Enhancement of edge channel transport by a low frequency irradiation

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The magnetotransport properties of high mobility two dimensional electron gas have recently attracted a significant interest due to the discovery of microwave induced zero resistance states. Here we show experimentally that microwave irradiation with a photon energy much smaller than the spacing between Landau levels can induce a strong decrease in the four terminal resistance. This effect is not predicted by the bulk transport models introduced to explain zero resistance states, but can be naturally explained by an edge transport model. This highlights the importance of edge channels for zero resistance state physics that was proposed recently.

PACS numbers: 89.20.Hh, 89.75.Hc, 05.40.Fb

High frequency transport in high purity two dimensional electron gases (2DEG) reveal many intriguing and unexpected phenomena of which microwave induced zero resistance (ZRS) states are probably the most striking manifestation. As experiments in Refs. [1, 2] show, microwave irradiation can lead to a complete disappearance of longitudinal resistance R_{xx} for particular values of the ratio $j = \omega/\omega_c$ between the driving frequency ω and the cyclotron frequency ω_c. Until 2010 this dissipationless effect was only observed in GaAs heterostructures of ultra high purity [1, 2] or high densities [3] . However the recent observation of ZRS for electrons on the liquid helium surface indicates that it is actually a generic effect that may appear in very different physical systems [4]. Despite the important theoretical efforts that were made to understand this effect, the physical origin of ZRS is still controversial. Most cited models [5–7] argue that microwave irradiation creates a negative resistance state which is unstable and gives rise to a zero resistance state through the formation of current domains. However many experimental features can not be explained with the above picture. In the ZRS regime resistance decreases exponentially with microwave power [8] and inverse temperature [2], instead of a direct switching to a non-dissipative state. Also it was shown that zero resistance states are not affected by the sense of circular polarization which questions mechanisms relying explicitly on transitions between Landau-levels [9]. Moreover ZRS disappear in Hall bars with a small channel size of a few microns, which indicates the importance of edge effects [10]. These experimental properties highlight the difficulties encountered by conventional theoretical descriptions of ZRS, other arguments against these theories are described in [11, 12].

In this Letter we investigate the adiabatic limit $\omega \ll \omega_c$ where transitions between Landau-levels are excluded. We show experimentally that even in this case microwave irradiation can lead to a strong suppression of R_{xx} in a wide range of magnetic fields. We then propose a semiclassical model that explains the observed effect through the enhancement of the drift velocity of trajectories skipping along sample edge and argue that this effect can not be explained properly in a bulk transport model. Our results support the recent theory [11] which proposed that ZRS appears due to microwave stabilization of electron transport along sample edges.

We have investigated magneto-transport under microwave irradiation in a $GaAs/Ga_{1-x}Al_xAs$ 2DEG with density $n_e \simeq 3.3 \times 10^{11}\mathrm{cm}^{-2}$, mobility $\mu \simeq 10^7$ $\mathrm{cm}^2/\mathrm{Vs}$ corresponding to transport time $\tau_{tr} \simeq 1.1$ ns. The Hall bar with a 100 μm wide channel was patterned using wet etching (see Fig. 1 inset). A micro-bonding wire was positioned on the Hall bar chip, parallel to the current channel at a distance of 100 μm from the nearest edge. One of the extremities of the wire was connected to a coaxial cable, which allowed to send microwave irradiation in a broad frequency range from 1 GHz to 40 GHz. The sample was cooled in a He^3 insert to a temperature of around 500 mK. We compared the effect of microwaves on the magnetoresistance at two different frequencies $f = 38.65$ GHz and $f = 2.3$ GHz. As shown, on Fig. 1, the high frequency irradiation leads to microwave induced resistance oscillations (MIRO) similar to those reported in [13]. The magnetoresistance under irradiation is characterized by a series of peaks and dips as a function of magnetic field whose position are determined by the ratio $j = \omega/\omega_c$ of the microwave frequency ω to the cyclotron frequency ω_c. At higher microwave power these oscillations are expected to develop into ZRS, however in our experiments this regime was out of reach due to the limited cooling power of the He^3 insert. The magnetoresistance under high frequency microwave irradiation $f = 38.64$ GHz contrasts sharply with the behavior under irradiation at $f = 2.3$ GHz. In this case the magnetoresistance does not exhibit oscillations anymore but presents a significant drop under irradiation in a large range of magnetic fields ($H \geq 0.05$ Tesla). This drop can not be explained by an increase in electron temperature since resistance increases with temperature in the explored range of magnetic fields.

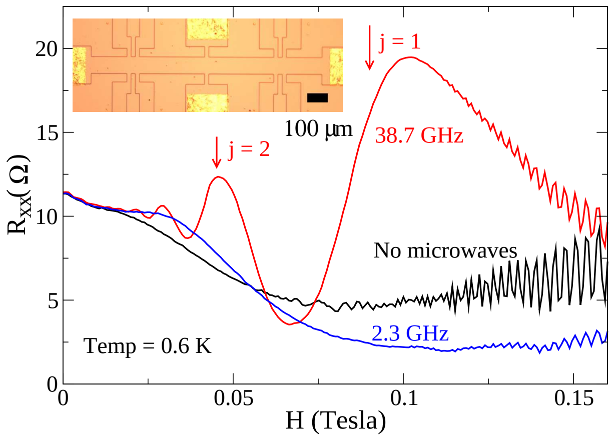

FIG. 1: Magnetoresistance of a high mobility Hall bar (optical photograph of the sample is shown in the inset) in the absence of microwaves and under irradiation at $f = 38.7$ GHz and $f = 2.3$ GHz. The high frequency irradiation induces oscillations in the magnetoresistance (MIRO), whereas the low frequency driving leads to an homogeneous drop in R_{xx} for $H > 0.1$ Tesla.

Let us first try to of explain the effect of the low frequency irradiation through a bulk mechanism. For a frequency $f = 2.3$ GHz and a typical magnetic field of $H \simeq 0.1$ Tesla, we find $j \simeq 0.05$. In this adiabatic limit, the microwave field can not give rise to transitions between Landau levels, thus neither elastic nor inelastic ZRS theories can justify a strong drop in resistance of around 50%. The only expected effect is that of a weak heating leading to a thermal broadening of the Landau levels. Also for $\omega\tau_{tr} \gg 1$ the electric field penetrates the sample in the form of plasmon excitations. At frequencies $\omega < \omega_c$ bulk-magnetoplasmons excitation are evanescent thus we expect the excitation field to be screened in the bulk of the sample [15]. On the contrary edge magnetoplasmon excitations are gapless and appear even at frequencies $\omega \ll \omega_c$ which may lead to an enhancement of the microwave field near the edges of the sample. As a consequence the effect of irradiation should be confined to the sample edges. This and recent results from [11] lead us to develop a model explaining the observed drop of resistance through the dynamics of orbits skipping along the sample edge under adiabatic microwave fields.

We first use the Landauer formula to make a connection between the four terminal resistance R_{xx} and the drift velocities of the skipping orbits along sample edges. This formula relates R_{xx} to the transmission T_n of the channels propagating along sample edges: $R_{xx} = \frac{h}{2e^2N}\sum(1-T_n)/\sum T_n$, where N is the number of occupied Landau levels; $N \simeq 70$ at $H = 0.1$ Tesla [14]. For this magnetic field, the typical transmission $T = 1 - NR_{xx}(2e^2/h) \simeq 0.985$ is very close to unity ($R_{xx} \simeq 2.5\ \Omega$). Since N is high in our experiments, we can make a semi-classical approximation for the transmissions: $T_n \simeq 1 - \frac{L}{v_g(n)\tau_n}$ where L is the distance between voltage probes, $v_g(n)$ is the group velocity of the channel which is given by the drift velocity in the semi-classical limit. Here τ_n is the typical time after which an electron from channel n is scattered into the sample bulk (it is however longer than τ_{tr} because the probability of scattering back to the edge is high after a collision on an impurity [16]); this yields

$$R_{xx} = \frac{h}{2e^2N^2}\sum_n \frac{L}{v_g(n)\tau_n} \tag{1}$$

This expression shows that orbits with low drift velocity give the main contribution to R_{xx}. We thus start by investigating the effects of microwave irradiation on a typical channel propagating along the edge with a drift velocity $v_g \ll v_F$ where v_F is the Fermi velocity (a typical trajectory is shown on Fig. 2). The relation Eq. (1) allows us to compute the resistance R_{xx} from the knowledge of the drift velocities under irradiation. This avoids the direct computation of the transmissions from a classical billiard model [11] which is numerically more expensive.

The polarization of the field is chosen along the y axis, perpendicular to the edge of the sample. This choice is related to the experimental geometry, another motivation is that the ratio between the perpendicular and longitudinal components of the electric field at the edge is given by the Hall parameter $\alpha = \sigma_{xy}/\sigma_{xx} \simeq 270$ at $H \simeq 0.1$ Tesla.

Two classical trajectories with and without microwave irradiation are compared on Fig. 2, they start with the same initial conditions but progressively diverge due to the effect of microwaves. The trajectory with irradiation propagates on average faster, which on the basis of our previous arguments, will lead to a decrease of R_{xx}. We will now show that this enhancement of drift velocity under irradiation is actually a general feature of edge transport and derive a simple analytical estimation for the increase in drift velocity. Our theoretical analysis is based on the conservation of the action S under adiabatic driving, which reflects the absence of transitions between Landau levels in the limit $\omega \ll \omega_c$. In absence of irradiation the drift velocity v_g is a function of the action S and of the position Y_c of the guiding center with respect to the wall $v_g = v_g(S, Y_c)$. The dependence on Y_c is illustrated in the bottom panel of Fig. 2; the expression for S and calculation details are given in [17]. The application of a microwave irradiation induces a modulation of the position of the guiding center $Y_c \to Y_c + \delta Y \cos\omega t$, where $\delta Y = \frac{eE_\omega}{m\omega_c^2}$; E_ω is the amplitude of the microwave field, $\omega_c = eH/m$ is the cyclotron frequency and m is the electron mass. Thus the time-averaged drift velocity under irradiation becomes $< v_g > = < v(S, Y_c + \delta Y\cos\omega t) >$. The results of this averaging procedure are displayed Fig. 3 and show the dependence of $< v_g > /v_F$ (where v_F is the Fermi ve-

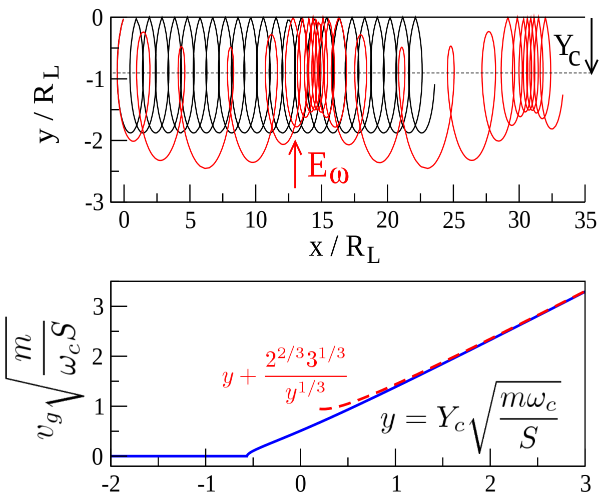

FIG. 2: Top panel: Comparison between two classical trajectories propagating along the edge with the same initial conditions but with (red) and without (black) microwaves. The propagation is faster in presence of driving (simulation parameters are $\omega/\omega_c = 0.1$ and $\epsilon_\omega = eE_\omega/(m\omega_c v_F) = 0.6$). Bottom panel: dependence of the group velocity v_g on the distance of the orbit guiding center to the wall Y_c at fixed action S. The rescaled variables allow to obtain a functional dependence valid for all action S (continuous curve), the asymptote for high Y_c is shown in dashed lines [17]. This high Y_c limit corresponds to trajectories almost tangent to the wall.

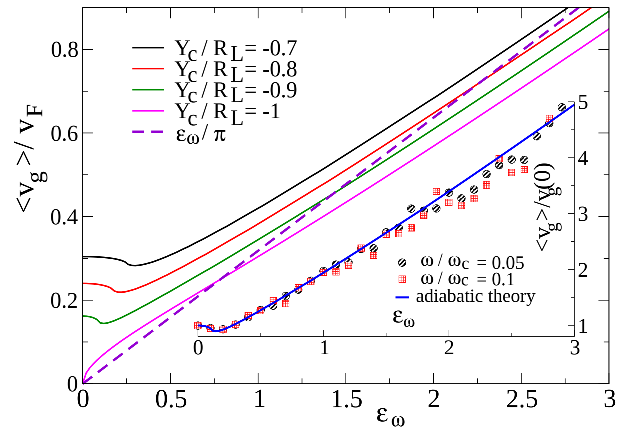

FIG. 3: Time averaged drift velocity as a function of the reduced driving field $\epsilon_\omega = eE_\omega/(m\omega_c v_F)$ computed using the adiabatic theory for several values of the ratio between Y_c and the Larmor radius $R_L = v_F/\omega_c$. The behavior at large fields is well described by the relation $< v_g >\simeq \epsilon_\omega v_F/\pi$ represented by the dashed line. The inset shows the good agreement between the adiabatic theory (continuous line) and direct numerical simulations of the classical dynamics for $Y_c = -0.9R_L$ (symbols).

locity) on dimensionless field $\epsilon_\omega = eE_\omega/(m\omega_c v_F)$ which is also the ratio between δY and the Larmor radius $R_L = v_F/\omega_c$. It confirms the increase of the drift velocity for a large range of driving field amplitudes. A comparison with the drift velocities extracted from direct numerical integration of the dynamics along the sample edge shows that the adiabatic theory gives a good quantitative prediction (see Fig. 3 inset).

The following simple argument gives a good approximation for the average drift velocity under irradiation. The quasistatic transverse electric field $E_\omega \cos\omega t$ induces a drift along the wall with velocity $(eE_\omega/m\omega_c)\cos\omega t$. The equilibrium drift velocity $v_g(0)$ is enhanced when $(eE_\omega/m\omega_c)\cos\omega t > 0$. However when $(eE_\omega/m\omega_c)\cos\omega t + v_g(0) < 0$, the electron does not move efficiently in the direction opposite to its equilibrium propagation direction and the drift freezes. This behavior can be seen directly on the trajectory on Fig. 2. By keeping the positive contribution only, we find $< v_g >\simeq v_g(0) + \frac{eE_\omega}{m\omega_c\pi}$ (see dashed line Fig. 3). This expression can be also be obtained within the adiabatic formalism by retaining only the contribution from the asymptotes $v_g(S, Y_c) = 0$ when $Y_c \to -\infty$ and $v_g(S, Y_c) \simeq Y_c\omega_c$ for high Y_c in the time averaging. The above compact expression is compared with exact adiabatic theory on Fig. 3 and provides a satisfactory agreement. Moreover the results of adiabatic theory are well described by straight lines even if the numerical coefficient derived from our heuristic argument is only approximate. This allows to derive a scaling for the magnetoresistance under irradiation which can be compared with our experimental data. For simplicity we keep the contribution of only a single typical channel propagating with drift velocity $v_g(0) \ll v_F$ in Eq. (1), which leads to:

$$\frac{R_{xx}(0)}{R_{xx}} - 1 = \frac{< v_g >}{v_g(0)} - 1 \propto \frac{E_\omega}{\omega_c} \propto \frac{\sqrt{\mathcal{P}_\omega}}{H} \qquad (2)$$

where $\mathcal{P}_\omega$ is the injected microwave power. Note that a scaling with the square root of microwave power was derived for ZRS in [11] and observed experimentally for low temperature MIRO in [18].

The equation Eq. 2 predicts that the quantity $\rho = \mathcal{P}_\omega^{-1/2}(R_{xx}(0)/R_{xx} - 1)$ should vary linearly with inverse magnetic field and be independent of microwave power. The magnetoresistances at different microwave powers indeed collapses on a single curve according to this scaling. This is represented on Fig. 4 for $f = 10.3$ GHz, note that a similar collapse was observed at other frequencies including $1.66, 2.3, 3.9$ and 5.5 GHz. Thus this model is successful at describing the observed decrease of magnetoresistance under irradiation in the regime of adiabatic driving $\omega \ll \omega_c$ at sufficiently strong magnetic fields where the $1/H$ decay is observed (see Fig. 4 inset). At lower magnetic fields the scaling breaks down as the guiding along sample edges is destroyed by disorder. Our explanation relied on the enhancement of the drift velocity of skipping orbits along sample edge under adiabatic irradiation ($\omega \ll \omega_c$) and is not suited to describe this regime. In the following, we will emphasize several

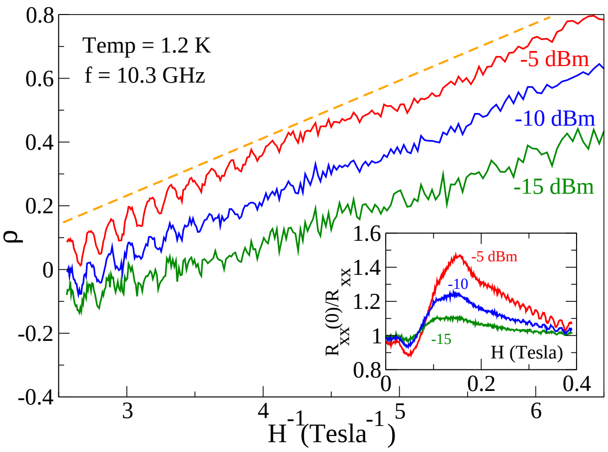

FIG. 4: The solid lines show the dependence of $\rho = (P_\omega/mW)^{-1/2}(R_{xx}(0)/R_{xx} - 1)$ on the inverse magnetic field for three power values ($-15, -10$ and -5 dBm) at frequency 10.3 GHz. The curves are shifted for clarity and collapse on a single straight line (dashed curve) with slope independent on microwave power as predicted by Eq. (2), power was varied by an order of magnitude. Inset shows the dependence of $R_{xx}(0)/R_{xx}$ on magnetic field for the same values of power.

experimental observations that appear to us relevant for constructing a theory valid at all magnetic fields.

The disorder potential in a high mobility 2DEG induces mainly small angle scattering. As a consequence it is characterized by two time scales the elastic lifetime $\tau_e \simeq 20$ ps which is the average time between two collisions and the transport lifetime $\tau_{tr} \simeq 1.1$ ns which measures the time needed for an electron to loose memory of its momentum [19]. While τ_{tr} is extracted from the mobility, τ_e is obtained from the decay of the Shubnikov-de Haas oscillations. By varying microwave frequency we found that the decrease of resistance due to the enhancement of drift velocity occurred only for $\omega\tau_{tr} \gg 1$ (the effect was present for $f = 1.67$ GHz but absent for $f = 1.13$ GHz). However the lowest magnetic field for which resistance still decreases under irradiation ($H \simeq 0.06$ Tesla for data on Fig. 1) does not seem determined by τ_{tr} but rather by τ_e. Indeed at $H \simeq 0.06$ Tesla, $\omega_c\tau_{tr} \simeq 160$ while $\omega_c\tau_e \simeq 3$ is of the order of unity. Compared to the adiabatic effect, MIRO appear only at higher frequencies in our experiments. They could be observed only for frequencies larger than 30 GHz, suggesting that they require the absence of scattering during a microwave oscillation period $\omega\tau_e \geq 1$. However MIRO can persist down to very low magnetic fields around 10 mTesla [2], which corresponds to $\omega_c\tau_{tr} \geq 1$. Therefore enhancement of guiding and MIRO/ZRS seem to appear in complementary regimes of magnetic fields and frequency. These observations demonstrate the importance of the two timescales τ_e and τ_{tr} for understanding phototransport in 2DEG.

In conclusion we report a strong reduction of sample resistance under microwave irradiation in a high mobility two dimensional electron gas. This effect occurs in the regime where the irradiation energy $\hbar\omega$ is much smaller than the spacing between Landau levels $\hbar\omega_c$ and does not induce interlevel transitions. We explain our results through the enhancement of the mean drift velocity along sample edges by a low frequency electric field. The theoretical analysis of this enhancement leads to a scaling relation between power and magnetic field which is confirmed experimentally, whereas bulk transport theories predict a vanishing effect for $\omega \ll \omega_c$. Thus our results strongly support the important role played by edge channel transport in zero resistance state physics which was recently put forward theoretically. After completion of this work one of us, A.C., paricipated in experiments with electrons trapped on a helium surface which also support this conclusion [20].

We thank D.L. Shepelyansky for fruitful discussions and acknowledge ANR NanoTERRA for support.

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APPENDIX: ADIABATIC THEORY

In this appendix we give the main formulas used for the calculation of the mean drift velocity under low frequency microwave irradiation within the adiabatic approximation. In the Landau-Gauge the Hamiltonian for the motion of an electron along an edge in presence of magnetic field reads [17]

$$H = \frac{p_y^2}{2m} + U(y) \tag{3}$$

the potential $U(y)$ is created by a hard specular wall located at $y = 0$:

$$U(y) = \begin{cases} \frac{m\omega_c^2}{2}(y - Y_c)^2 & \text{if } y < 0 \\ \infty & \text{if } y \geq 0 \end{cases} \tag{4}$$

where $Y_c = k\hbar/(eH)$ is the position of the guiding center; and k is the wavenumber in the direction parallel to the wall.

The oscillation period in this potential for a particle with energy E and guiding center Y_c reads:

$$T(E, Y_c) = \frac{2}{\omega_c}\text{Arccos}(t) \tag{5}$$

$$t = \omega_c Y_c \sqrt{\frac{m}{2E}} = \frac{Y_c}{R_L} \tag{6}$$

and integration over energy yields the expression for action:

$$S(E, Y_c) = \frac{2E}{\omega_c}\sigma(t) \tag{7}$$

$$\sigma(t) = \text{Arccos}(t) - t\sqrt{1 - t^2} \tag{8}$$

In the semiclassical approximation valid for levels with number $n \gg 1$, the positions of the energy levels are given by the equation:

$$S(E_n(Y_c), Y_c) \simeq 2\pi\hbar n \tag{9}$$

Using this expression we find the value of the group velocity

$$v_g = \frac{1}{\hbar}\frac{\partial E_n}{\partial k} \tag{10}$$

$$= -\frac{1}{m\omega_c}\frac{\partial_{Y_c} S}{\partial_E S} \tag{11}$$

$$= \frac{2\sqrt{R_L^2 - Y_c^2}}{T(Y_c, E)} \tag{12}$$

As expected the group velocity coincides with the drift velocity of a classical trajectory propagating along the sample edge with guiding a center Y_c.

The above equation gives an expression of v_g as a function of E, Y_c, however to apply the adiabatic theory we need to evaluate v_g as a function of S, Y_c. For this purpose we use the following expression :

$$S(E, Y_c) = \frac{2E}{\omega_c}\sigma(t) = m\omega_c Y_c^2 \sigma(t) t^{-2} \tag{13}$$

Inverting this equation we would find an expression of t as a function of $S/(m\omega_c Y_c^2)$, however this quantity does not depend on the sign of Y_c as a result Eq. (13) has in general two solutions of opposite sign. The correct solution can then be chosen by noting that t and Y_c have the same sign. Thus we instead invert numerically the relation:

$$\sqrt{\frac{m\omega_c}{S}} Y_c = \frac{t}{\sqrt{\sigma(t)}} \tag{14}$$

which gives an expression of t as a function of $\sqrt{\frac{m\omega_c}{S}} Y_c$:

$$t = t(\sqrt{\frac{m\omega_c}{S}} Y_c) \tag{15}$$

As a result the rescaled group velocity $v_g \sqrt{\frac{m}{\omega_c S}}$ depends only on $\sqrt{\frac{m\omega_c}{S}} Y_c$ through the relation:

$$v_g \sqrt{\frac{m}{\omega_c S}} = \frac{\sqrt{t^{-2} - 1}}{\text{Arccos}(t)} \sqrt{\frac{m\omega_c}{S}} Y_c \tag{16}$$

which is displayed on Fig. 2. In the limit of high values of $\sqrt{\frac{m\omega_c}{S}} Y_c$ Eqs. (14,16) can be expanded in power series to lead the asymptotic behavior shown on Fig. 2.

We now determine of conductance in presence of adiabatic microwave driving. Let $Y_F(S)$ be the value of the guiding center Y_c for which the Landau levels tilted by the presence of the wall potential intersect the Fermi level $E(S, Y_c) = E_F$ where E_F is the Fermi energy. Without microwaves the particles at the Fermi energy with action S move at a mean velocity $v_g(S, Y_F(S))$. When the low frequency irradiation is turned on, the action is not changed (adiabatic limit) however the group velocities are modified by the presence of a the quasi-static field $E_\omega \cos \omega t$

$$v_g(S, Y_F(S)) \to v_g(S, Y_F(S) + Y_\omega) - \frac{E_\omega}{m\omega_c}\cos \omega t \tag{17}$$

where $Y_\omega = E_\omega/(m\omega_c^2)\cos \omega t$. Indeed the electric field $E_\omega \cos \omega t$ changes the energy levels to :

$$E_n(Y_c) \to E_n(Y_c + Y_\omega) - E_\omega \cos \omega t Y_c - \frac{E_\omega^2}{2m\omega_c^2} \tag{18}$$

Averaging over the oscillations of the electric field $E_\omega \cos \omega t$, yields the expression for the mean drift velocity:

$$< v_g >=< v_g(S, Y_F(S) + Y_\omega \cos \omega t) >_t \tag{19}$$

This average was computed numerically leading to the results displayed on Fig. 3.