Chirikov criterion

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Chirikov criterion or Chirikov resonance-overlap criterion was introduced in 1959 by Boris Chirikov and successfully applied by him to explain the confinement border for plasma in open mirror traps observed in experiments at the Kurchatov Institute (http://en.wikipedia.org/wiki/Kurchatov_Institute) . This was the very first physical and analytical criterion for the onset of chaotic motion in deterministic Hamiltonian systems.

Contents

Description

According to the Chirikov criterion [1], a deterministic trajectory will begin to move between two nonlinear resonances in a chaotic and unpredictable manner as soon as these unperturbed resonances overlap. This occurs when the "perturbation" or "chaos" parameter becomes larger than unity:

$$K \sim S^2 > 1 \ , \ S = \Delta\omega_r / \Omega_d \qquad (1)$$

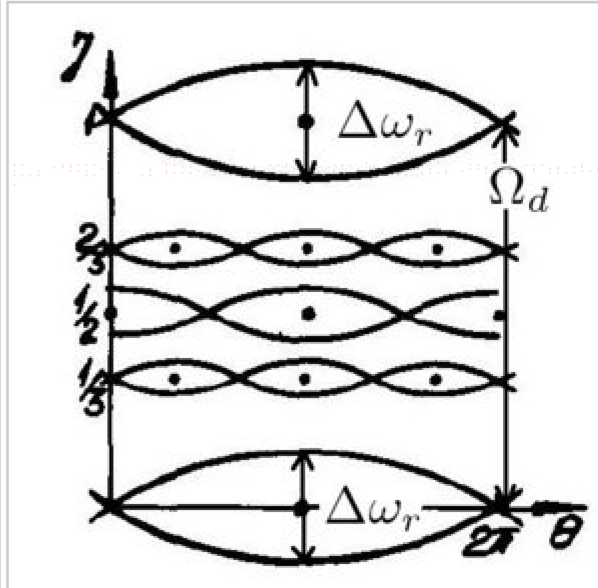

Figure 1: Diagram of separatrices of resonances at $J_r = 0; 1/3; 1/2; 2/3; 1$ with distance Ω_d between main resonances and their width $\Delta\omega_r$

Here S is the resonance-overlap parameter. S is the ratio of the sum of the unperturbed resonance half-widths in frequency $\Delta\omega_r$ and the frequency distance Ω_d between two unperturbed resonances (see Fig.1). The width $\Delta\omega_r$ is often computed in the pendulum approximation and is typically proportional to the square-root of the perturbation amplitude. Since its introduction, the Chirikov criterion has become an important analytical tool for the determination of the chaos border in Hamiltonian systems (see [2,3]). This criterion, while supplying important insight to the onset of chaos, is not rigorous. A more rigorous approach was supplied by John Greene [4,5], who identified the chaos boarder with the destabilization of higher order fixed points in a vicinity of an invariant curve with a chosen fixed rotation number [4,5]. Although it is more precise, the approach of Greene requires significant numerical computation and is not often suitable for a derivation of analytical dependences on system parameters. The accuracy of the Chirikov criterion can also be improved using a renormalization approach [6] that takes into account resonances on smaller and smaller scales, provided the overlapping resonances are of similar size. For unequal size overlapping resonances the criterion becomes less accurate. Also, it should be noted that the criterion uses the pendulum approximation for a nonlinear resonance which is valid in a regime of moderate nonlinearity [2,3]. Thus, the criterion is not directly applicable when the unperturbed Hamiltonian is linear in actions and its linear frequencies are degenerate. For a example, a three-dimensional oscillator with three equal frequencies has approximately half of phase space being chaotic at an arbitrarily weak quartic nonlinear coupling between modes [7]. But such linearly degenerate systems are special; moreover, Kolmogorov-Arnold-Moser theory also does not apply naively in these cases.

Example

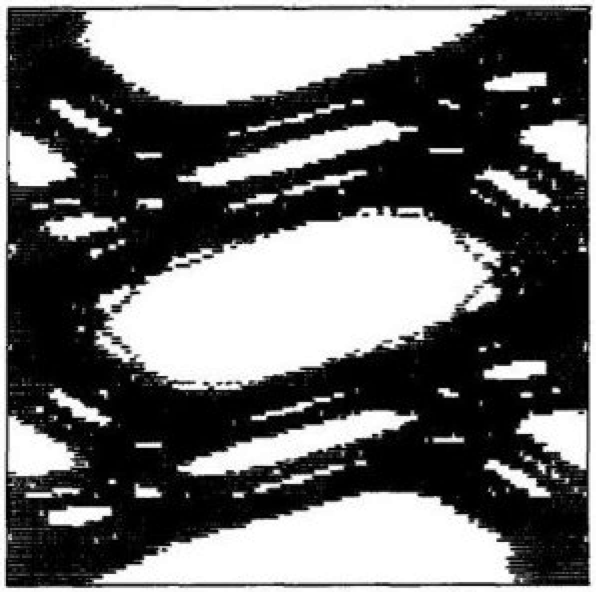

Figure 3: Same as above at $K = 1.13$

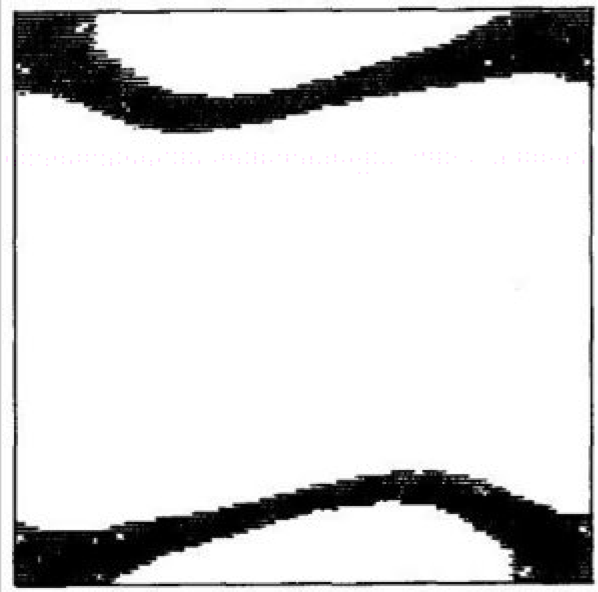

Figure 2: Phase space (θ, I) of the Chirikov standard map at $K = 0.96$

To determine the chaos border for the Chirikov standard map using the Chirikov criterion we write the system Hamiltonian

$$H(I,\theta,t) = I^2/2 + K\cos\theta\delta_1(t),$$

where $\delta_1(t)$ is a periodic $\delta-$ function with period 1 in time t and I,θ are action-angle variables. After the expansion of the $\delta-$ function in a Fourier series this Hamiltonian takes the form

$$H(I,\theta,t) = I^2/2 + K\sum_m \cos(\theta - 2\pi m t),$$

where summation is over all integers m. Assuming the perturbation to be small we obtain the positions of the "principal" or "forced" resonances at $I = 2\pi J_r = 2\pi m$ (see Fig.1). These resonances are identical and the distance between two neigbhoring resonances is $\Omega_d = 2\pi$. Taken by itself, dynamics of an individual resonance is described by the pendulum Hamiltonian $H(I,\theta) = I^2/2 + K\cos\theta$ with the separatrix width at energy $H(I,\theta) = E = K$ being $\omega_r = \Delta I = 4\sqrt{K}$. Consequently, the overlap of these unperturbed resonances occurs when $S = \Delta\omega_r/\Omega_d = 2\sqrt{K}/\pi > 1$. Due to higher order resonances and chaotic separatrix layers, overlap, leading to global chaos, actually occurs when $K \approx 2.5S^2 > 1$. The transition to global chaos in the vicinity of critical parameter $K \approx 1$ is clearly seen in Figs.2,3. Using this numerical adjustment factor, the overlap criterion gives the global chaos border in various dynamical systems with a few percent accuracy.

Counter-Example

The Chirikov criterion determines the chaos border in generic nonlinear systems. A certain class of nonlinear systems can be completely integrable and in this case the criterion does not work. A well known example is the Toda lattice [8] which remains integrable even at very strong nonlinearity. Apart of these specific completely integrable systems, the Chirikov criterion usually works well for generic systems.

History and applications

In 1954 G.I.Budker (http://en.wikipedia.org/wiki/Budker) proposed a mirror magnetic trap for plasma confinement. Particle confinement in such an open trap is provided by the conservation of an adiabatic invariant, the magnetic moment of a particle, which is known, however, to be an approximate motion integral only. The experimental studies of such a system have been done by S.N.Rodionov [9]. The confinement border observed in experiments was explained by Boris Chirikov on the basis of invented by him resonance-overlap criterion [1]. More details about chaos border for particle confinement in magnetic traps can be find in [10,11]. The Chirikov criterion finds applications for the dynamics of solar system, particle dynamics in accelerators, magnetic plasma traps, microwave ionization of Rydberg atoms and various other systems.

Chaotic stories

- Boris Chirikov first presented his results on the stochastic instability of magnetically confined plasma at the Kurchatov seminar in Moscow in 1958, when the plasma research was classified secret. Only after the London plasma conference of 1958 did the results become public, and Kurchatov ordered the plasma results to be published quickly. This led to Chirikov's celebrated 1959 theoretical paper [1] in a special issue of the journal Atomic Energy. Boris Chirikov had started his career as an experimenter, but the world would now know him as the theorist who invented the resonance overlap criterion. What is less known is the story of the writing of the paper in the same journal issue that describes the related plasma experiments of S. Rodionov [9]. Though Rodionov's name appears as the sole author, the paper was written by Chirikov. Why? The story goes that Rodionov had broken his right hand (in an overcrowded public bus) and was in the hospital. Chirikov was ordered by the KGB to take his secret notes, go to the hospital, and write the paper from the words of Rodionov. The KGB orders included that Chirikov take a weapon, a revolver, to ensure the security of the secret documents, but Chirikov refused, arguing that it would be too dangerous to take a revolver on the public buses that, in those days, were always very overcrowded with people. Finally, the KGB agreed that Chirikov would not have to carry the revolver, but he was obliged to return all his notes, including the "Rodionov manuscript" back to the secure place. What we now know as the Chirikov criterion came as a result of Chirikov's generalizing the theoretical analysis he had first performed for the stochastic instability of confined plasma. (from Reminiscences of Boris Chirikov (http://www.scholarpedia.org/wiki/images/ftp/chirikov2008s.pdf)).

- Other stories can be found in the Scholarpedia article Boris Chirikov and in the Reminiscences of Boris Chirikov (http://www.quantware.ups-tlse.fr/chirikov/reminis.html)

References

(see more at http://www.quantware.ups-tlse.fr/chirikov/publications.html)

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- Reminiscences of Boris Chirikov (http://www.scholarpedia.org/wiki/images/ftp/chirikov2008s.pdf)

Recommended Reading

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L.E.Reichl, "The Transition to chaos in conservative classical systems and quantum manifestations", Springer, Berlin (2004).

External links

- website dedicated to Boris Chirikov (http://www.quantware.ups-tlse.fr/chirikov/)
- Boris Chirikov at Wikipedia (http://en.wikipedia.org/w/index.php?title=Boris_Chirikov)
- Chirikov criterion at Wikipedia (http://en.wikipedia.org/wiki/Chirikov_criterion)

See also internal links

Chirikov standard map, Boris Valerianovich Chirikov , Hamiltonian systems, Mapping, Chaos, Kolmogorov-Arnold-Moser Theory, Kolmogorov-Sinai entropy, Aubry-Mather theory, Quantum chaos

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