance $\omega_B/\omega \approx 1$ while in the experiments ZRS exist also near integer low resonances j with $\omega_B/\omega \approx 1/j$. We suppose that these resonances may appear due to an additional effective 2DEG potential and surface modulation in space. This may generate higher harmonics of the Larmor motion and produce synchronization also at integers $j > 1$. Such a modulation apparently appears during molecular epitaxial growth [14]. This modulation also produces a frequency shift in the rotational frequency that may be responsible for a resonance shift of the ZRS domain compared to the Larmor resonance (see [2, 3, 13]). A coherent rotation of electrons in the crystal generates a rotating magnetic field $B_W \sim \mu_0 e v_F \rho \sim 1G$ parallel to 2DEG which can be detected experimentally.

Our theory is based on the classical dynamics and it is crucial to analyze the contribution of quantum effects. In principle it is known that at small effective values of Planck's constant $\hbar_{eff}$ the synchronization is preserved while at large values $\hbar_{eff}$ it is destroyed by quantum fluctuations [15]. For 2DEG $\hbar_{eff} \sim 1/n_L$ and at $n_L \sim 100$ it is natural to expect that the synchronization is robust against quantum fluctuations. However, a reduction of n_L by an order of magnitude due to an increase of ω to a THz range may significantly enhance quantum noise and destroy ZRS. Further theory development is required to study quantum effects properly. The most important question is about the amount of electrons which are involved in the rotating Wigner crystal. Indeed, our classical studies show that all electrons are involved in this state but in the quantum case it is rather possible that only a finite fraction of electrons near the Fermi level contributes to the rotating crystal, while all other electrons will stay as a non-interacting background.

Finally we make a conjecture that the mechanism described here may be responsible for enormously long life time ($\sim 10^{12}$ rotations) and sharp edges of planetary rings (e.g. $\sim 10m$ for Saturn) [16, 17]. Indeed, a temperature there is very low and in the rotational frame the 2D dynamics of particles is similar to motion of electrons in a magnetic field [16]. Hence, moons inside a ring and near to a resonance may produce synchronization and diffusion suppression with emergence of ZRS in space.

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