

Time reversal of complex evolution on a quantum computer

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The Boltzmann-Loschmidt dispute in 1876-1877 discussed the problem of time reversal of thermalization from time reversible classical equations of motion. Here, 150 years later, we highlight this problem in the frame of quantum computing. A quantum protocol is proposed that allows to perform a time reversal of complex evolution in the regime of many-body quantum chaos with many qubits. The system represents an evolution of qubits on a square lattice with inter-qubit next-nearest static couplings with a driven pulsed magnetic field. The system entropy grows rapidly to maximal values but returns to initial small values after time reversal. This time reversal is shown to be stable with respect to quantum gate imperfections. However, similar to the Lorenz butterfly effect, there is the butterfly effect of qubit when inversion of only one qubit breaks time reversibility of the whole system. It is argued that this protocol is accessible to nowadays quantum computers and annealers with hundreds of qubits.

I. INTRODUCTION

The statistical theory of entropy growth and thermalization had been formulated by Boltzmann in 1872 on the basis of dynamical equations of motion of atomic gas [1]. This theory was objected to by Loschmidt in 1876 [2] who pointed out that the reversible dynamical equations allow to invert velocities of atoms returning them back to the initial state. The reply of Boltzmann was published in 1877 [3] and the legend tells that on the direct question on what happens with entropy and thermalization if all atom velocities are inverted he replied *then go and invert them* [4].

In the frame of classical mechanics the resolution of Boltzmann-Loschmidt dispute is related to emergence of dynamical chaos with its exponential instability of non-linear motion that leads to an exponential growth of errors thus breaking time reversal on logarithmically short times [5–8]. An example of such time reversal breaking is given in [9].

To illustrate such an exponential instability of chaotic motion E. N. Lorenz in 1972 illustrated this by a flap of a butterfly's wings in Brazil setting off a tornado in Texas [10]. This became known as the butterfly effect [11].

In the frame of quantum mechanics the time evolution is described by the linear Schrödinger equation and the Lyapunov exponential instability of motion (with exponent Λ) exists only on a logarithmically short Ehrenfest time scale $t_E \sim |\ln \hbar|/\Lambda$ after which mixing stops at a small phase space scale determined by the Planck constant $\hbar$ (taken here in dimensionless units) [12–16]. Hence for a quantum chaos evolution the time reversal is preserved at a moderate strength of computer errors while the corresponding classical evolution is nonreversible on the same computer even at tiny round-off computer errors [9]. The effects of errors and perturbations on time

reversal of evolution in the regime of quantum chaos, known as the Loschmidt echo, have been studied by different groups with detailed description given in [17–22].

The experimental realization of time reversal was first performed in spin systems as spin echo [23–25]. For acoustic and electromagnetic waves time reversal was realized in [26, 27] leading to important useful applications. Time reversal technique is also broadly used for a seismic analysis in geophysics [28]. For atomic Bose-Einstein condensate a time reversal of a part of wave packet was achieved in experiments with atoms in kicked optical lattice [29, 30] following the theoretical proposal [31]. An experiment with time reversal of almost all wave packet of cold atoms, or ions, in a harmonic trap was recently proposed in [32].

In recent decades, there has been a significant progress in the development of quantum computation and information (see e.g. [33–35]). Quantum computing now is done with several tens of qubits and even with more than hundreds of qubits (see e.g. [36–40]). Even though a significant number of qubits has been reached now in quantum computing, still the realization of several thousands of two-qubit gates remains a challenge due to decoherence effects. For certain systems it is easier to realize quantum simulators of specific physical effects like e.g. the time evolution of Ising Hamiltonians or kicked Ising chains (see e.g. [38–40]).

It is important to note that many interesting mathematical and numerical results for kicked Ising chains have been obtained in the regime of quantum chaos (see [41–44] and Refs. therein). The quantum simulations of such kicked Ising chains in certain respects are more accessible for nowadays quantum computers or simulators (see e.g. [38, 39]).

While in [41–44] the studies are mainly done for kicked Ising chains here we consider a quantum computer with qubits forming a two-dimensional (2D) square lattice. Such quantum computer hardware models with static imperfections in 2D have been studied e.g. in [45–47]. Here

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we show that a specific case of kicked Ising 2D model allows to realize efficiently a complex evolution in a regime of quantum chaos when a rapid entropy growth is inverted in time with full recovering of the initial quantum register state. We argue that such type of quantum computation on a quantum computer allows to highlight the Boltzmann-Loschmidt dispute in a new frame of quantum reality.

We also point out that the proposed Time Reversal Kicked Ising (TIRKI) model can be also realized on quantum annealers that can operate with thousands of qubits solving difficult physical problems (see e.g. [48–50] and Refs. therein). The time reversal in a system with several hundreds of qubits would allow to realize the inversion of time arrow for a macroscopically huge number of Hilbert states.

The paper is organized as follows: the TIRKI model is presented in Section 2, the properties of many-body quantum chaos in this model are described in Section 3, the results for time reversal in the model are described in Section 4, the effects of static imperfections on time reversal are considered in Section 5, the butterfly effect of qubit for time reversal is analyzed in Section 6, and discussion of the results is given in Section 7.

II. DESCRIPTION OF TIRKI MODEL

The TIRKI model of a driven quantum computer is described by the Hamiltonian:

$$H = H_0 + V\delta(t-T) = J \sum_{i,j} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x \times \sum_m \delta(t-mT), \quad (1)$$

where H_0 is unperturbed Hamiltonian and V describes a time-dependent kick perturbation with a periodic δ -function of period T ; σ^x and σ^z are Pauli matrices of qubits, sum over i, j runs over next-nearest neighbor qubits located on a square lattice with hard boundary conditions; J and h describe the qubit coupling strength and the kick amplitude. The number of qubits is $n_q = L_x \times L_y$ with lattice sizes L_x, L_y in x, y -directions. The size of the Hilbert space is $N = 2^{n_q}$ and the quantum register states are $|b_i\rangle = |10 \dots 1\rangle$ with 0 or 1 for spin down or up.

The time evolution propagator of wavefunction ψ of (1) on one period T has the form:

$$\bar{\psi} = U\psi = U_H(\varepsilon)U_K(h)\psi = \exp(-i\varepsilon \sum_{i,j} \sigma_i^z \sigma_j^z) \exp(-ih \sum_i \sigma_i^x) \psi, \quad (2)$$

where $\bar{\psi}$ is the wavefunction after one perturbation period and $\varepsilon = JT$. The unitary propagator U_K can be considered as a pulsed magnetic field acting simultaneously on all qubits while the operator U_H simply describes a free evolution of coupled qubits. The quasienergy eigenfunctions ψ_λ of the evolution operator U are determined by the equation $U\psi_\lambda = \lambda\psi_\lambda$ with $|\lambda| = 1$.

The time reversal protocol is the following: the evolution is done during t_r periods (kicks) with the propagator (2); after that the time reversal evolution operator takes the form

$$U_r = U_K(-h)U_H(T') = U_K(h)^*U_H(\varepsilon)^* = U^\dagger, \quad (3)$$

where $U_K(-h)$ is the kick operator with h replaced by $-h$ (magnetic field changes sign); and the period between kicks being $T' = (2\pi - \varepsilon)/J$ so that we have $\varepsilon \rightarrow 2\pi - \varepsilon$. However, at $\varepsilon = 2\pi$ the operator $U_K(2\pi) = I$ where I is unit operator. As a result we obtain complex conjugation operator and after a back preparation of t_r periods of $U_r = U^\dagger$ the whole system returns to the initial state at the time moment $t = 2t_r$. The same time reversal protocol works also for any unperturbed Hamiltonian H_0 being an integer function of σ_i^z, σ_j^z multiplied by J (so that eigenvalues of H are integers multiplied by J). For example this can be the unperturbed Hamiltonian $H_0 = J(\sum_{i,j} \sigma_i^z \sigma_j^z + A \sum_i \sigma_i^z)$ where A is any integer.

Thus the time reversal of quantum evolution (2) is rather simple for realization: one simply needs to replace $\varepsilon \rightarrow 2\pi - \varepsilon$ by increasing the time interval between kicks and by changing sign of pulsed magnetic field $h \rightarrow -h$.

To study how robust is the phenomenon of time reversal we consider how it is influenced by static imperfections following the approach used in [19]. With imperfections the Hamiltonian takes the form:

$$H = H_0 + V\delta(t-T) = \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z + \sum_i h_i \sigma_i^x \times \sum_m \delta(t-mT), \quad (4)$$

with $J_{ij} = J + \eta_{ij}$ and $h_i = h + \mu_i$ where random static perturbations are distributed in narrow intervals $-\eta/2 \leq \eta_{ij} \leq \eta/2$ (they act only for nearest-neighbor qubits as in (1)) and $-\delta_x/2 \leq \mu_i \leq \delta_x/2$. Thus with imperfections we have the evolution operator for $t \leq t_r$:

$$U = U_H(\varepsilon)U_K(h)\psi = \exp(-i \sum_{i,j} (\varepsilon + \varepsilon_{ij}) \sigma_i^z \sigma_j^z) \exp(-i \sum_i (h + \mu_i) \sigma_i^x) \psi, \quad (5)$$

where amplitudes of weak random imperfections are distributed in narrow intervals $-\delta_z/2 \leq \varepsilon_{ij} \leq \delta_z/2$ and $-\delta_x/2 \leq \mu_i \leq \delta_x/2$.

For the time reversal part $t_r < t \leq 2t_r$ we have $T \rightarrow (2\pi - \varepsilon)/J$ and $h \rightarrow -h$ so that the backward evolution propagator with imperfections is:

$$U_r = \exp(-i \sum_i (-h + \mu_i) \sigma_i^x) \exp(-i \sum_{i,j} (-\varepsilon + (2\pi - \varepsilon) \varepsilon_{ij}) \sigma_i^z \sigma_j^z) \psi. \quad (6)$$

Thus for $t_r < t \leq 2t_r$ the strength of imperfections is increased for $\varepsilon < \pi$ with $\delta_z \rightarrow (2\pi - \varepsilon)\delta_z$. We assume that μ_i do not change sign.

We characterize the precision of time reversal by fidelity defined in a usual way as $F(t) = |\langle \psi(t=0) | \psi(t) \rangle|^2$. In absence of imperfections we have exact time reversal and $F(t = 2t_r) = 1$. The fidelity $F(2t_r)$ decreases with increase of imperfections.

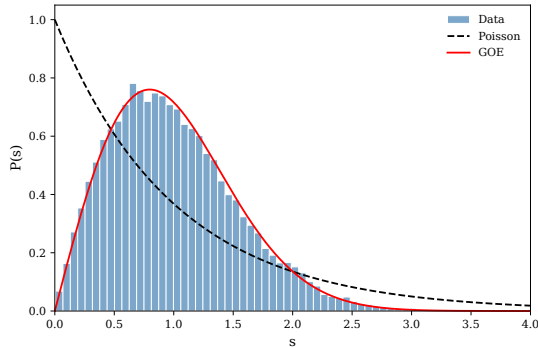


FIG. 1. Level spacing statistics of quasienergies λ of unitary operator U at parameters $\varepsilon = 2$, $h = 1$, $n_q = 4 \times 4 = 16$; it is obtained by diagonalizing all ten symmetry sectors separately, there are approximately 4.9×10^4 spacings. The Poisson (dashed curve) and Wigner surmise of RMT (red solid curve) distributions are also shown.

Also we study the level spacing statistics of quasienergies λ . Following [45–47] we also characterize the eigenstates ψ_λ by their von Neumann entropy defined as $S_q = -\sum_i W_i \log_2 W_i$ where W_i is the quantum probability to find the noninteracting multiqubit quantum register state $|\varphi_i\rangle$ of Hamiltonian H_0 in the eigenstate $|\psi_\lambda\rangle$ of the evolution operator U at $h > 0$. The same definition of S_q can be used for a evolution of wavefunction $\phi(t)$ at some time moment t (instead of $|\psi_\lambda\rangle$).

We also note that without imperfections the kick operator at $h = \pi/2$ is just a global spin-flip operator $U_K(\pi/2) = \exp(-i\pi/2 \sum_i \sigma_i^x) = -i\Pi_i \sigma_i^x$ and since the term $\sigma_i^z \sigma_j^z$ in the U_H is invariant under a global spin flip, $U_K(\pi/2)$ commutes with H . As a result the characteristics of time evolution have periodic dependence on h with $\pi/2$ period (see below). For a square lattice and without imperfections, the lattice has the symmetry of the dihedral group D_4 with five irreducible symmetry sectors A_1 , A_2 , B_1 , B_2 , and E . In addition, the time evolution operator U is invariant under the global spin flip operator $[U, \Pi_i \sigma_i^x] = 0$. So in total, we have ten symmetry sectors $D_4 \times \mathbb{Z}_2$. Finally, U is diagonalized exactly in each sector separately to calculate the quasienergy spacings in Figure 1.

We note that a quantum time reversal protocol for quantum computing of the classical chaotic Arnold cat map had been discussed in [51, 52]. However, this protocol requires a significant number of two-qubit and one-qubit gates and is still not accessible to nowadays quantum computers (about 10^3 gates for one map iteration at 27 qubits). In contrast, the protocol proposed here is easy for implementation both with quantum computers and quantum annealers: for one evolution period one keeps fixed static inter-qubit couplings and makes a short pulse of global magnetic field performing fixed phase rotation for all qubits.

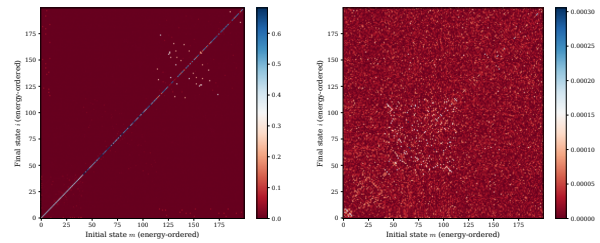


FIG. 2. Matrix of excited state W_{im} after $t_r = 30$ perturbation periods at $n_q = 4 \times 4$, $\varepsilon = 2$, and $h = 0.1$ (left); $h = 1$ (right); only 200 initial register states with minimal energy $E_0 = H_0$ are shown; m/i is index of initial/excited register state.

III. MANY-BODY QUANTUM CHAOS IN TIRKI MODEL

Here we present results showing that in the absence of imperfections at typical parameter values $\varepsilon \sim 1$, $h \sim 1$ the quantum register states of Hamiltonian H_0 (1) are mixed by the kick potential V and the system is the regime of quantum chaos (see e.g. [15, 45–47]). Indeed, for the typical parameters ε, h we show that the level spacing statistics $P(s)$ of nearest quasienergies λ is described by the Random Matrix Theory (RMT) [15] (see Fig. 1, here s is rescaled level spacing). Indeed, the data shows that $P(s)$ nicely follows the Wigner surmise $P_W(s) = (\pi s/2) \exp(-\pi s^2/4)$. This justifies that the TIRKI model at such parameters is in the regime of developed quantum chaos.

In Fig. 2 we show that after a time evolution during $t = 30$ the initial register states (200 of them at lowest energy $E_0 = H_0$ are shown) the kick perturbation V does not excite significant number of other register states at $h = 0.1$ while for $h = 1$ many of register states are excited. Thus at $h = 0.1$ the system (1) is close to an integrable case while at $h = 1$ the system is in the regime of developed quantum chaos in agreement with the RMT level spacing statistics shown in Fig. 1.

Another confirmation of the emergence of quantum chaos is given in Fig. 3 where we show the dependence of entropy S_q after $t_r = 30$ kicks as a function of kick strength $0 < h < \pi/2$ and initial register energy $E_0 = H_0$ for $\varepsilon = 2$ and $n_q = 5 \times 5 = 25$. Approximately, for weak values of $h < 0.1$ the entropy S_q remains relatively small $S_q < 2$ while for $h > 0.5$ it reaches its maximal value $S_q \approx 25$. Thus the system is in the regime of strong chaos for $h > 0.5$. Also due to periodicity of kick strength with $h = \pi/2$ the values of S_q are periodic and symmetric in respect to $h \rightarrow \pi/2 - h$.

The results of Figs. 1, 2, 3 show that at typical parameter values $\varepsilon \sim h \sim 1$ the system (1) is in the regime of developed quantum many-body chaos characterized by complex evolution in time.

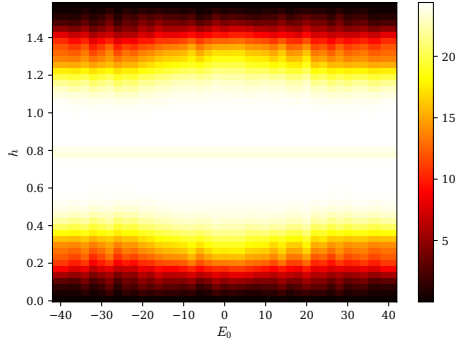


FIG. 3. Quantum register melting: color bar shows entropy values S_q after $t_r = 30$ evolution periods in dependence on h and initial energy $E_0 = H_0$ for $n_q = 5 \times 5 = 25$ qubits, $\varepsilon = 2$, $t_r = 30$, $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$. Melting sets in above a threshold in h and is strongest in the high-density central band ($E_0 \approx 0$), invading the low-density band edges later; the horizontal dark stripes are the fidelity register preserving areas.

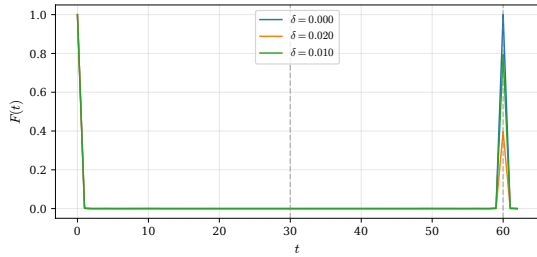


FIG. 4. Fidelity $F(t)$ dependence on time t for model parameters $\varepsilon = 2$, $h = 1$, $n_q = 5 \times 5$, $t_r = 30$ (vertical dashed line), initial state is $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$; amplitude of static imperfections is $\delta_z = \delta_x = \delta$ with $\delta = 0.02$ (orange curve); 0.01 (green curve); 0 (blue curve, no imperfections).

IV. TIME REVERSAL FOR TIRKI MODEL

Here we discuss the properties of time reversal protocol described in Section II (mainly in absence of imperfections). The effects of imperfections are discussed in Section V.

In Fig. 4 we show the time dependence of fidelity $F(t)$ on time with the time reversal done at time moment $t = t_r = 30$ at system parameters $\varepsilon = 2$, $h = 1$ and $n_q = 5 \times 5 = 25$ qubits. Already after one period the fidelity drops to a very small value but at the return moment $t = 2t_r = 60$ it is perfectly recovered at $F(2t_r) = 1$. The effect of static imperfections is shown by the dashed curves at $\delta_z = \delta_x = \delta$. These imperfections reduce the fidelity revival at $t = 2t_r$ but still the fidelity remains relatively high at $\delta = 0.01$ (imperfections effects are discussed in detail in Section V).

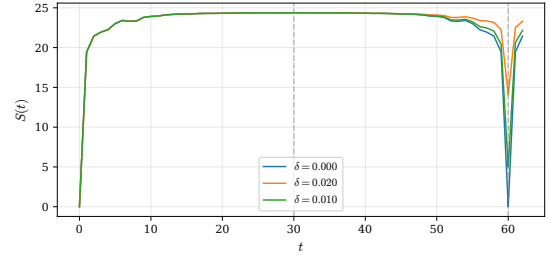


FIG. 5. Entropy S_q dependence on time t for the same system parameters as in Fig. 4.

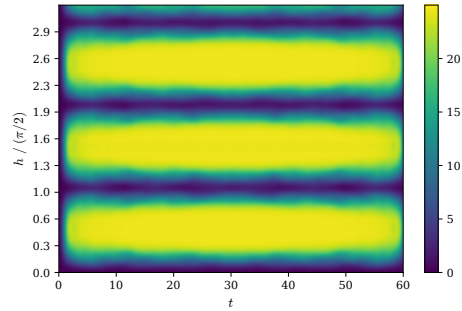


FIG. 6. Entropy S_q dependence on h and time t shown by color (color bar with S_q values); here $\varepsilon = 2$, $n_q = 5 \times 5 = 25$, $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$, $t_r = 30$ and no imperfections. The kick strength expressed as $h/(\pi/2)$ (vertical). Bright (yellow) bands mark fast saturation to a large entropy; the dark horizontal stripes at $h/(\pi/2) = 0, 1, 2, 3$, are the register-preserving resonances where the kick is trivial up to a phase.

The time reversal effect for entropy $S_q(t)$ is shown in Fig. 5 for the same system parameters. After first 5 periods of time the entropy grows from its initial value $S_q = 0$ almost up to its maximal value $S_q = 25$ remaining close to this value on longer times. At the return time $t = 2t_r = 60$ the entropy returns to its initial value $S_q = 0$. The backward return of entropy $S_q(2t_r)$ is only weakly affected by static imperfections (see more detail in Section V).

The dependence of $S_q(t)$ on time and parameter h is shown in Fig. 6 at $\varepsilon = 2$ for the time interval $0 \leq t \leq 2t_r = 60$. The results clearly show the time reversal of entropy $S_q(t)$. As in Fig. 3 the entropy remains small $S_q < 2$ for $h < 0.1$ being in a quasi-integrable phase while the phase of developed quantum chaos appears at $h > 0.1$ (as discussed above there is periodicity and symmetry for h variation).

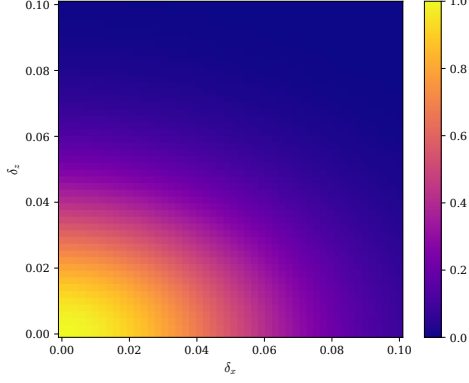


FIG. 7. Fidelity $F(2t_r)$ shown by color bar vs. imperfection amplitudes δ_x, δ_z at $n_q = 4 \times 4 = 16$, $\varepsilon = 2$, $h = 1$, $t_r = 30$, initial state is $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$; δ_x is the spread of the kick angle per site, and δ_z is the spread of the Ising coupling per bond, data is for a single fixed realization. The reversal is exact only at $\delta_x = \delta_z = 0$.

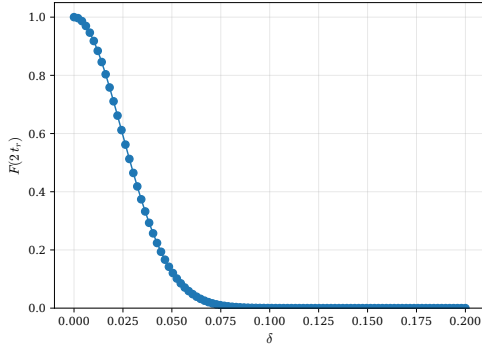


FIG. 8. Fidelity $F(2t_r)$ vs $\delta_z = \delta_x = \delta$; here $\varepsilon = 2$, $h = 1$, $n_q = 16$, $t_r = 30$, $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$. This figure is a cut through the diagonal of Fig. 7.

V. EFFECTS OF IMPERFECTIONS FOR TIME REVERSAL

The effects of static imperfections for fidelity $F(2t_r)$ are shown in Fig. 7 with its dependence on imperfection amplitudes δ_x, δ_z . The results show that the fidelity of time reversal remains robust against these imperfections.

The dependence of fidelity at the return time $F(2t_r)$ on the amplitude of imperfections $\delta_x = \delta_z = \delta$ is shown in Fig. 8. It is approximately described by the expression $F(2t_r) \sim \exp(-G\delta^2 t_r)$ where G depends on system parameters h, ε, n_q being in agreement with the general properties of Loschmidt echo decay. The Loschmidt echo decay is studied and described in detail in [18–22, 45] and due to that we do not discuss this behavior in detail here.

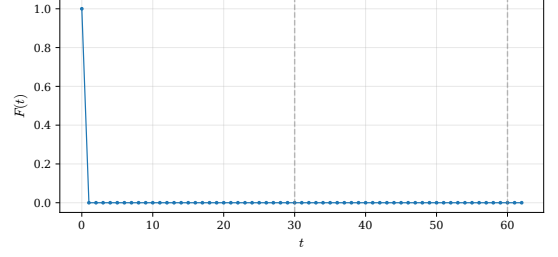


FIG. 9. Fidelity $F(t)$ dependence on time t for the butterfly effect of qubit: one central qubit is inverted at $t_r = 30$. Here $n_q = 5 \times 5 = 25$, $\varepsilon = 2$, $h = 1$, $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$ and there are no imperfections.

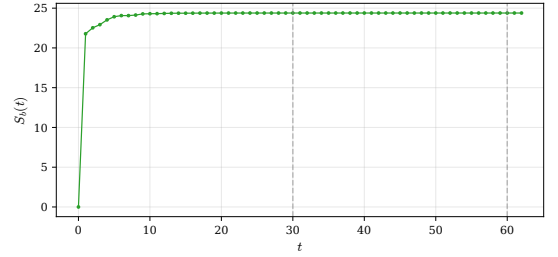


FIG. 10. Same as in Fig. 9 but for system entropy $S_q(t)$ vs t .

VI. BUTTERFLY EFFECT OF QUBIT

The above results show that quantum computing of complex evolution (2) remains stable with respect to quantum gate imperfections. This stability is also preserved in respect to noise phase gate errors as it is shown e.g. in [19]. However, it is possible to ask if the time reversal operation would remain stable if at a time reversal moment t_r there is inversion of only one qubit (apart of this the time reversal protocol remains unchanged). This time reversal operation can be viewed as the butterfly effect of qubit. The butterfly effect in the classical world with classical equations of motion and chaotic dynamics was discussed by E.N.Lorenz [10] (see also the story written by R.Bradbury [53]).

The butterfly effect of qubit is shown in Fig. 9 and Fig. 10 for fidelity $F(t)$ and entropy $S_q(t)$. Here the time reversal protocol for the system with 25 qubits, without imperfections, is applied at the time moment $t_r = 30$ with inversion of central qubit (located at the lattice position $x = y = 3$). This one qubit inversion breaks completely the time reversal: there are no signs of return at $t = 2t_r = 60$ for F and S_q . Also the time reversal is broken for the system total spin squared $S^2(t)$ as it is shown in Fig. 12 while in absence of qubit inversion $S^2(t)$ returned exactly to its initial value.

To illustrate the butterfly effect of qubit in a better

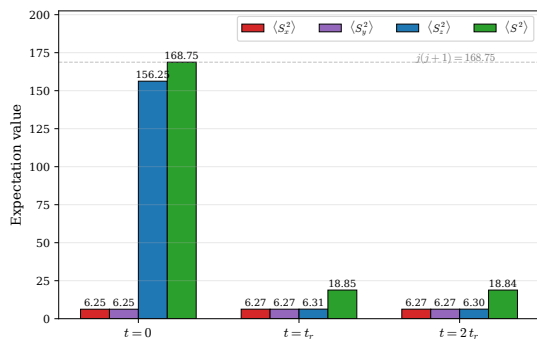


FIG. 11. Spin square $S^2(t)$ of the whole system shown at time moments $t = 0; t_r = 30; 2t_r = 60$ for the butterfly effect of qubit at the same system parameters as in Fig. 9.

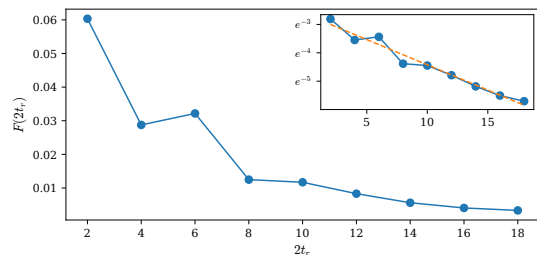


FIG. 12. Fidelity $F(t = 2t_r)$ vs time return moment $t = 2t_r$ for the butterfly effect of qubit when only one central qubit is inverted at time t_r , $n_q = 5 \times 5 = 25$, $\varepsilon = 2$, $h = 1$, $|\psi_0\rangle = |\uparrow\uparrow\uparrow \dots \uparrow\rangle$ and there are no imperfections. Insert shows $\ln F(t = 2t_r)$ vs $2t_r$, and a fitted dashed line gives $\ln F = -0.178 \times 2t_r - 2.64$.

way we show in Fig. 12 the dependence of fidelity F on the return time $2t_r$ when only one qubit is inverted at a time reversal moment t_r . The results show that $F(2t_r)$ drops significantly already after one evolution period with $t_r = 1$. For $2t_r > 2$ the decay of fidelity continues approximately exponentially ($F \sim \exp(-0.178 \times 2t_r)$) for parameters of Fig. 12 insert). We note certain similarity of this decay with the results reported in [51] (see Fig.3 there).

Thus we conclude that due to many-body quantum chaos and related induced rapid entropy growth (see e.g. Fig. 5) there is a strong butterfly effect of qubit for the complex quantum evolution of our system (1) that com-

pletely breaks time reversibility. We consider this butterfly effect of qubit as an analog of the classical butterfly effect discussed in [10, 11].

VII. DISCUSSION

In this work we propose a simple quantum protocol that allows one to perform time reversal of complex time evolution on a quantum computer in a regime of quantum chaos. In this protocol we consider a quantum computer qubits on a square lattice with next-nearest neighbor static interactions of qubits. The time evolution is considered for a short periodically pulsed global magnetic field acting simultaneously on all qubits. This corresponds to one-qubit phase rotation for all qubits. The periodic time evolution is shown to be in a regime of quantum chaos with the level spacing statistics of quasienergies described by RMT. The time reversal is done by a change of period of pulses and by inversion of magnetic field at a certain moment of time. As a result the system exactly returns to its initial quantum register state. The time reversal is shown to be robust against quantum imperfections. The present abilities of quantum computers [36–40] with up to 144 qubits or quantum annealers [48–50] allow to perform this time reversal protocol with about 100 or even larger number of qubits with rather simple quantum operations. For a quantum simulator with 100 qubits a macroscopic number of quantum trajectories in the Hilbert space of size $N_H \sim 10^{30}$ will be reversed in time returning back to their initial state. This number is significantly larger than the Avogadro number $N_A \approx 6 \times 10^{23}$.

Even if the time reversal is robust with respect to gate imperfections we show that the many-body quantum chaos leads to the butterfly effect of qubit when inversion of only one qubit breaks time reversal of the whole system.

We argue that the experiments on quantum computers or quantum annealers will highlight the Boltzmann-Loschmidt dispute within the quantum reality 150 years later.

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