Opinion formation in Wikipedia Ising networks

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We study properties of opinion formation on Wikipedia Ising Networks. Each Wikipedia article is represented as a node and links are formed by citations of one article to another generating a directed network of a given language edition with millions of nodes. Ising spins are placed at each node and their orientation up or down is determined by a majority vote of connected neighbors. At the initial stage there are only a few nodes from two groups with fixed competing opinions up and down while other nodes are assumed to have no initial opinion with no effect on the vote. The competition of two opinions is modeled by an asynchronous Monte Carlo process converging to a spin polarized steady-state phase. This phase remains stable with respect to small fluctuations induced by an effective temperature of the Monte Carlo process. The opinion polarization at the steady-state provides opinion (spin) preferences for each node. In the framework of this Ising Network Opinion Formation model we analyze the influence and competition between political leaders, world countries and social concepts. This approach is also generalized to the competition between three groups of different opinions described by three colors, for example Donald Trump, Vladimir Putin, Xi Jinping or USA, Russia, China within English, Russian and Chinese editions of Wikipedia of March 2025. We argue that this approach provides a generic description of opinion formation in various complex networks.

I. INTRODUCTION

The process of opinion formation in human society gains higher and higher importance with the development of social networks which start to produce an important impact on political views and elections (see e.g. [1, 2]). The statistical properties of such social networks typically have a scale-free structure as reviewed in [3, 4]. Various voter models have been proposed and studied by different groups with a development of physical concepts and their applications to sociophysics [3, 5–13].

Recently, we proposed the Ising Network Opinion Formation (INOF) model and analyzed its applications to Wikipedia networks for 6 language editions of 2017 [14]. This model allows to determine opinion polarization for all Wikipedia articles (or nodes) induced by two groups of nodes with fixed opposite opinions (red or blue, spin up or down). In this INOF model the initial two groups of one or a few nodes have fixed opposite opinions represented by spin up (red color) or spin down (blue color). All other nodes have initially an undecided opinion (spin zero or white color). The formation of a steady-state opinion of each node emerges as a result of an asynchronous Monte Carlo process in which an opinion of a given node i is determined by a majority vote of his friends presented by spins up or down or zero from all network nodes j that have links to node i. Such spin flips, induced by local majority votes, are done for all nodes without repetitions in random order over all N nodes. These procedure is repeated up to convergence to a steady-state for a sufficiently long time τ and corresponds to a particular random path pathway realization for the order of spins to be flipped. Finally, an average over a high number of pathways realizations is done to obtain averages and distributions of the opinions for nodes or the whole network (see next section for more technical details). A somewhat similar procedure is used in the studies of problems of associative memory (see e.g. [15, 16]) even if there are significant differences from the INOF model due to the absence of certain fixed nodes and other initially white nodes and the use of positive/negative transition elements between nodes while all of them are positive for the INOF case considered here.

We also note that the INOF approach is generic and can be applied to various directed networks. In particular, it has also been applied to the analysis of fibrosis progression in the MetaCore network of protein-protein interactions [18]. A similar approach, without white nodes, was used to study the competition of dollar and possible BRICS currencies in the world trade network [19].

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In this work, we extend the studies of the INOF model [14] to more recent Wikipedia networks collected either in October 2024, as described in [17], or collected at 20 March 2025 and to new specific initial groups with fixed opinions. For example, we analyze the competition between Apple Inc. vs. Microsoft, Donald Trump vs. Vladimir Putin and others. Furthermore, the competition of three entries is also considered for several cases and finally, we also examine the effects of fluctuations appearing as a result of a certain effective temperature in the voting process.

Wikipedia networks have rather exceptional features as compared to other networks: the meaning of their nodes is very clear, they represent all aspects of nature and human activity and the presence of multiple language editions allows to analyze various cultural views of humanity. A variety of academic research of Wikipedia with analysis of different aspects of nature and society was reviewed in [17, 20–23]. Therefore we hope that the INOF approach to Wikipedia Ising networks (WIN) will find multiple and divers applications.

The article is composed as follows: Section 2 describes the INOF model and the used Wikipedia data sets, Section 3 presents results for the confrontation of opinions for two groups of entries, Section 4 presents results for a three groups contest, Section 5 analyzes effects of fluctuations induced by an effective temperature and Section 6 contains discussion and conclusion. Finally, the Appendix provides some additional Figures and data.

II. MODEL DESCRIPTION AND DATA SETS

In this work, we use mostly use three very recent Wikipedia editions (English EN, Russian RU, Chinese ZH) collected at 20 March 2025 with the number of network nodes/articles $N = 6969712$, 2035086, 1468935 and the number of links $N_\ell = 190031938$, 44188839, 21160179 for EN, RU, ZH respectively. For certain cases, we also use the English (EN) and French (FR) Wikipedia network collected at 1 October 2024 and already used [17] with $N = 6891535, 2638634$ and $N_\ell = 185658675, 76118849$ for EN (2024), FR (2024) respectively. A description of the extraction procedure to create the wikipedia networks from raw dump files is given in [17].

Wikipedia articles correspond to network nodes and citation from a given article j to another article i correspond to a directed link with the adjacency matrix element $A_{ij} = 1$ (and $A_{ij} = 0$ in absence of a link from j to i); multiple citations from j to i are considered as only one link. Then the matrix of Markov transitions is defined by $S_{ij} = A_{ij}/k_j$ where $k_j = \sum_i A_{ij}$ is a number of out-going links from node j to any other node i (such that $A_{ij} \neq 0$); for the case of dangling nodes without out-going links (i.e. with $k_j = 0$), we simply define $S_{ij} = 1/N$ implying the usual column sum normalization $\sum_i S_{ij} = 1$ for all j. For later use, we also introduce the modified matrix $\tilde{S}_{ij}$ which is identical to S_{ij} for $k_j > 0$ and with $\tilde{S}_{ij} = 0$ for dangling nodes.

Usually, in other typical types of network studies (see e.g. [17]) one introduces the Google matrix of the network defined as $G_{ij} = \alpha S_{ij} + (1-\alpha)/N$ where alpha is the damping factor with the standard value $\alpha = 0.85$ [24–26]. Here the network nodes can be characterized by the PageRank vector which is the eigenvector of the Google matrix G [24–26] with the highest eigenvalue $\lambda = 1$, i.e. $GP = \lambda P = P$, and the damping factor $\alpha < 1$ ensures that this vector is unique and can be computed efficiently. Its components $P(i)$ are positive and normalized to unity ($\sum_{i=1}{}^N P(i) = 1$). The network nodes i can be ordered by monotonically decreasing probabilities $P(i)$ which provides the PageRank index K with with highest probability at $K = 1$ and smallest at $K = N$. Some results for the PageRank vector and its index for recent Wikipedia editions of 2024 can be found at [17]. However, in this work, we do not use the Google matrix neither the PageRank and focus mostly on the modified matrix $\tilde{S}$ and also the adjacency matrix A_{ij} to define an asynchronous Monte Carlo process.

As in [14] a few selected nodes (wiki-articles) have assigned fixed spin values $\sigma_l = -1$ blue e.g. for *Microsoft* and $\sigma_k = 1$ red for *Apple Inc.*. These specific spin nodes always keep their polarization. All other nodes i are initially assigned with a white color (or spin $\sigma_i = 0$) and have no definite initial opinion. However, once they acquire a different color red or blue (spin value $\sigma_i = \pm 1$) during the asynchronous Monte Carlo process they can flip only between ± 1 and cannot change back to the white opinion.

To define the asynchronous Monte Carlo process, we choose a random spin i among the non-fixed set of spins, and compute its influence score from ingoing links j:

$$Z_i = \sum_{j \neq i} \sigma_j V_{ij}. \tag{1}$$

where the sum is over all nodes j linking to node i. Here V_{ij} is the element of the vote matrix, defined by one of two options: $V_{ij} = A_{ij}$ (the adjacency matrix element, option OPA), or $V_{ij} = \tilde{S}_{ij}$ (the modified Markov transition matrix element, option OPS). For the OPS option, the matrix $\tilde{S}_{ij}$ is used, in which columns corresponding to dangling nodes contain only zero elements, ensuring these nodes do not contribute to Z_i. We discuss both options OPA and OPS with a primary focus on the OPS case.

In Eq. (1) $\sigma_j = 1$ if the spin of node j is oriented up (red color), or $\sigma_j = -1$ if it is oriented down (blue color), or $\sigma_j = 0$ if the node j has no opinion (if it has still its initial white value). After the computation of Z_i, the spin σ_i of node i, is updated: it becomes $\sigma_i = 1$ if $Z_i > 0$, $\sigma_i = -1$ if $Z_i < 0$, and remains unchanged if $Z_i = 0$. This operation is repeated for all non-fixed nodes i' following a predetermined random order (shuffle) such that there is no repetition at this level and each spin is updated only once. Note that due to the possibility of $Z_i = 0$ it is possible that a node i keeps his initial white value $\sigma_i = 0$. After the update the modified value of σ_i is used for the computation of $Z_{i'}$ of subsequent values $\sigma_{i'}$.

One full pass of updating all non-fixed spins constitutes a single time step, $\tau = 1$. The procedure is then repeated for subsequent time steps $\tau = 2, 3, \ldots$ using a new random shuffle for the update order at each step. We find that the final steady-state is reached after $\tau \approx 20$ steps with only a very small number of spin flips in the $\tau = 20$. There is a certain fraction of nodes that remain white for $\tau \geq 20$ which we attribute to their presence in isolated communities (about 12% for EN 2025, 15% for RU 2025 and 30% for ZH 2025 and 10% for FR 2024). These nodes are not taken into account when determining the opinion polarization of other nodes and all statistical quantities such as averages, fractions and histograms are computed with respect to the set of non-white nodes. We point out that compared to the usual case of Wikipedia networks the size of the configuration space of the INOF model is drastically increased to 2^N instead of N.

The physical interpretation of the OPA case corresponds to the situation where a node j gives an unlimited number of votes to the nodes i to which he has links while for the OPS case the node j has only a limited vote capacity (since the total probability in column j is normalized to unity). Therefore these two options OPA and OPS describe two different possibilities for the voting process. We note that due to a misprint in [14] the analysis was performed for OPA case and not with OPS one as it is declared in [14].

Repeating this asynchronous Monte Carlo process, with the same initial condition and different random orders (or pathways) for the spin flip defined by the rule (1), we obtain various random realizations leading to different final steady-state distributions in each case. Using this data we perform an average over up to $N_r = 10^5$ pathway realizations ($N_r = 10^6$ for the case of FR 2024 to obtain a reduced statistical error for this case; see below) that provides an average opinion polarization μ_i of a given spin (node, article). The further average of μ_i over all (non-white) network nodes gives the global polarization μ_0 with a deviation $\Delta\mu_i = \mu_i - \mu_0$ for each article. This deviation $\Delta\mu_i$ represents the opinion preference of a given article i to red or blue entries as compared to the average global Wikipedia opinion μ_0. The set of white nodes in the final steady-state distribution contains about 10% - 30% of the total number of nodes (30% only for ZH Wiki2025 and at most 15% for the other cases) and this set is extremely stable with respect to different pathway realizations and also with respect to the different choices of initial fixed nodes. These white nodes are not taken into account in the computation of μ_0 and μ_i is only computed for non-white nodes (those which have nearly always either red or blue values depending on the pathway realization).

The voting process for the case of a competition between three groups of entries is an extension of this procedure and its details are be explained later.

III. RESULTS FOR COMPETITION OF TWO GROUPS OF ENTRIES

A. Comparison of OPA and OPS

The relaxation of Ising spins to the steady-state in Wiki2024, Wiki2025 networks is very similar to those found for Wiki2017 networks studied in [14] (see e.g. Figure 1 there). Thus the steady-state is reached at $\tau \geq 20$, the fractions of red and blue nodes at the final state are concentrated mainly at all red or at all blue nodes.

For each pathway realization we compute the fraction of red nodes f_r as the ratio $f_r = n_r/(n_r + n_b)$ where n_r (n_b) is the number of red (blue) nodes in the network for this pathway realization at $\tau = 20$. In particular, white nodes are not taken into account in this fraction and the corresponding fraction of blue nodes is simply by symmetry $f_b = n_b/(n_r + n_b) = 1 - f_r$. We show examples for the probability density of f_r (histogram for many different pathway realizations) in Figure 1 for two groups of entries *socialism, communism* (red) vs.

capitalism, imperialism (blue) SCCI for the two cases OPA and OPS of EN Wiki2025.

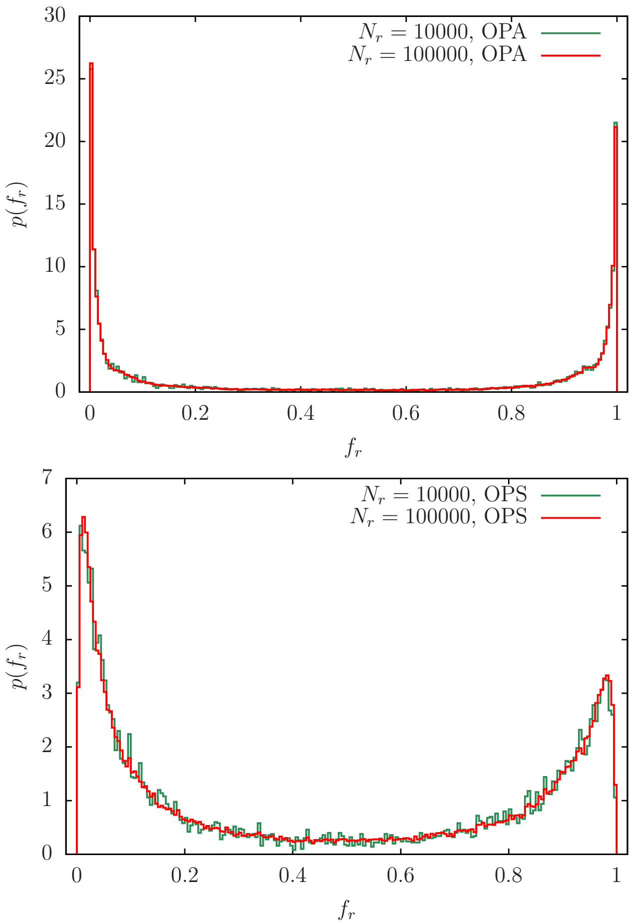


FIG. 1. Probability density $p(f_r)$ of fraction of red nodes f_r for SCCI for EN Wiki2025 versus f_r using the matrix A (OPA case, top panel) or the matrix $\tilde{S}$ (OPS case, bottom panel). The value of f_r for a given pathway realization gives the fraction of red outcome of network nodes for Socialism/Communism (fraction computed with respect to all non-white network nodes). The histograms show the distribution of f_r with respect to all N_r different random pathway realizations and all nodes. The normalization is fixed by $\int_0^1 p(f_r)\, df_r = 1$ with bin width 0.005 (i.e. 200 bins in the full interval for $f_r \in [0, 1]$). The red (green) curves correspond to $N_r = 100000$ ($N_r = 10000$). Note that the distribution $\tilde{p}(f_b)$ for blue outcome for Capitalism/Imperialism (not shown in the figure) can be obtained by symmetry $\tilde{p}(f_b) = p(1 - f_b)$ since $f_r + f_b = 1$.

We note that the probability distribution $p(f_r)$ in Figure 1 is obtained as a histogram using N_r values of f_r for the obtained for the different pathway realizations. The shape of distributions for OPA and OPS cases are similar with strong maxima close to $f_r \approx 0$ and $f_r \approx 1$ but for the OPA case the distribution has sharper peaks. In both cases, it is very likely that either one or the other opinion is a strong winner for a given random pathway realization but this effect is somewhat stronger for the OPA case where the peak maxima are roughly four times larger than for the OPS case. Furthermore, the comparison of the two curves for $N_r = 10^4$ and $N_r = 10^5$ indicates the reduction of statistical fluctuations with increasing N_r.

In this work, we compute for many cases and situations the average spin polarization μ_i of a node i with respect to the N_r random pathway realizations by the equation $\mu_i = (n_r(i) - n_b(i))/(n_r(i) + n_b(i))$ where $n_r(i)$ ($n_b(i)$) is the number of red outcome of node i (blue outcome of node i) for the N_r different random pathway realizations of the Monte Carlo procedure. Here, for the non-white nodes i we typically have $n_r(i)+n_b(i) \approx N_r$ and the number of white outcome n_w of these nodes is very small $n_w(i) = N_r - (n_r(i) + n_b(i)) \ll N_r$ (it is not always exactly zero due a limited iteration time $\tau = 20$ in the Monte Carlo procedure). For the fraction of white nodes (about 12% for EN Wiki2025), we have typically $n_w(i) \approx N_r$ and we do not compute μ_i for these nodes for which μ_i is either not even defined (if $n_b(i) + n_r(i) = 0$) or has a large statistical error (if $0 < n_b(i) + n_r(i) \ll N_r$). Once μ_i is known for a given non-white node, we also compute the difference $\Delta\mu_i = \mu_i - \mu_0$ where μ_0 is the global network average of μ_i over all (non-white) nodes i. This difference represents the opinion preference of the node i in comparison to the average global network opinion μ_0 and it will be used in several of the subsequent figures and tables to present data for the case of a two-way competition.

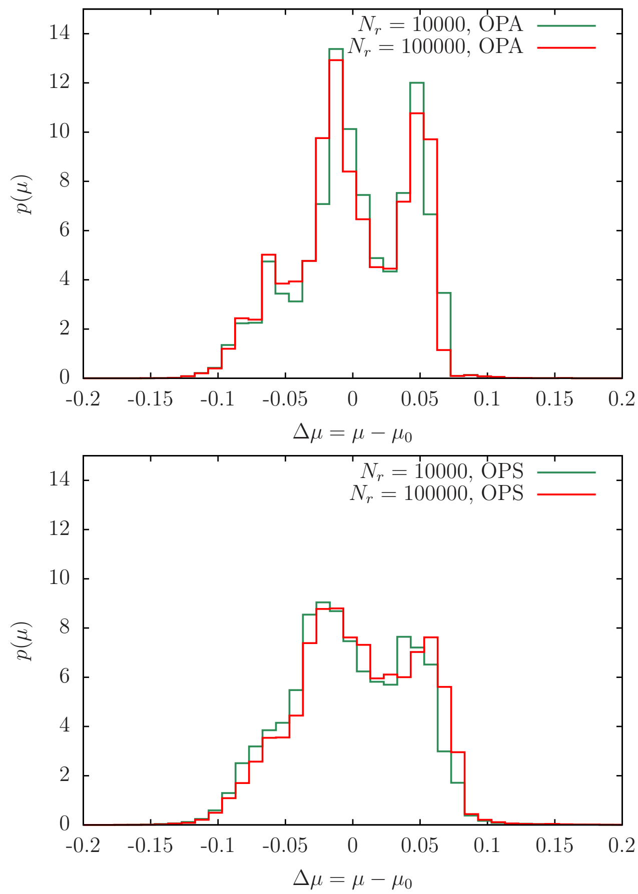


FIG. 2. Probability density $p(\mu)$ of polarization μ for SCCI for EN Wiki2025 versus $\Delta\mu = \mu - \mu_0$ using the matrix A (OPA case, top panel) or the matrix $\tilde{S}$ (OPS case, bottom panel). Here a red (blue) outcome with $\mu_i \approx 1$ ($\mu_i \approx -1$) corresponds to Socialism/Communism (Capitalism/Imperialism). The histograms show the distribution of μ_i for all non-white nodes i in the network with with $n_r(i) + n_b(i) \approx N_r$ and white nodes with $n_r(i) + n_b(i) \ll N$ are not taken into account. The normalization is fixed by $\int_{-1}^{1} p(\mu)\, d\mu = 1$ with bin width 0.01 (i.e. 200 bins in the full interval for $\mu_i \in [-1, 1]$). Values of $p(\mu)$ outside the shown interval $[-0.2, 0.2]$ are zero on graphical precision. The red (green) curve corresponds to $N_r = 100000$ ($N_r = 10000$). The value μ_0 is the average of μ_i with respect to all (non-white) nodes i (computed for $N_r = 100000$) and has the values $\mu_0 = -0.02333$ for OPA and $\mu_0 = -0.10886$ for OPS.

In particular the probability density $p(\mu)$ of μ_i for the SCCI case of EN Wiki2025 (red for Socialism/Communism and blue for Capitalism/Imperialism) is shown in Figure 2 for both cases OPA and OPS.

Here the $p(\mu)\, \delta\mu$ represents the fraction of (non-white) network nodes i with $\mu \le \mu_i \le \mu + \delta\mu$ for some small value of $\delta\mu$ (bin width $\delta\mu = 0.01$ in the histograms of Figure 2). In global the profiles of OPA and OPS distributions are similar even if there are certain differences related to different voting procedures. As in Figure 1, the curves of Figure 2 at $N_r = 10^5$ seem to be rather stable with respect to statistical fluctuations as the comparison with the curves for $N_r = 10^4$ shows. In Figure 2, these fluctuations are a bit higher as in Figure 1 the computation of $\Delta\mu_i$ indeed requires at least $N_r = 10^5$ while $N_r = 10^4$ is not really sufficient.

The statistical accuracy of the polarization μ_i of a given node i computed for N_r realizations can be obtained in the following way: we know that $\mu_i = \langle\sigma_i\rangle$ where $\sigma_i = \pm 1$ is the spin value after $\tau = 20$ Monte

Carlo iterations for one specific pathway realization and $\langle\sigma_i\rangle$ is simply the average of σ_i with respect to the N_r random pathway realization. Since $\sigma_i^2 = 1$, we find that $\langle\sigma_i^2\rangle = 1$ and the variance of σ_i is simply $\mathrm{Var}(\sigma_i) = 1-\mu_i^2$. From this, we obtain the statistical error of the average μ_i as $\sqrt{(1-\mu_i^2)/(N_r-1)} \sim 1/\sqrt{N_r}$ for $\mu_i \approx 0$ (case of "largest" error). For $N_r = 10^5$ this gives a theoretical statistical error of μ_i being ≈ 0.003 for $\mu_i \approx 0$ (and a slightly reduced error by a factor $1-\mu_i^2$ if $\mu_i \neq 0$).

It is also possible to compute the statistical error quite accurately by a more direct numerical computation. For this we divide the full data of global $N_r = 10^5$ pathway realizations into 100 samples with reduced $N_r = 10^3$ and compute for each sample partial averages μ_i, μ_0 and $\Delta\mu_i$ over the reduced value of $N_r = 10^3$. This provides in particular partial averages μ_0 with quite significant fluctuations between the samples. Using the 100 partial average values $\Delta\mu_i$ for each sample it is straightforward to determine the statistical error of $\Delta\mu_i$ as $\approx \sqrt{\langle\Delta\mu_i^2\rangle - \langle\Delta\mu_i\rangle)^2/(100-1)}$ where $\langle\ldots\rangle$ is the simple average over the 100 samples (of the partial sample averages). It turns out that typical error values obtained in this way (at global $N_r = 10^5$ and for SCCI of EN Wiki2025 and a few other cases) are closer to $0.0008-0.0015$ which is about $2-3$ times smaller than the theoretical error ≈ 0.003. This reduction is apparently due to rather strong correlations between μ_i and μ_0 which are likely to have statistical fluctuations in the same direction (partial sample averages of both are likely to be "large" or "small" at the same time). These correlations are also visible in the two peak distribution of Figure 1 showing that in one given pathway realization it is rather likely to have either $f_r \approx 1$ (or $f_r \approx 0$) corresponding to a majority of network nodes i with either $\sigma_i = 1$ (or $\sigma_i = -1$ respectively). We note that the numerical statistical error of μ_0, which is obtained as byproduct of the above procedure, is $\approx 0.002-0.003$ which is closer to the theoretical error.

We also remind that to obtain f_r in Figure 1, we first perform an effective average over the network for a *fixed random pathway realization* (counting the number or fraction of red nodes in the network which is also the network average of $(\sigma_i+1)/2$) and then in Figure 1 we show histograms of this quantity using N_r values of f_r obtained from the N_r random pathways. For the data shown in Figure 2 this is essentially the other way round: first we compute μ_i as the average of σ_i over the N_r random pathway realizations and then we compute a histogram using the obtained μ_i values for all (non-white) network nodes i.

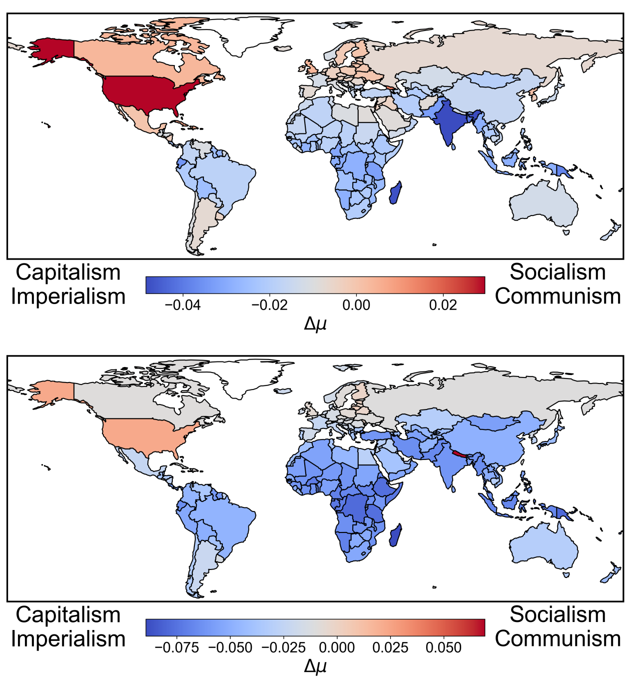


FIG. 3. Opinion polarization of world countries for *socialism, communism* ($\Delta\mu > 0$) vs. *capitalism, imperialism* ($\Delta\mu < 0$) for OPA case (top panel with $\mu_0 = -0.023$) and OPS case (bottom panel with $\mu_0 = -0.109$) for EN Wiki2025 and 197 countries.

The world map of countries (taken as Wikipedia article of a given country) with their polarization values μ_i

and $\Delta\mu_i = \mu_i - \mu_0$ is shown by color in Figure 3 for SCCI from EN Wiki2025. For both cases OPA and OPS the global polarization $\mu_0 < 0$ is in favor of *capitalism, imperialism* but for OPS $|\mu_0|$ is by a factor 4 higher compared to the OPA case. The global distribution of colors on the world map is qualitatively similar for both OPA and OPS cases. However, in the OPS case the preference to *capitalism, imperialism* is significantly more pronounced with almost all countries of Africa, dominance in Latin America, India, China and other countries in a south of Asia. In Europe lowest $\Delta\mu$ values are for France, Italy being however significantly larger compared to those of China, India, Africa. For Russia $\Delta\mu$ is closer to zero which is similar to the cases of Canada and Germany. USA has clearly a positive value of $\Delta\mu$. We note that similar opinion polarization of countries have been seen for EN Wiki2017 in [14].

After the comparison of OPA and OPS cases we conclude that they provide qualitatively similar results even if there are quantitative differences between these two voting options. In the following, we present results mainly for the OPS case.

B. Competition Apple Inc. vs. Microsoft

In Figure 4, we present the opinion polarization of world countries with respect to two companies *Microsoft* (blue $\sigma = \mu = -1$) vs. *Apple Inc.* (red $\sigma = \mu = 1$) for the OPS case of the EN Wiki2025. The global opinion polarization $\mu_0 = -0.076$ (average of σ_i over all N nodes and and all $N_r = 10^5$ pathway realizations) is in favor of *Microsoft*. The top countries with highest preferences $\Delta\mu > 0$ for Apple Inc. are *India, Nigeria, Bangladesh, Nepal* and those with preferences $\Delta\mu < 0$ for Microsoft are *Russia, Georgia, Ukraine, Kyrgyzstan.* USA has a slight preference for *Microsoft* and China has a slighter one also for *Microsoft*. We attribute the significant negative $\Delta\mu$ value of Russia to the fact that at the disappearance of the USSR the Russian personal computer market was dominated by Microsoft and this influence remained till recent times, Apple computers were too expensive at those times. A significant preference of India for *Apple Inc.* is related to the presence of several direct links pointing from Apple-related articles to India.

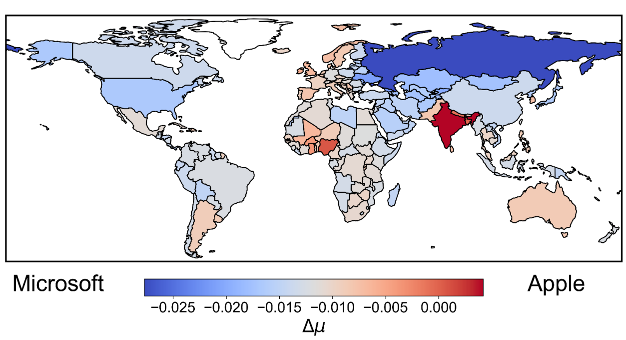


FIG. 4. Opinion polarization of world countries for *Apple Inc.* ($\Delta\mu > 0$) vs. *Microsoft Corporation* ($\Delta\mu < 0$), $\mu_0 = -0.076$ following OPS for EN Wiki2025.

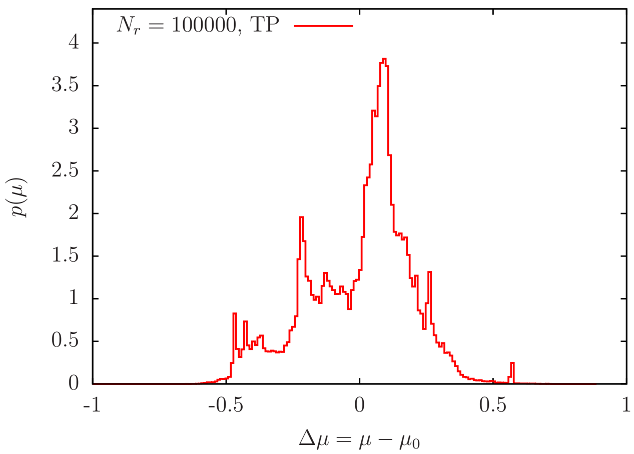


FIG. 5. Probability density $p(\mu)$ for the OPS case in Eq. (1) for two fixed nodes with *Vladimir Putin* (red) vs. *Donald Trump* (blue) for EN Wiki2025 (TP competition) with same bin number and normalization as in Figure 2. Here only one red curve of $p(\mu)$ versus $\Delta\mu = \mu - \mu_0$ for $N_r = 100000$ is shown, $\mu_0 = 0.11262$.

Another example is the competition between two pharmacological companies *Pfizer* (red $\mu = 1$) vs. *Johnson & Johnson* (blue $\mu = -1$) is presented in Appendix Figure A1 showing the polarization of countries on the world map. The obtained results show a stronger influence of Pfizer with $\mu_0 = 0.049$ with its dominance in Canada, India, China even if Johnson & Johnson is richer but Pfizer is more ancient and more influential in WIN.

C. Contest Donald Trump vs. Vladimir Putin

Another example is a contest between two groups with one red fixed node *Vladimir Putin* and one fixed blue node *Donald Trump* for OPS case at EN Wiki2025.

Country	Article	$\Delta\mu$ (Putin - Trump)
Argentina	Javier Milei	0.071
Australia	Anthony Albanese	0.048
Brazil	Luiz Inácio Lula da Silva	0.066
Canada	Mark Carney	-0.042
China	Xi Jinping	0.120
France	Emmanuel Macron	0.120
Germany	Friedrich Merz	0.200
India	Narendra Modi	0.165
Indonesia	Prabowo Subianto	0.161
Italy	Giorgia Meloni	0.166
Japan	Shigeru Ishiba	0.124
Mexico	Claudia Sheinbaum	-0.040
Russia	Vladimir Putin	0.887
Saudi Arabia	Salman of Saudi Arabia	0.164
South Africa	Cyril Ramaphosa	0.111
South Korea	Lee Jae-myung	0.076
Turkey	Recep Tayyip Erdoğan	0.234
United Kingdom	Keir Starmer	0.082
United States	Donald Trump	-1.113
European Union	Ursula von der Leyen	0.155
African Union	Mahamoud Ali Youssouf	0.176

TABLE I. Opinion polarization expressed by $\Delta\mu$ (following OPS for EN Wiki 2025), for leaders of G20. *Vladimir Putin* corresponds to $\mu = 1$ and *Donald Trump* corresponds to $\mu = -1$ with $\mu_0 = 0.113$.

The probability density distribution $p(\mu)$ for this contest in shown in Figure 5 with $\mu_0 = 0.113$ being in the favor of Putin. This corresponds to $f_p = (1 + \mu_0)/2 = 0.5565$ votes for Putin and $f_b = (1 - \mu_0)/2 = 0.4435$ votes for Trump over all nodes of EN Wiki2025 network (white nodes are excluded). The highest peak in Figure 5 at $\mu \approx \mu_0 + 0.1$ corresponds to such articles as *Breton language, French Polynesia, Field (mathematics), Moses, Mao Zedong, Nairobi, Table tennis.*

In Table I we show the opinion polarization $\Delta\mu_{G20} = \mu_{G20} - \mu_0$ for the contest Putin vs Trump for $G20$ political leaders representing most influential world countries (including representatives of European Union and African Union). It is surprising to see that only leaders of Canada and Mexico have a polarization $\Delta\mu$ in favor of Trump while all others have polarization $\Delta\mu$ in favor of Putin with highest $\Delta\mu$ values for Turkey and Germany.

Here we should note that polarization in favor of one or another entry, as the case Trump vs Putin, does not mean that a given entry is favorable to Putin ($\Delta\mu > 0$) or Trump ($\Delta\mu < 0$). Indeed, it is difficult to think that Macron, Merz or Starmer are favorable to Putin. The polarization μ measures the strength of links between entries or articles but it does not take into account if a link has positive (like) or negative (dislike) attitude. Thus it is more correct to interpret articles with a high positive $\Delta\mu > 0$ value as strongly linked with or influenced by Putin, and high negative $\Delta\mu < 0$ as those strongly linked with Trump. Indeed, Canada and Mexico are strongly linked/influenced with/by USA and thus with Trump while Germany and Turkey are strongly linked/influenced with/by Russia and thus with Putin. The information if links hold like/dislike (positive/negative) attitude is not accessible by the present network construction and INOF approach.

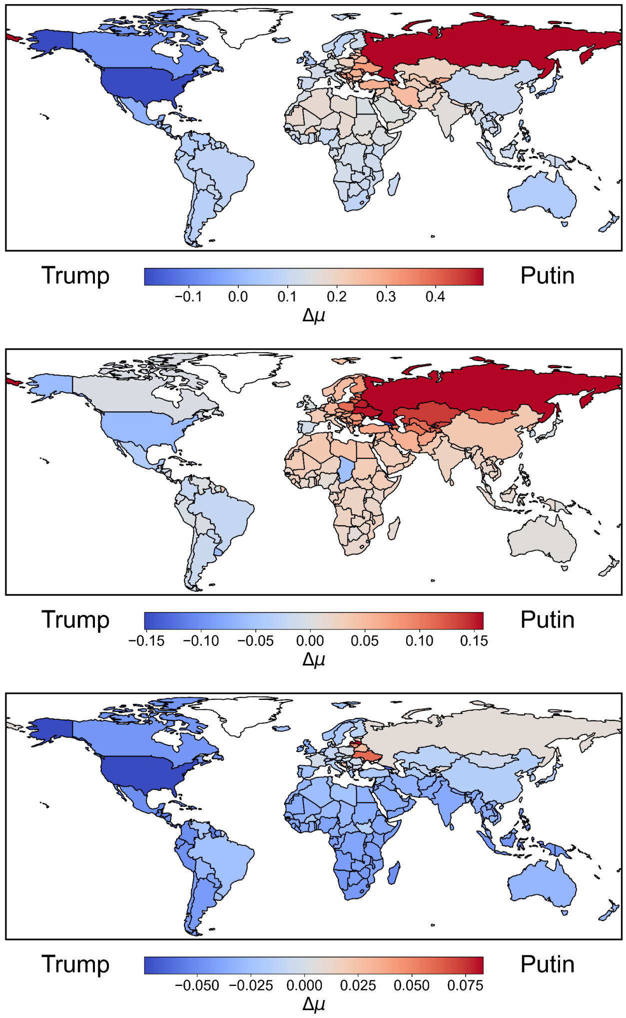


FIG. 6. Opinion polarization of world countries for *Vladimir Putin* ($\Delta\mu > 0$) vs. *Donald Trump* ($\Delta\mu < 0$) following OPS for EN Wiki2025 (top panel with $\mu_0 = 0.113$), RU Wiki2025 (middle panel with $\mu_0 = 0.592$) and ZH Wiki2025 (bottom panel with $\mu_0 = -0.340$).

The influence of Trump vs Putin for the world countries (represented by the Wikipedia articles of these countries) is presented via the world map in Figure 6 with $\Delta\mu$ values of all countries and for three editions EN, RU (for Russian), ZH (for Chinese) of Wikipedia 2025. For EN Wiki2025 the positive polarization $\Delta\mu$ for Putin extends to Russia, former republics of USSR, with strong influence for Turkey, Iran, Bosnia and Herzegovina, Serbia, Albania. The country polarization in favor of Trump includes USA, Canada, Mexico, Latin America, UK, Australia, South Africa, Japan, Ireland, New Zealand. From Figure 6 middle panel for RU Wiki2025 it follows that the polarization in favor of Putin is significantly increased propagating to higher number of countries including West Europe, Northern Africa, slightly positive polarization of China. Polarization in favor of Trump extends from USA to Mexico, Brazil, Argentina, Chad. This is rather natural since the Russian Wikipedia edition favors the importance of Russia and Putin. The polarization from ZH Wiki2025 has almost all countries polarized in favor of Trump; Russia has zero polarization $\Delta\mu \approx 0$. Positive polarization for Putin exists only in Latvia, Ukraine, Lithuania, Belarus, Bosnia and Herzegovina, Estonia, Moldova and Luxembourg. This corresponds to the fact that events of special military operation of Russia in Ukraine are strongly linked with Putin explaining his significant influence on Ukraine. However, it is somewhat surprising that the ZH edition shows so strong polarization for Trump with $\mu_0 = -0.340$. We attribute this to very strong commercial exchange between China and USA. Also one should take into account that mainland China has its one analog of Wikipedia in Chinese known as Baidu Baike. Also the

Chinese government has cut off access to the Chinese Wikipedia for residents of mainland China since 2019. Thus the contributions to the ZH edition at Wikipedia are coming mainly from outside of the mainland China and thus they may be rather different from the view points of the majority of China's population.

D. Competition Emmanuel Macron vs. Marine Le Pen

We also consider the INOF competition between Emmanuel Macron ($\mu = 1$) vs. Marine Le Pen ($\mu = -1$) with $\mu_0 = -0.028$ from FR Wiki2024. These corresponds to rather close vote fractions with $f_r = (1+\mu_0)/2 = 0.4860$ for Macron and $f_b = (1-\mu_0)/2 = 0.5140$ for Le Pen (number of pathway realizations $N_r = 10^6$). The polarization opinions of 14 French political figures and top 10 French richest persons from the Forbes list 2015-2024 are presented in Appendix Table A1. The results for these 24 persons show that only two of them have a polarization in favor of Emmanuel Macron and other 22 in favor of Marine Le Pen. At the same time the values of $\Delta\mu$ are mainly located at relatively small values $|\Delta\mu| \sim 0.01$ being approximately by a factor 10 smaller of values of for the contest of Trump-Putin shown in Table I. Due to these smaller $\Delta\mu$ values we with also compute an additional data set with $N_r = 10^6$ to reduce the statistical errors (note that the FR Wiki2024 network is roughly 2 times smaller in number of nodes and links as compared to those of EN Wiki2025 thus reducing the numerical effort). Table A1 shows two columns of $\Delta\mu$ computed for $N_r = 10^5$ and $N_r = 10^6$ (the smaller data set is statistically independent and not included in the larger date set). The values are rather close and the small differences indicate the size of the typical fluctuations at $N_r = 10^5$.

The theoretical error of the μ_i values is $(1-\mu_i^2)/\sqrt{N_r} \approx 0.003$ (for $N = 10^5$) or ≈ 0.001 (for $N = 10^6$). However, the more precise error estimation for $\Delta\mu_i = \mu_i - \mu_0$ using a subdivision of the data in 100 samples (described above after the discussion for Figure 2) provides typical statistical errors of $\Delta\mu_i$ for the entries in Table A1 which are roughly 3 times smaller than the theoretical error, i.e. ≈ 0.001 (for $N = 10^5$) or ≈ 0.0003 (for $N_r = 10^6$) which is due to rather strong correlations between μ_i and μ_0. Therefore, despite the typical small values of $\Delta\mu_i \approx 0.01$ in Table A1 their relative error is mostly only 3% (for the $N_r = 10^6$ data). Note, that the error 0.0023 of μ_0 itself is closer to the theoretical error 0.003 which is confirmed by the value $\mu_0 = -0.025$ for $N_r = 10^5$ while $\mu_0 = -0.028$ for $N_r = 10^6$.

For the world countries only a few of them have a favorable opinion polarization for Emmanuel Macron (e.g. *Andorra, Ivory Coast, San Marino, Finland, Qatar*) while all others have $\Delta\mu < 0$ in favor of Marine Le Pen with typical values $\Delta\mu \sim -0.01$. This result is surprising since Emmanuel Macron, as a president, has much more activity on the international level as compared to Marine Le Pen. Our interpretation is similar to those discussed for the Trump-Putin case in Table I: a high opinion polarization in a favor of an entry does not necessary mean positive or negative attitude to this entry from the viewpoint of a given Wikipedia article but shows that this entry produces a significant influence on this article (positive or negative).

IV. RESULTS FOR COMPETITION OF THREE GROUPS OF ENTRIES

It is possible to generalize the competition between 2 groups to a competition between 3 groups. A similar case for a competition of 3 currencies in the world trade has been considered in [19]. However, in [19] there were no white nodes in the initial distribution of nodes and the network size was very small representing only about 200 countries (nodes).

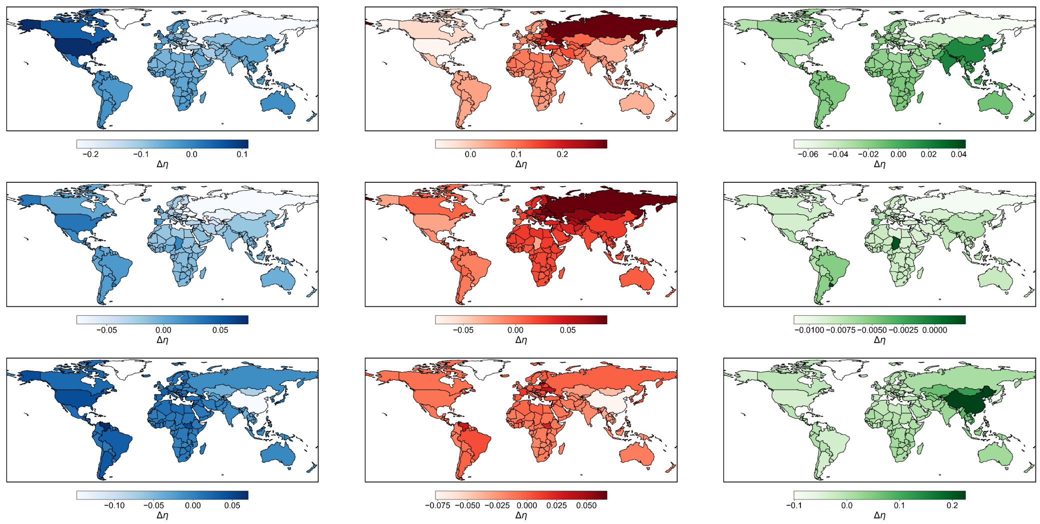


FIG. 7. Opinion polarization of world countries for *Donald Trump*, *Vladimir Putin* and *Xi Jinping* following OPS for EN Wiki2025 (top panels), RU Wiki2025 (center row panels) and ZH Wiki2025 (bottom panels). Column panels show the cases of *Donald Trump* in blue color on the left, *Vladimir Putin* in red color in the center and *Xi Jinping* in green color on the right. The corresponding values of $\eta_{0,Trump}$, $\eta_{0,Putin}$ and $\eta_{0,Jinping}$ are: $\eta_{0,Trump} = 0.398$, $\eta_{0,Putin} = 0.4597$, $\eta_{0,Jinping} = 0.1424$ for EN; $\eta_{0,Trump} = 0.193$, $\eta_{0,Putin} = 0.779$, $\eta_{0,Jinping} = 0.0282$ for RU; and $\eta_{0,Trump} = 0.223$, $\eta_{0,Putin} = 0.104$, $\eta_{0,Jinping} = 0.673$.

For the case of 3 competing groups, we compute for a given node i three scores $Z_i(C)$ for three color values C by:

$$Z_i(C) = \sum_{j \neq i} \sigma_j(C) V_{ij}. \tag{2}$$

Here $\sigma_j(C) = 1$ if node j has color C otherwise $\sigma_j(C) = 0$ and in the computation of $Z_i(C)$ only nodes i with color C contribute. Note that the white color counts as an effective fourth color which has also a score but this fourth white score is not used in the spin update process of the Monte Carlo procedure. If among the three score values $Z_i(C)$ (for the three non-white colors) there is a single clear maximum color $C_{\max}$ with $Z_i(C_{\max}) > Z_i(C)$ for $C_{\max} \neq C$, the node i will acquire the new color $C_{\max}$. If there is no clear maximum, i.e. with at least two maximal identical values $Z_i(C_1) = Z_i(C_2) \geq Z_i(C_3)$ (for $C_1 \neq C_2 \neq C_3 \neq C_1$) the color of node i will not be changed (it may also stay white if it was white before). Note that we consider the three group competitions only for the OPS case with $V_{ij} = \tilde{S}_{ij}$ having fractional values. Therefore the scenario of two equal maximal scores and a third strictly smaller score $(Z_i(C_3) < Z_i(C_{1,2}))$ is very rare. However, the scenario of having three identical values being zero $Z_i(C_1) = Z_i(C_2) = Z_i(C_3) = 0$ may happen quite regularly if all nodes j with non-zero values of V_{ij} in the sum (2) have still their initial blank color.

After the color update of node i the Monte Carlo procedure is performed in the same way as for the above case (1) of two colors: the small number of nodes of the three groups with initially fixed color are never updated (they have a "frozen"color) and the update procedure is done in random order for all other non-fixed nodes which have the white color as initial condition. A full update run is repeated up to $\tau = 20$ iterations at which nearly all node color values are stable and in a steady state distribution. Finally, this procedure is repeated with the same initial condition but for N_r different random pathway realizations in the update order which allows to compute averages and distributions of the obtained network color fractions.

As for the two group competition, once a node switches from white to another color it cannot go back to the initial white color but even with this there is still a significant fraction of nodes which stay (nearly) always white for all N_r pathway realizations. The sets of of "white" nodes essentially only depend on the used Wikipedia edition (and not on the selected fixed color nodes for the competition) and these sets are also the same as for the two group competition.

Formally the competition of 3 colors is different from the Ising case of two colors with spins up or down but we still keep notations INOF, WIN for the case of 3 color competition since it appeared originally from the Ising type spin relation (1) with 2 colors. We note that in both procedures the spin/color information propagates from the initial groups with frozen spin/color through the network and after $\tau = 20$ update iterations (per node) essentially all non-white nodes have a stable spin/color value which no longer changes. However, the final spin/color value of each node depends strongly on the selected random pathway for the update order (see also Figure 1).

We attribute to each of the three groups its own color $C \in \{R, G, B\}$ being red, green or blue (RGB) and compute for each (non-white) node the *color polarization* of color C as the fraction $\eta_C(i) = n_C(i)/\sum_{C' \in \{R,G,B\}} n_{C'}(i)$ where $n_C(i)$ is the number of color C outcomes of node i in the N_r pathway realizations. (Note that for the non-white nodes i we typically have $\sum_{C' \in \{R,G,B\}} n_{C'}(i) \approx N_r$ while for the white nodes j we have $\sum_{C' \in \{R,G,B\}} n_{C'}(j) \ll N_r$.)

From $\eta_C(i)$ we compute for $C \in \{R, G, B\}$ its global network average (over non-white nodes) which is noted $\eta_{0,C}$ and we characterize the node preference for color C by the difference $\Delta\eta_C(i) = \eta_C(i) - \eta_{0,C}$ which represents the color preference of node i in comparison to the global network color preference (both for color C).

For the 3-entry analysis, where each of the 3 entries is mapped to an RGB color channel, we present two types of world map visualizations: *Monochromatic Maps:* Each map displays a single color channel (Red, Green, or Blue). The color intensity is scaled by the $\Delta\eta_C$ value, ranging from zero to maximum saturation. *Multicolor Maps:* These maps use an RGB color triangle to represent the combination of the 3 entries. The triangle is constructed such that each vertex represents a pure color, corresponding to the maximum value of one component (η_R, η_G, or η_B) while the other two are at their minima.

Concerning the statistical error of $\eta_C(i)$ or $\Delta\eta_C(i)$, we mention that the theoretical error can be obtained in a similar way as for the case of two group competitions. Now, we use that $\eta_C(i)$ is the average (over the N_r random pathway realizations) of $\sigma_i(C)$ a quantity which has values 0 or 1 such that $\sigma_i(C)^2 = \sigma_i(C)$. This allows to compute the variance from its average and gives the theoretical error of $\eta_C(i)$ as $\sqrt{\eta_C(i)[1 - \eta_C(i)]/(N_r - 1)} \approx 1/(2\sqrt{N_r}) \approx 0.0015$ for $N_r = 10^5$ and $\eta_C(i) \approx 0.5$ (value of maximal theoretical error) which is similar to the two color case. (The factor 1/2 is a trivial effect of the formula $\eta_C(i) = (1 \pm \mu)/2$ for the two color case.) We have also verified by the method of sample averages that the error of $\Delta\eta_i(C)$ is typically reduced by the same factor $2 - 3$ as for the two color case.

In the following, we present results for the competition of three groups in next subsections for 3 types of groups being political leaders, countries and society political concepts.

A. Contest Trump, Putin, Xi Jinping

In Figure 7 we present the world map of (monochromatic) color polarization of countries for *Donald Trump, Vladimir Putin, Xi Jinping* from the view of the three Wiki2025 editions EN, RU, ZH. The values of global color polarization $\eta_{0,Trump}, \eta_{0,Putin}, \eta_{0,Jinping}$ of these 3 political leaders are given in the caption of Figure 7 (of course $\eta_{0,Trump} + \eta_{0,Putin} + \eta_{0,Jinping} = 1$).

For EN Putin has the highest color polarization being ahead of Trump and then Jinping. For the pair Trump-Putin their relative polarization remains approximately as in their own two group contest presented in the previous Section. It is interesting to note that in this edition there are more countries with $\Delta\eta$ in favor of Putin, also in this case the maximal positive value $\Delta\eta(C) \approx 0.3$ is by a factor 3 higher than for the case of Trump and by a factor 7 higher than for the case of Jinping.

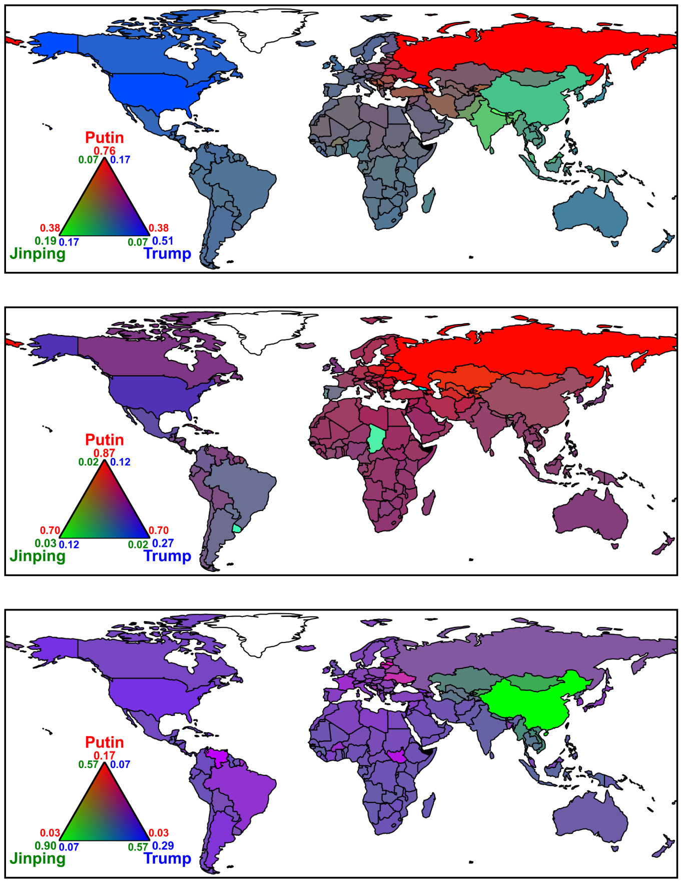


FIG. 8. Opinion polarization of world countries $(\eta_{Trump}, \eta_{Putin}, \eta_{Jinping})$ for *Donald Trump*, *Vladimir Putin* and *Xi Jinping* following OPS for EN Wiki2025 (top panel), RU Wiki2025 (middle panel) and ZH Wiki2025 (bottom panel) (same as Figure 7). The color mapping uses an **RGB color triangle** to visualize the 3-component polarization data $(\eta_{\text{Putin}}, \eta_{\text{Jinping}}, \eta_{\text{Trump}})$. The vertices of the triangle are defined by the observed extrema in the data of countries. Specifically, the pure **Red vertex** represents the state where η_{Putin} is at its maximum observed value $(\eta_{\text{Putin, max}})$ while both η_{Jinping} and η_{Trump} are at their minimums $(\eta_{\text{Jinping, min}}, \eta_{\text{Trump, min}})$. The **Green** and **Blue** vertices are defined analogously for Jinping and Trump, respectively. The resulting color for each country is an interpolation within this triangle, where mixtures like magenta indicate high polarization towards both Putin (Red) and Trump (Blue), and neutral colors represent a more balanced polarization.

For RU the color polarization in favor of Putin extends even over more countries than expected. We attribute this to the fact that RU Wiki naturally gives a higher preference to the president of Russia.

The ZH edition naturally places Jinping at the highest global color polarization followed by Trump and then Putin. The maximal polarization of countries $\Delta\eta(C)$ is also by a factor 4 higher as compared to the cases of Trump and Putin showing a high influence of Jinping of world countries from the view point of the ZH edition.

Any color can be presented as a combination of three colors red, green, blue (RGB). Taking into account this property we can make a summation of 3-colors world map in Figure 7 for each country using its corresponding color average of $\eta_{Trump}, \eta_{Putin}, \eta_{Jinping}$ (for each edition) and as a result to obtain a color RGB world map of countries. The result of this operation is presented in Figure 8 for three editions EN, RU, ZH of Wiki 2025.

From the EN edition we see that the influence of Putin of course completely dominates in Russia and also propagates to former USSR republics (but it is not strong in the countries of Central Asia) and few countries of East Europe such as Romania, Hungary, Serbia and also with a smaller strength to Turkey, Iran . The influence of Trump naturally dominates USA and extends to Canada, Mexico, Latin America, UK, Australia, New Zealand, Japan. South Korea. The influence of Xi Jinping from China extends to India, Pakistan and countries of South East Asia.

In the case of the RU edition the influence of Putin propagates from Russia to former Soviet republics, Afghanistan and in a less strong way to Turkey, Iran, Poland. The influence of Trump is restricted to USA extending to Brazil, Argentina, Mexico. The clear influence of Xi Jinping is well seen for Chad and Uruguay and is not well visible for other countries that can be considered as a significant exaggeration of the RU edition. There are many countries with color being a mixture of red and blue (between USA and Russia).

For the ZH edition the influence of Xi Jinping propagates from China to Mongolia, Kazakhstan, Kyrgyzstan as most obvious cases. The country colors for influence of Trump and Putin are mixed and give no clear preferences.

In Appendix Figures A2, A3 we show the density, or frequency, of articles in the planes of color values $\eta_{Trump}, \eta_{Putin}, \eta_{Jinping}$ for the EN, RU, ZH editions that allows to see in a better way the distribution of articles and their color polarizations. (more technical details in the captions of these figures). Note that for the case of a pure two group competition these type of figures would give straight lines on the antidiagonal (from $(0,1)$ to $(1,0)$), since e.g. $1 = \eta_{Trump} + \eta_{Putin}$ for a the pure *Trump-Putin* competition. In Appendix Figure A2, the data are indeed somewhat concentrated close to this antidiagonal showing the that modifications due to the influence of the third group of *Jinping* are rather modest for the editions of EN and RU.

B. Contest USA, Russia, China

We present the results of contest between USA, Russia and China in Figures 9, 10. The presentation is similar to the previous case of contest of Donald Trump, Vladimir Putin, Xi Jinping with the same three colors of opinion polarization. Here the situation is more standard with a dominance of USA in EN, Russia in RU, China in the ZH edition. Each of the three editions seems to emphasize its color polarization of other world countries in favor of their country of native language (USA for EN, Russia for RU, China for ZH) as it is well visible in corresponding panels of Figure 9. This effect is also clearly seen in Figure 10 in the RGB representation. Thus in its native edition each of three countries considers that it has a dominant influence on the main part of world countries. In Appendix Figure A4 we show the density distribution of articles (nodes) in the planes of color opinion polarization ($\eta_{USA} - \eta_{Russia}$-plane). Here in two out of three cases (EN and RU) the data is also somewhat close to the antidiagonal indicating a reduced influence of the third group for *China*.

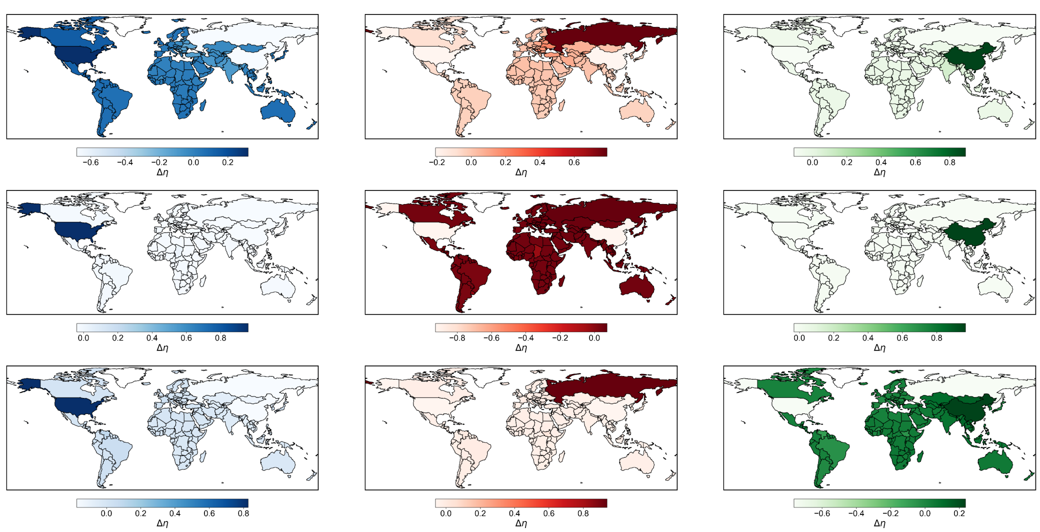


FIG. 9. Opinion polarization of world countries for *USA*, *Russia* and *China* following OPS for EN Wiki2025 (top panels), RU Wiki2025 (center row panels) and ZH Wiki2025 (bottom panels). Column panels show the case of *USA* in blue color on the left, *Russia* in red color in the center and *China* in green color on the right. The corresponding values of $\eta_{0,USA}$, $\eta_{0,Russia}$ and $\eta_{0,China}$ are: $\eta_{0,USA} = 0.682$, $\eta_{0,Russia} = 0.209$, $\eta_{0,China} = 0.109$ for EN; $\eta_{0,USA} = 0.040$, $\eta_{0,Russia} = 0.927$, $\eta_{0,China} = 0.033$ for RU; and $\eta_{0,USA} = 0.174$, $\eta_{0,Russia} = 0.062$, $\eta_{0,China} = 0.764$.

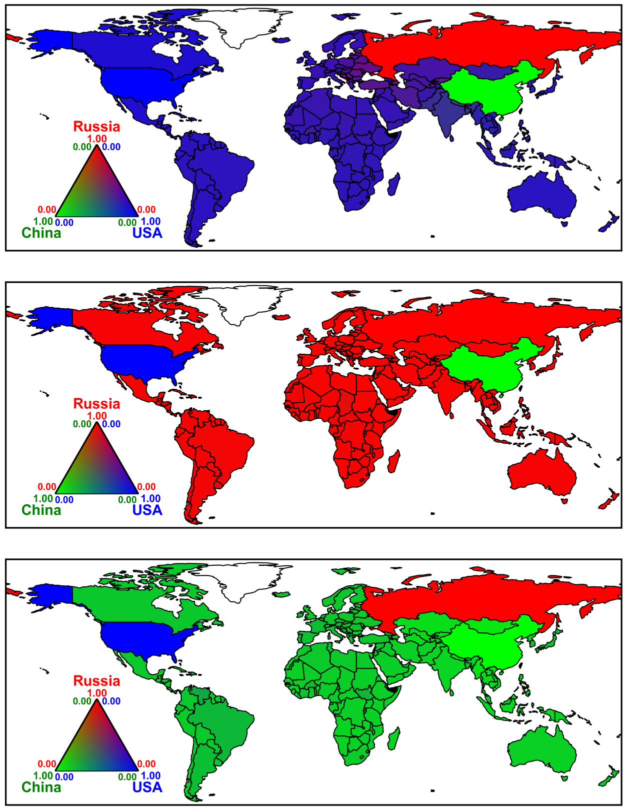


FIG. 10. Opinion polarization of world countries ($\eta_{USA}, \eta_{Russia}, \eta_{China}$) for *USA*, *Russia* and *China* following OPS for EN Wiki2025 (top panel), RU Wiki2025 (middle panel) and ZH Wiki2025 (bottom panel) (same as Figure 8) in color scale is given by normalized RGB value of Russia, China and USA respectively.

It it interesting to compare the correlations of polarization influence on 197 countries by political leaders: Donald Trump with those of USA, Vladimir Putin with Russia and Xi Jinping with China. To do this we present in Appendix Figure A5 the plane of polarizations of 197 countries for editions EN, RU, ZH; the values of Spearman correlation coefficient ρ are given for each pair USA-Trump, Russia-Putin, China-Jinping for these three editions. The two smallest correlator values are for the the pair China-Jinping (EN edition) with $\rho = 0.59$ and the pair USA-Trump (ZH edition) with $\rho = 0.89$ while for all other pairs and editions we have $0.9 \leq \rho \leq 0.97$. Such high correlator values clearly show the close link between the influence of political leaders on 197 world countries to those of their own country influence on these 197 countries.

C. Contest Liberalism, Communism, Nationalism

As a final example, we consider for the EN Wiki2025 edition the influence competition of the three social concepts *Liberalism* (blue), *Communism* (red) and *Nationalism* (green).

Their color influence are shown in Figure 11 with 3 monochromatic color maps in the top 3 panels and an RGB map in the bottom panel.

From the RGB map, we see that the strongest influence of Nationalism is for Serbia, Bosnia Herzegovina, Albania, Turkey, Romania, Bulgaria and in a less respect Pakistan, India, Bangladesh. The influence of Communism is rather local being represented only in Nepal. The influence of Liberalism is mostly spread over the world with the strongest adepts being UK, USA, Somaliland, Belgium, Netherlands and many others. The color of Russia is at the middle between Communism and Nationalism. This distribution of influence of the three concepts corresponds to the political orientations of these countries. Furthermore, in Appendix Figure A6 we present the histogram distribution in $\eta_{Liberalism} - \eta_{communism}$-plane. Here the articles are approximately distributed around the coordinate $(0.33, 0.23)$ corresponding to $\eta_{Nationalism} = 0.44$ which appears to be the typical polarization of these three concepts for 197 countries and also all other Wikipedia articles. In particular, here the distribution is not concentrated close to the antidiagonal as for certain other cases, showing that none of the three groups has a significantly reduced influence in comparison to the other two groups.

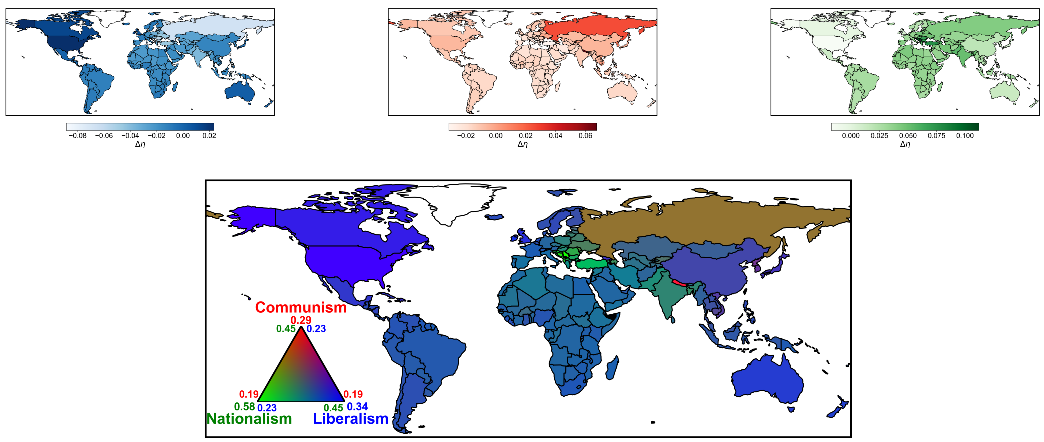


FIG. 11. Opinion polarization of world countries for *Liberalism*, *Communism* and *Nationalism* following OPS for EN Wiki2025 (top panels). Column panels show the case of *Liberalism* in blue color on the left, *Communism* in red color on the center and *Nationalism* in green color on the right. In bottom panel the values of $\eta_{Liberalism}$, $\eta_{Communism}$ and $\eta_{Nationalism}$ are shown in color scale given by normalized RGB values. The corresponding values of $\eta_{0,Liberalism}$, $\eta_{0,Communism}$ and $\eta_{0,Nationalism}$ are: $\eta_{0,Liberalism} = 0.316$, $\eta_{0,Communism} = 0.219$ and $\eta_{0,Nationalism} = 0.466$ for EN.

V. EFFECTS OF FLUCTUATIONS AT EFFECTIVE FINITE TEMPERATURE

For the competition of two groups with different opinions (red vs, blue) the relation (1) for Z_i determines the condition of spin updates with $\sigma_i = 1$ if $Z_i > 1$, $\sigma_i = -1$ if $Z_i - < 0$ and no spin change if $Z_i = 0$. Such a condition corresponds in the Monte Carlo process to the effective temperature $T = 0$ since it gives a firm choice for the updated spin. It is interesting to analyze how stable this procedure is in presence of fluctuations produced by a finite effective temperature T. A finite T value physically corresponds to the presence of finite probabilities $W_\pm(i)$ $(W_+(i) + W_-(i) = 1)$ to obtain the new spin value $\sigma_i = \pm 1$.

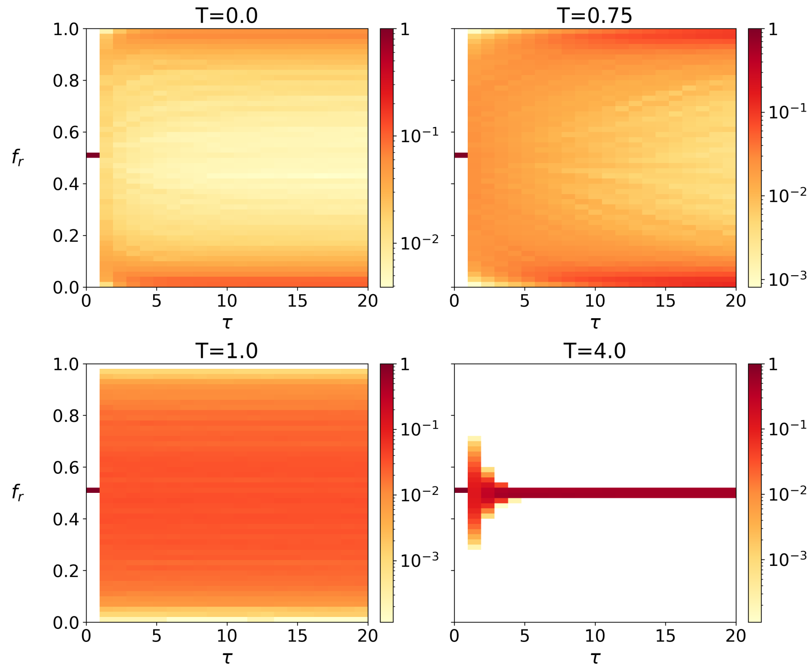


FIG. 12. Density of the fraction of red nodes, f_r, as a function of τ, for the competition between *Socialism, Communism* (red) and *Capitalism, Imperialism* (blue), using the EN Wiki2024 dataset. Each panel shows a density plot for 10^4 pathway realizations on a logarithmic scale. The distribution is normalized to one and uses 50 bins for the range $f_r \in [0, 1]$. Panels correspond to different temperatures: $T = 0$ (top left), $T = 0.75$ (top right), $T = 1$ (bottom left), and $T = 4$ (bottom right).

To model this situation we write $Z_i = Z_+(i) - Z_-(i)$ as a difference of two positive quantities $Z_\pm(i) = \sum_{j\neq i} V_{ij}\delta_{\sigma_j,\pm 1} \geq 0$ (i.e. sum only over all j with either $\sigma_j = 1$ or $\sigma = -1$ for the two cases $+$ or $-$ respectively). This is similar to the color score $Z_i(C)$ used in (2) if we use only two colors for spins ± 1. Then the probabilities $W_\pm(i)$ are determined by the relations

$$W_+(i) = \frac{{Z_+}^\beta(i)}{{Z_+}^\beta(i) + {Z_-}^\beta(i)} \quad , \quad W_-(i) = \frac{{Z_-}^\beta(i)}{{Z_+}^\beta(i) + {Z_-}^\beta(i)} \quad , \quad T = \frac{1}{\beta} \tag{3}$$

where during a Monte Carlo step the spin i takes the value $\sigma_i = \pm 1$ with probability $W_\pm(i)$. At $T = 0$ ($\beta \to \infty$) we have $W_+(i) = 1$ and $W_-(i) = 0$ if $Z_i = Z_+(i) - Z_-(i) > 0$ ($W_+(i) = 0$ and $W_-(i) = 1$ if $Z_i = Z_+(i) - Z_-(i) < 0$) which reproduces the previous spin update condition based on $Z_i > 0$ or $Z_i < 0$. At high temperature $T \gg 1$ ($\beta \ll 1$) we have $W_+(i) \approx W_-(i) \approx 1/2$ such that the new spin value $\sigma_i = \pm 1$ is purely random with equal probabilities.

We mention that (3) can be understood by introducing two virtual "energy levels" $\varepsilon_\pm(i) = -\ln(Z_\pm(i))$ such $Z_\pm(i)^\beta = e^{-\beta\varepsilon_\pm(i)}$ (for each node i there is a different two level system). In this case the probabilities (3) are just the usual probabilities of the levels $\varepsilon_\pm(i)$ in the canonical ensemble at temperature T for this two level system: $W_\pm(i) = e^{-\beta\varepsilon_\pm(i)}/(e^{-\beta\varepsilon_+(i)} + e^{-\beta\varepsilon_-(i)})$.

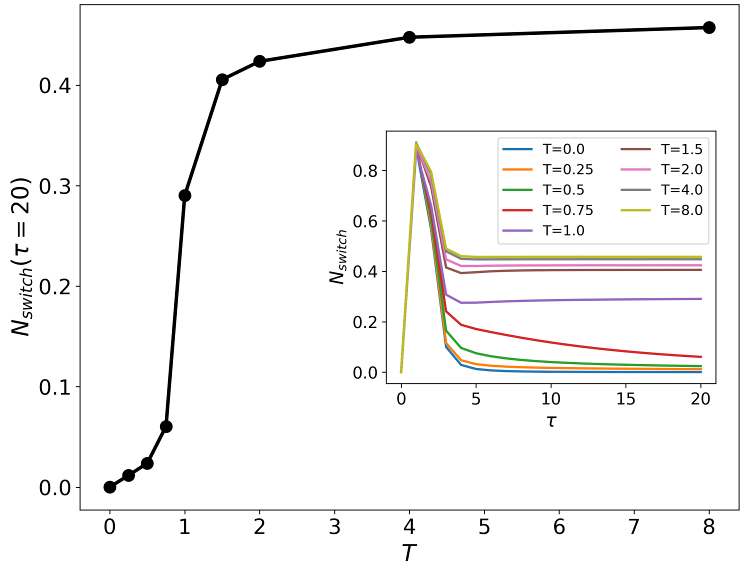


FIG. 13. Normalized number of switches, N_{switch}, between white, red, and blue nodes for the competition between *Socialism, Communism* (red) and *Capitalism, Imperialism* (blue), using the EN Wiki2024 dataset. The normalization is such that $N_{\mathrm{switch}} = 1$ corresponds to a number of switches equal to the total number of nodes. Main panel: N_{switch} at maximal iteration time $\tau = 20$ as a function of temperature T. Inset panel: The time evolution of N_{switch} for several temperature values.

The results for this finite temperature model of fluctuations are shown in Figures 12 and 13. Figure 12 shows that at $T \leq 0.75$ the density distribution of the (network) fraction of red nodes f_r in at maximal iteration time $\tau = 20$ is concentrated to the two regions $f_r \approx 0$ and $f_r \approx 1$ (similarly as Figure 1 which corresponds to $T = 0$). In contrast at $T = 1$ this density has a broad homogeneous distribution approximately in the range $0.2 \leq f_r \leq 0.8$ and at $T = 4$ all density is located in a narrow range around $f_r = 0.5$. Indeed, at such a high temperature the probabilities to have spin up or down from (3) are very close $W_+ \approx W_- \approx 0.5$ and hence we have approximately half od spins up and half down. The transition from a spin polarized steady-state at low temperatures to a non-polarized one takes place in the vicinity of a certain critical temperature T_c. Its value can be approximately determined by measuring the normalized number of spin flips or spin switches N_{switch} at *maximal iteration time* $\tau = 20$ as a function of temperature T. Up to now this quantity (at $T = 0$) is essentially zero since at $\tau = 20$ for a specific given pathway realization the spins of individual nodes are mostly in stable steady state.

This dependence of N_{switch} on temperature T (and also on iteration time τ) is shown in Figure 13. This Figure shows that the critical temperature is $T_c \approx 1$ where we have a sharp increase of number of flips (at $\tau = 20$) and a rapid growth of the normalized switch number N_{switch}. Thus the obtained results of Figures 12 and 13 show that the spin polarized phase remains stable for the temperature range $0 \leq T \leq T_c \approx 1$ while above T_c there is a melting of the polarized phase and we obtain a non-polarized liquid state at $T > T_c \approx 1$ at which individual spins no longer have stable values with respect to iteration time even at $\tau \geq 20$. In particular, we see that for a *specific given random pathway realization* at $T = 0$ or $T \ll T_c$ the spin values of individual nodes become stable in time (fluctuations discussed in the previous section are entirely due to the many *different* random pathway realizations which produce *different* steady states) while at $T \geq T_c$ there is no real spin-steady state (for a given pathway realization) and spins continue to be flipped even at $\tau \geq 20$.

We argue that the main result of this effective temperature model (3) is the fact that the polarized phase remains stable with respect to fluctuations at small or modest temperatures.

VI. DISCUSSION AND CONCLUSION

In this work we described the process of opinion formation appearing in Wikipedia Ising Networks (WIN) being based on an asynchronous Monte Carlo procedure. This INOF approach is determined by a simple natural rule that the opinion of a given node (article, user) in a network is determined by a majority opinion of other nodes connected to this given node. We discussed two possible voting procedures: OPA case where vote contributions are given by a sum over elements of the adjacency matrix A_{ij} going to a selected given node i or the OPS case when the the weight of a vote is given by an element of normalized matrix of Markov transitions S_{ij} (see 1). Only the OPA case was considered in previous studies [14]. We show that these two vote options give similar results but specific vote polarizations may be different. We think that both vote options OPA and OPS can be suitable for the description of opinion formation on networks. Thus for the protein-protein interaction networks we think that the OPS case, used for the MetaCore protein network in [18], is more correct since the interaction capacity of a given protein is bounded by various chemical processes. The important new element of the INOF approach is the presence of white nodes with undefined opinion at the initial stage of the asynchronous Monte Carlo process. Our results show that the spin polarized steady-state remains stable with respect to small fluctuations at an effective temperature below a certain critical border while above this border there is a melting of this phase and a transition to a liquid non-polarized spin phase.

We also demonstrated that the situation with competition between two groups with fixed red/blue opinions can be generalized to the case of three competing groups with fixed opinions (red, green, blue) and that in this case the generalized INOF approach leads to fair results for WIN,

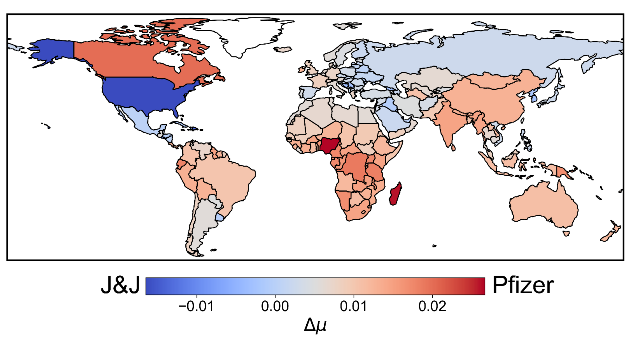
For the EN, RY, ZH editions of Wikipedia 2025 we compared opinions of different cultural views of these editions with respect to political leaders Donald Trump, Vladimir Putin, Xi Jinping and determined their influence on 197 world counties. Surprisingly Putin happens to produce a higher polarization influence in the EN edition. With the INOF approach we also determined the influence of USA, Russia, China on other countries for these 3 editions. We also showed that other types of contests can be studied like the competition between Liberalism, Communism, Nationalism.

The described INOF approach is generic and can be applied to various directed networks. Thus in [18] this approach allowed to describe myocardial fibrosis progression in the MetaCore network of protein-protein interactions. Also, a variation of this approach (without white nodes) determines dominant features of trade currencies in the World Trade Network from the UN COMTRADE database [19].

Of course, the Wikipedia networks have important exceptional features as compared to other networks: the meaning of their nodes is very clear, they enclose all aspects of nature and human activity and the presence of multiple language editions allows to analyze various cultural views of humanity. Thus we hope that the INOF approach to Wikipedia Ising networks will find diverse interesting applications.

APPENDIX

Here we present additional Figures and one additional data table related to the main part of the article. Below, we mostly only give a short description of them and for a more detailed discussion of this materiel we refer to the main text.



Appendix Figure A1. Opinion polarization of world countries for *Pfizer* ($\Delta\mu > 0$) vs. *Johnson & Johnson* ($\Delta\mu < 0$), $\mu_0 = 0.049$ following OPS for EN Wiki2025.

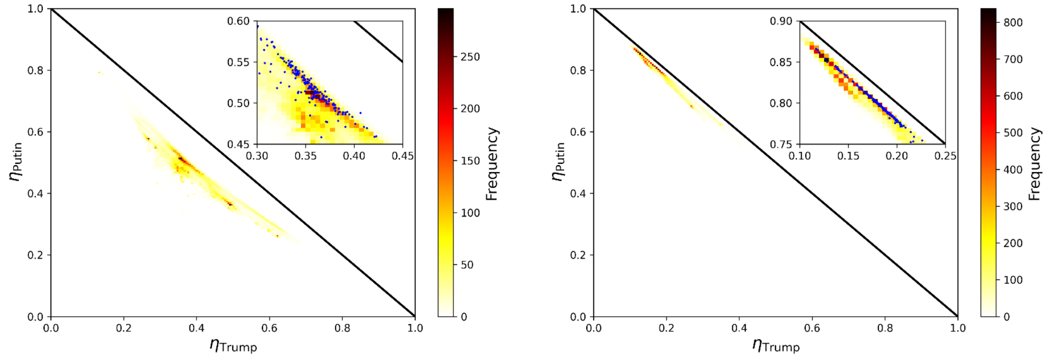
Appendix Figure A1 shows the opinion polarization of world countries for *Pfizer* ($\Delta\mu > 0$) vs. *Johnson*

& Johnson ($\Delta\mu < 0$) using the EN Wiki2025 edition. Globally, the influence of Pfizer seems to be stronger, however Johnson & Johnson are dominating the USA.

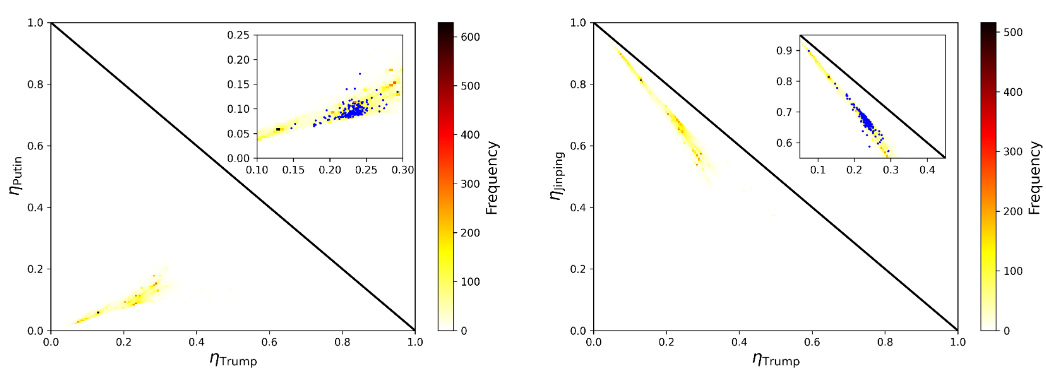
Article	$\Delta\mu$ (Macron - Le Pen) $N_r = 10^5$	$\Delta\mu$ (Macron - Le Pen) $N_r = 10^6$
François Hollande	-0.0095	-0.0095
Jean-Luc Mélenchon	-0.0120	-0.0113
Édouard Philippe	-0.0104	-0.0099
Nicolas Sarkozy	-0.0098	-0.0099
Dominique Strauss-Kahn	-0.0098	-0.0101
Manuel Valls	-0.0103	-0.0103
Dominique de Villepin	-0.0099	-0.0100
Éric Zemmour	-0.0113	-0.0117
Gabriel Attal	-0.0105	-0.0102
François Bayrou	-0.0112	-0.0107
Éric Ciotti	-0.0141	-0.0135
Jordan Bardella	-0.0155	-0.0150
Rachida Dati	-0.0106	-0.0106
Bruno Retailleau	-0.0107	-0.0105
Bernard Arnault	-0.0078	-0.0077
Françoise Bettencourt Meyers	-0.0083	-0.0069
Alain Wertheimer	0.0289	0.0320
Gérard Wertheimer	0.0289	0.0320
François Pinault	-0.0075	-0.0074
Emmanuel Besnier	0.0172	0.0175
Nicolas Puech	-0.0006	-0.0004
Vincent Bolloré	-0.0104	-0.0095
Xavier Niel	-0.0088	-0.0084
Carrie Perrodo	-0.0019	-0.0005

Appendix Table A1. Opinion polarization expressed by $\Delta\mu$ (following OPS for FR Wiki2024), for important personalities from French politics (top) and French richest persons (following Forbes top 10 ranking 2015-2024) (bottom). *Emmanuel Macron* corresponds to $\mu = 1$ and *Marine Le Pen* corresponds to $\mu = -1$ with $\mu_0 = -0.0252 \pm 0.0023$ for $N_r = 10^5$ and $\mu_0 = -0.0279 \pm 0.0008$ for $N_r = 10^6$. The 2nd (3rd) column corresponds to data for $N_r = 10^5$ ($N_r = 10^6$) with a typical statistical error 0.001 (0.0003). The data for $N_r = 10^5$ is statistically independent and not a subset of the data for $N_r = 10^6$.

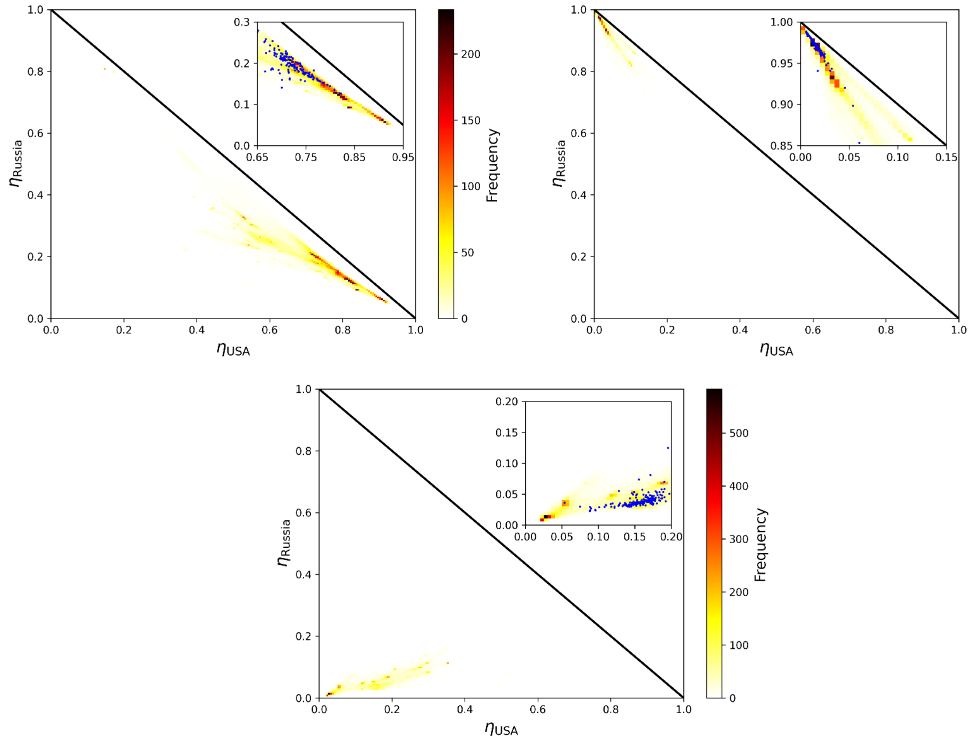
Appendix Table A1, provides the values of $\Delta\mu_i$ for certain French political or rich personalities for the competition *Emmanuel Macron* ($\mu = 1$) vs. *Marine Le Pen* ($\mu = -1$) in FR Wiki2024. For most entries in this table there is a slight preference for Le Pen with typical values $\Delta\mu_i \approx -0.01$. Since these values are close to zero two data columns for $N_r = 10^5$ and $N_r = 10^6$ are shown which clarifies that the statistical fluctuations are typically well below $|\Delta\mu_i|$.



Appendix Figure A2. Histogram of opinion polarization in the $(\eta_{Trump}, \eta_{Putin})$ plane for *Donald Trump*, *Vladimir Putin*, and *Xi Jinping* across all articles following OPS for EN Wiki2025 (left panel), RU Wiki2025 (right panel)). Note that for each article, the sum of polarization values is normalized to 1, and therefore $\eta_{Jinping} = 1 - \eta_{Trump} - \eta_{Putin}$. The corresponding insets show the same histogram zoomed in on the most populated region of the plane, with blue circles representing the country articles.

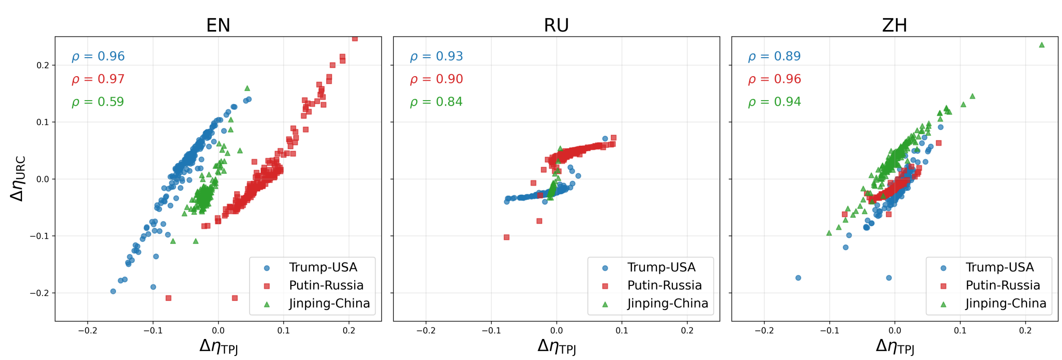


Appendix Figure A3. Histogram of opinion polarization in the $(\eta_{Trump}, \eta_{Putin})$ plane (left panel) and $(\eta_{Trump}, \eta_{Jinping})$ plane (right panel) for *Donald Trump*, *Vladimir Putin*, and *Xi Jinping* across all articles following OPS case for ZH Wiki2025. Note that for each article, the sum of polarization values is normalized to 1, $\eta_{Jinping} + \eta_{Trump} + \eta_{Putin} = 1$. The corresponding insets show the same histogram zoomed in on the most populated region of the plane, with blue circles representing the country articles.



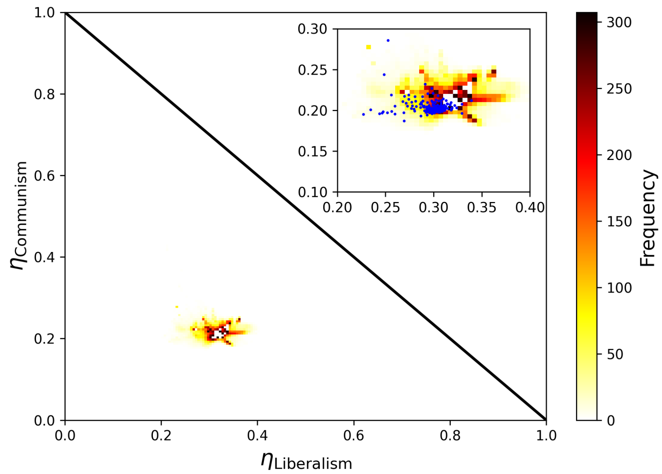
Appendix Figure A4. Histogram of opinion polarization in the $(\eta_{USA}, \eta_{Russia})$ plane for *USA*, *Russia*, and *China* across all articles following OPS for EN Wiki2025 (top left panel), RU Wiki2025 (top right panel), and ZH Wiki2025 (bottom panel). Note that for each article, the sum of polarization values is normalized to 1, and therefore $\eta_{China} = 1 - \eta_{USA} - \eta_{Russia}$. The corresponding insets show the same histogram zoomed in on the most populated region of the plane, with blue circles representing the country articles.

Appendix Figures A2-A4 provide color plots of histogram distributions for certain three group competitions in the plane of two color polarization values $\eta_{C_1} - \eta_{C_2}$ (see main text for a detailed discussion). In certain cases the data is close to the antidiagonal (with $1 \approx \eta_{C_1} + \eta_{C_2}$) indicating that typical values of the third color polarization $\eta_{C_3} = 1 - (\eta_{C_1} + \eta_{C_2})$ are rather small.



Appendix Figure A5. Plane of color polarizations $\Delta\eta_{URC}$ for the contest of USA, Russia, China of Figure 9 vs. those $\Delta\eta_{TPJ}$ for the contest of 3 political leaders of Figure 7 with data shown for all 197 countries. The metrics are derived from the triads (USA, Russia, China) and (Trump, Putin, Jinping), respectively. Panels correspond to the datasets from Wiki 2025 for: (left) English (EN), (center) Russian (RU), and (right) Chinese (ZH). The Spearman correlation coefficient, ρ, is reported for each panel.

Appendix Figure A5 illustrates correlations between the three group competitions of *USA, Russia, China* and *Trump, Putin, Jinping.*



Appendix Figure A6. Histogram of opinion polarization in the $(\eta_{Liberalism}, \eta_{Communism})$ plane for *Liberalism*, *Communism*, and *Nationalism* across all articles following OPS for EN Wiki2025. Note that for each article, the sum of polarization values is normalized to 1, and therefore $\eta_{Nationalism} = 1 - \eta_{Liberalism} - \eta_{Communism}$. The corresponding insets show the same histogram zoomed in on the most populated region of the plane, with blue circles representing the country articles.

Appendix Figure A6 is similar to Appendix Figures A2-A4 for the case of the three group competition *Liberalism, Communism, Nationalism* for EN Wiki2025.

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