Walking droplets: an analogy with quantum wave/particle duality

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QUANDROPS project in Liège

Combination of theoretical and experimental groups

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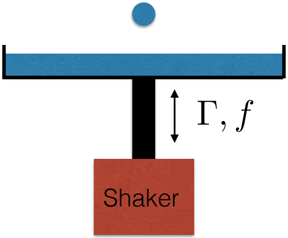
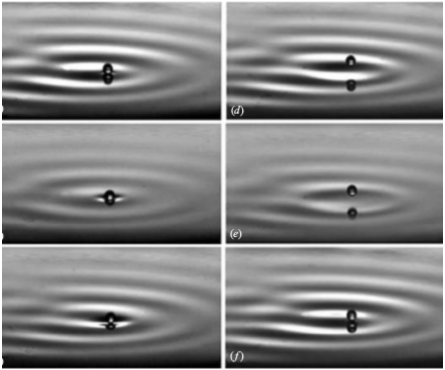
The founding experiment

Couder et al., Nature **437**, 208 (2005):
A droplet of oil falls on a vibrating oil bath: walker.

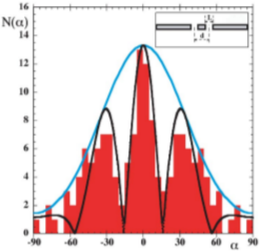


$\nu = 20cSt$

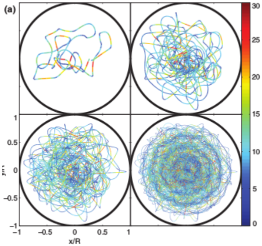


Viscosity: $\mu_{\text{oil}} = 20\mu_{water}$
Acceleration of the bath: $\Gamma(t) = 3.5g\cos(2\pi f\ t)$
$f = 80$ Hz.

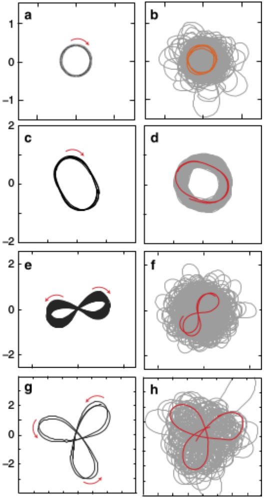
A versatile system



Double slit
Couder, Fort (2006)



Circular billiard
Harris et al (2013)

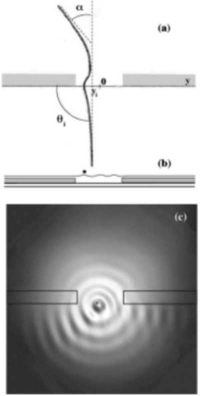


Harmonic potential
Perrard et al (2014)

And tunneling, Landau orbits, Zeeman effect,. . .

Interaction between a particle and a wave

The droplets pertubes the surface profile, which guides the droplets.



Macroscopic system

Main similarities

- wave and particles effects in the dynamics
- unpredictability of the droplet's trajectory $\Rightarrow$ probabilistic description of the droplet
- interference pattern in the distribution of droplet's trajectory

Main differences

- dissipation in the fluid system
- droplet's trajectory can be measured without perturbation

Bohm's formulation of quantum theory

A point particle moving under the influence of the wave function

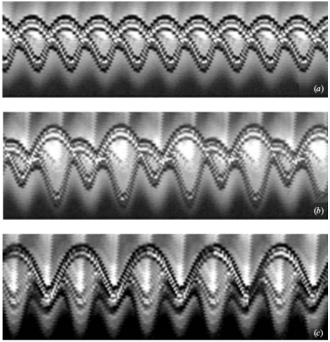
An alternative formulation of quantum theory (De Broglie, 1926, Bohm, 1952):

- identical distribution of observables
- no collapse of the wave function during/after measurement
- simple dynamical equations:
  for the wave: usual Schrödinger equation: $i\hbar\partial_t\psi = H\psi$
  for the associated particle:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \frac{\nabla S}{m}, \qquad \psi = |\psi| e^{iS/\hbar}$$

Description of a droplet: Hydrodynamics view

Protière et al, 2006; Bush, 2015
Separation between the vertical and the horizontal motions.
Vertically:
(approximate) Parabolic flight between two bounces.
Horizontally:
Momentum given by the local slope of the bath profile



Heuristic model for the walker's dynamics

Faraday waves with dispersion relation:

$$\omega^2 = (gk + \sigma/\rho k^3)\tanh kh$$

Description of the droplet trajectory on the horizontal direction

$$m\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = F(\mathbf{x}) - C\nabla\zeta$$

$F(\mathbf{x})$: external force
C: coupling between the particle and the wave
$\zeta(\mathbf{x},t)$: surfave profile. For a free droplet:

$$\zeta(\mathbf{x},t) = \mathrm{Re}\left[\sum_{j=0}^{\infty} G^{(0)}(\mathbf{x},\mathbf{x}_j)\exp\left(-\frac{t-jT_F}{MeT_F}\right)\right]$$

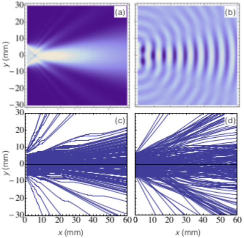
with $\mathbf{x}_j$: position of the bounce at time $t - jT_F$.
T_F: Faraday period, Me: memory parameter
$G^{(0)}(\mathbf{x},\mathbf{x}_0)$: free Green function in the plane

Our description for a single slit experiment

Quantify the quantum analogy (Richardson et al, 2014).
Insert the exact Green function in $\zeta(\mathbf{x}, t)$:

$$G(\mathbf{x}, \mathbf{x}_0; k) = \frac{1}{2\pi} \sum_{n=0}^{\infty} \frac{Me^{(1)}_{n/2}(u_>) Ce_{n/2}(u_<) ce_{n/2}(v) ce_{n/2}(v_0)}{ce_{n/2}(0) Me^{(1)\,\prime}_{n/2}(0)}$$

$Me^{(1)}_{\nu}, Ce_{\nu}, ce_{\nu}$: Mathieu functions. u, v: elliptic coordinates



Work in progress

- quantify the effect of memory
- diffractive effects in the high memory regime

Conclusion and perspectives

- macroscopic system, which realises a coupling between a wave and particle
- coherence effects for one or several “particles”
- range of validity of a quantum approach still in debate

For the future:

- comparison with *ab initio* numerical solution of the full $3D$ fluid problem: effects of depth, effective boundary conditions
- unique playground to visualise particle trajectories. Bohmian effects

More details: Richardson et al, arxiv:1410.1373