Propagating wave correlation functions in complex environments

In collaboration with

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The University of Nottingham

Aim: Modelling high-frequency wave dynamics including noise, interference and multiple reflections

Applications:

-Electromagnetic Compatibility

Spurious emissions from cirucits and cables in confined environment.

-Wireless Communication

multiple antenna arrangements in mobile phones, WLAN etc, but also for future technologies (on-chip and chip-to-chip communication)

-Noise and vibration issues in mechanical engineering.

Partners:

Nottingham Trent University
TU München
University of Nice Sophia Antipolis
University of Maryland

inuTech GmbH – Nürnberg
CDH AG - Ingolstadt
CST AG – Darmstadt
IMST GmbH - Duisburg
NXP Semiconductors

Aim: Modelling high-frequency wave dynamics including noise, interference and multiple reflections

Outline of the talk

Introduction: correlations, Green functions and classical dynamics.

I) Correlation functions: *free propagation in the Wigner-Weyl picture*.

II) Correlation functions: *multiple reflections - a semiclassical treatment*.

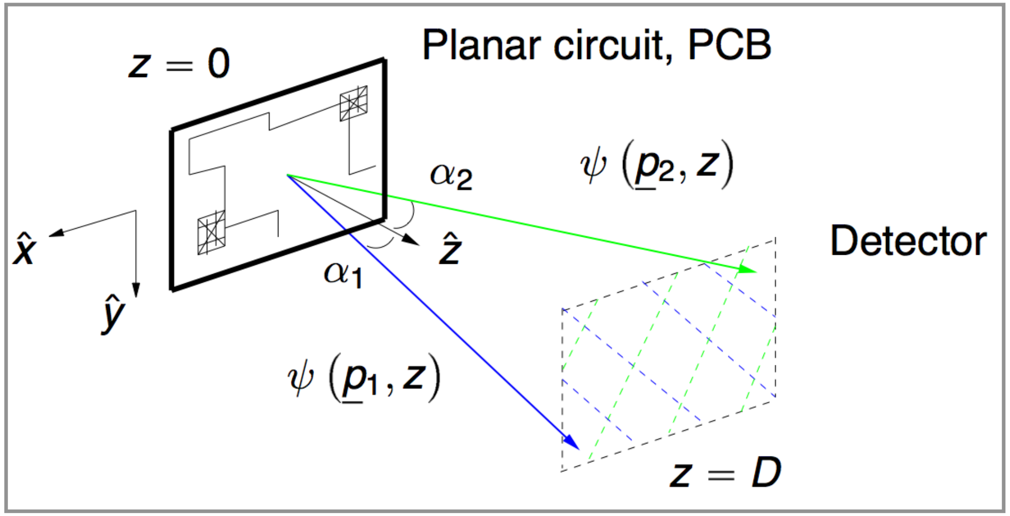
III) Propagating the classical flow – *Discrete Flow Mapping*.

Introduction:

Stochastic source – consider correlation function in plane parallel to $z = 0$.

$$\Gamma_z(p_1, p_2) = \langle \psi(p_1, z)\, \psi^*(p_2, z) \rangle$$

here in momentum space; $\langle . \rangle$ denotes, for example, time average.



Idea:

- Near-field correlation → far-field correlation;
- Wigner transform to describe waves in phase-space (position, momentum);
- Derive efficient propagation schemes in phase-space;
- Retrieve field-field correlation in configuration space.

Can we predict Γ_z over the whole domain including reflections?

Introduction:

Stochastic source – consider correlation function in plane parallel to $z = 0$.

$$\Gamma_z (p_1, p_2) = \langle \psi (p_1, z) \psi^* (p_2, z) \rangle$$

here in momentum space; $\langle . \rangle$ denotes, for example, time average.

Previous work:

- Connection between correlation function and (imaginary part of) Green function
  - *Creagh and Dimon (1997);*
  - *Hortikar and Srednicki (1998);*
  - *Weaver and Lobkis (2001);*
  - *Urbina and Richter (2006)*
- Connection between correlation function and phase space propagation
  - *Marcuvitz (1991)*
  - ***Optics:*** *Littlejohn and Winston (1993), ... , Alonso (2011)*
  - *Dittrich, Viviescas and Sandoval (2006)*
- Propagation of correlation function as numerical tool:
  - *Russer and Russer (2012)*

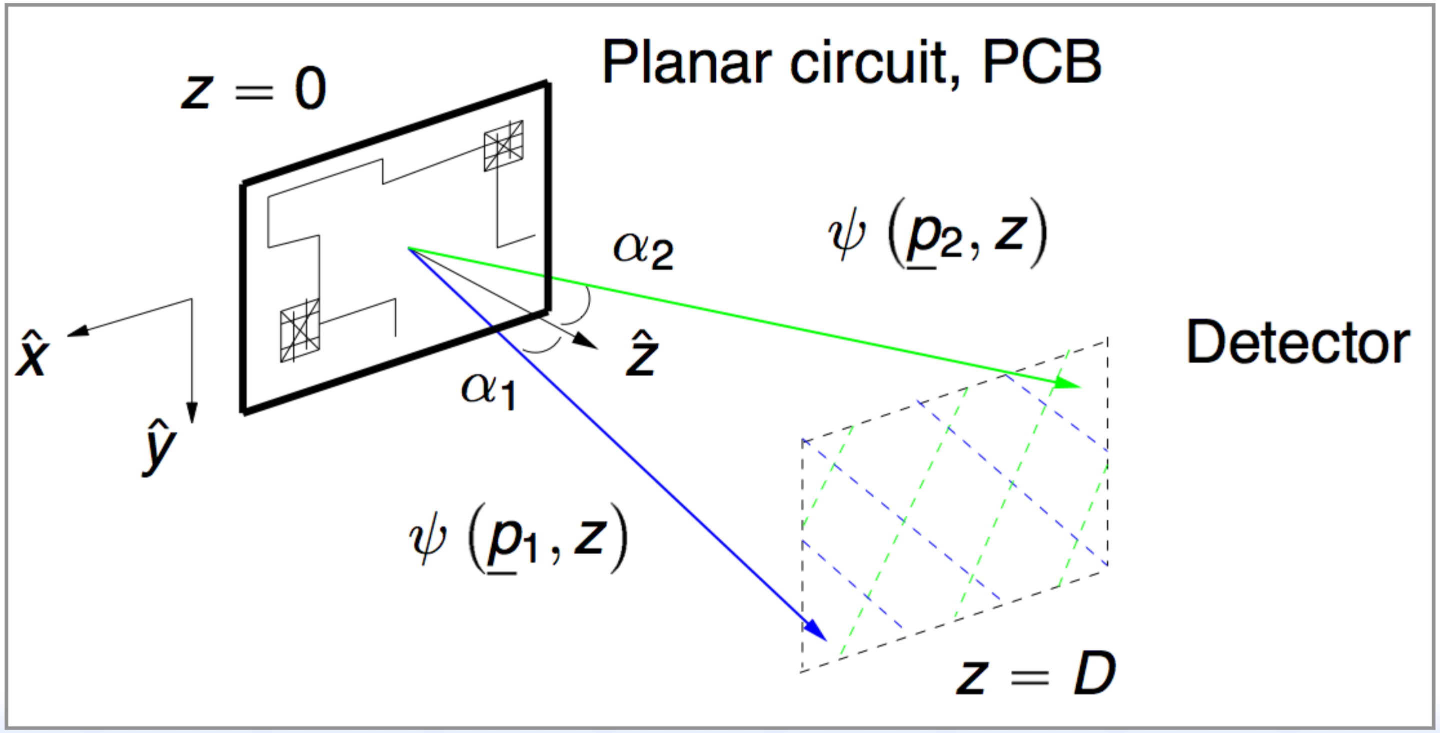
and many more …

Source Distribution:

Stochastic source – consider correlation function in plane parallel to $z = 0$.

$$\Gamma_z(p_1, p_2) = \langle \psi(p_1, z)\, \psi^*(p_2, z) \rangle$$

here in momentum space; $\langle . \rangle$ denotes, for example, time average.



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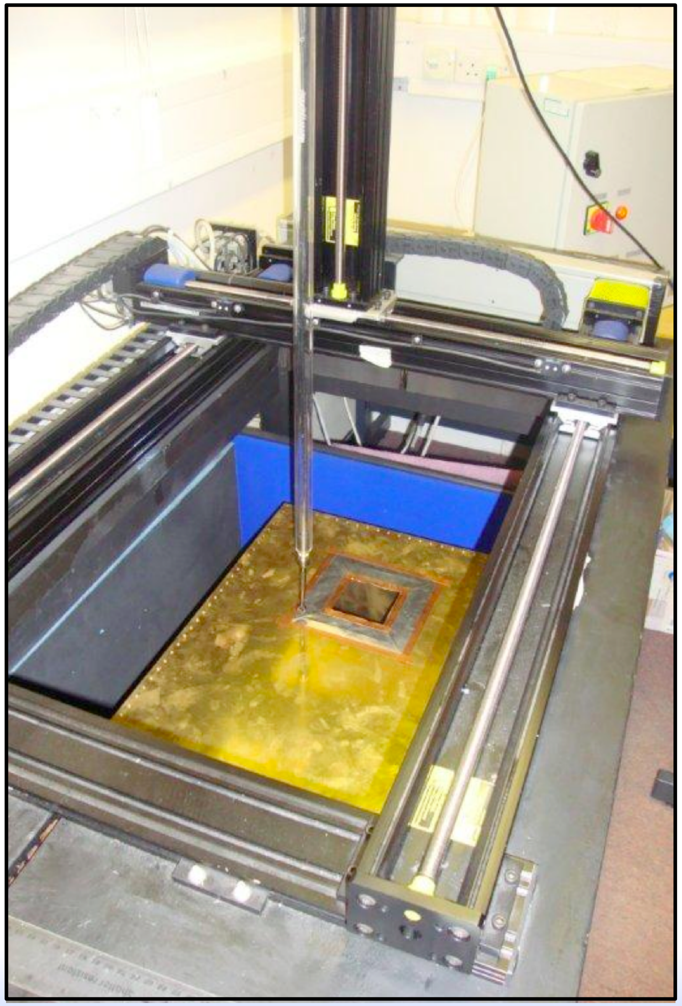
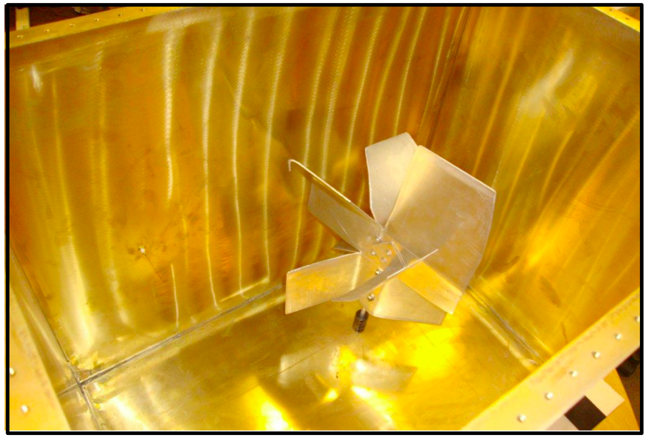
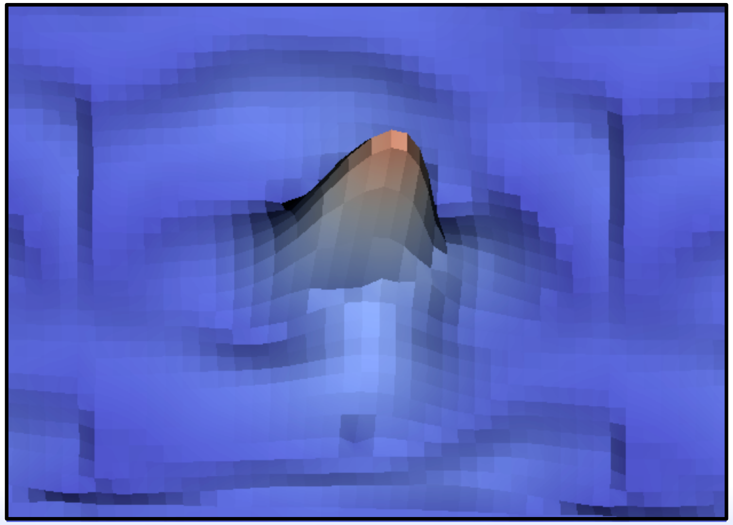
Source Distribution:

Measurement of source correlation function

Chris Smartt et al - GGIEMR





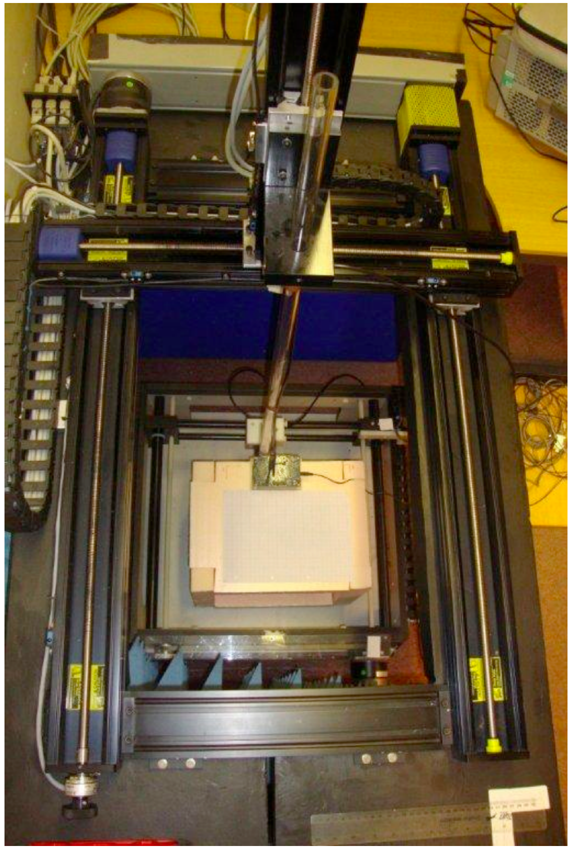


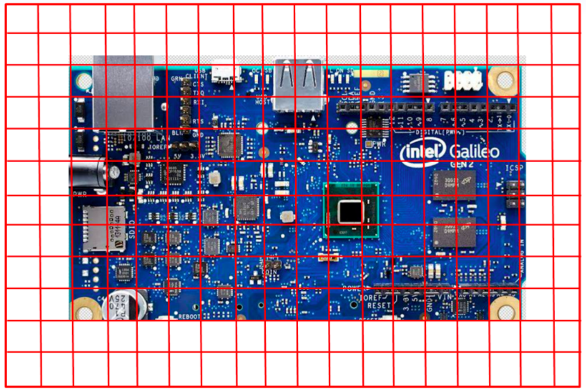
Cavity with aperture – single probe, single frequency

Source Distribution:

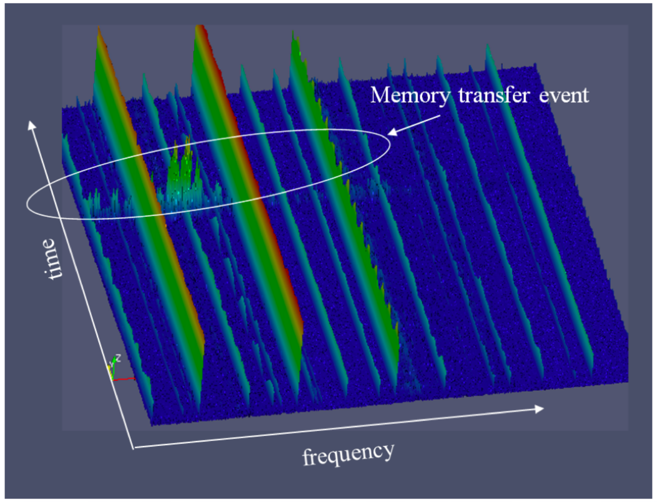
Measurement of source correlation function

Chris Smartt et al - GGIEMR





Arduino Galileo PCB



Arduino PCB – two-probe 1D time measurement

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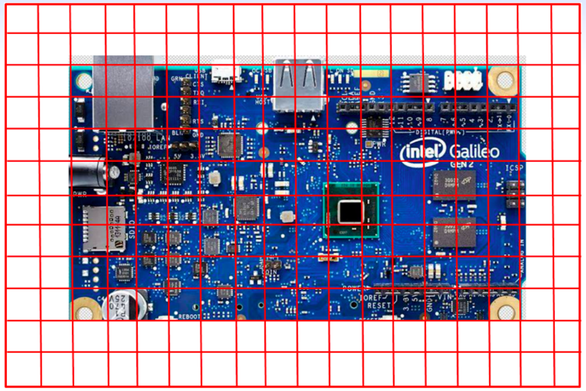
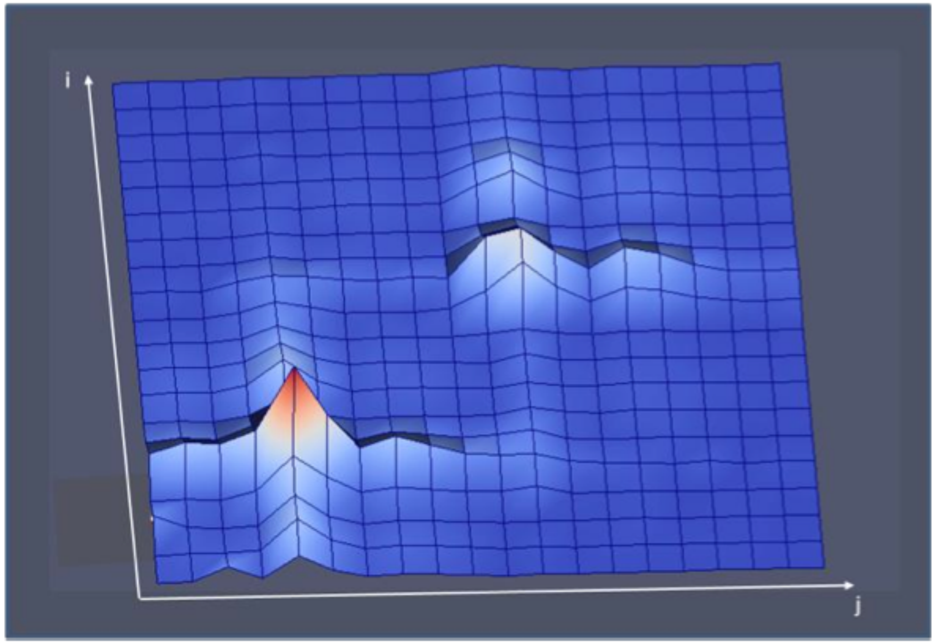
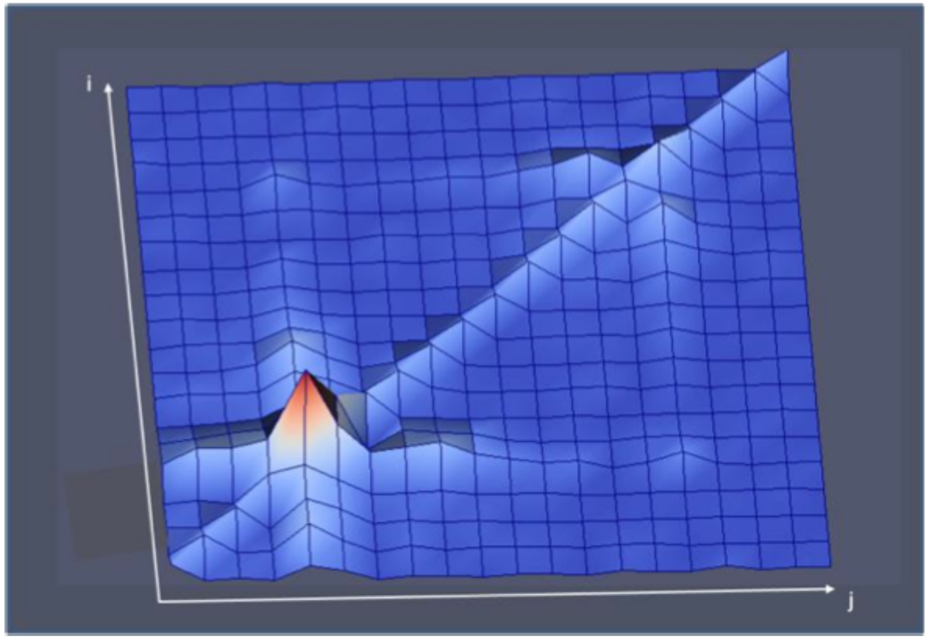
Source Distribution:

Measurement of source correlation function

Chris Smartt et al - GGIEMR







100 MHz **233 MHz**

Arduino PCB – two-probe 1D time measurement

Radiation into free space: Propagation rules

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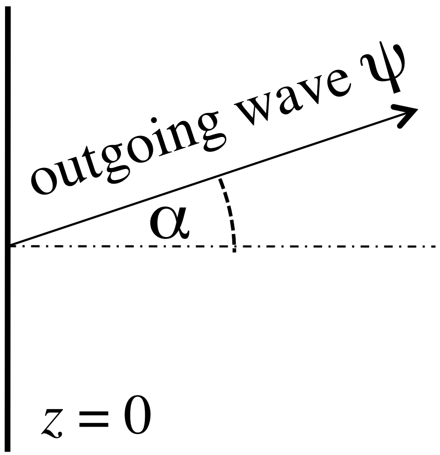
Propagation into free space: Huygens principle – Green's identity

$$\psi\left(p,z\right)=\underbrace{\frac{i}{2k\sqrt{1-p^{2}}}e^{ikz\sqrt{1-p^{2}}}}_{\hat{G}_{0}}\nu\left(p,z=0\right)+\underbrace{\frac{1}{2}e^{ikz\sqrt{1-p^{2}}}}_{\hat{G}_{1}}\psi\left(p,z=0\right)$$



$$\nu(p,0)=\frac{\partial\psi}{\partial n}\text{ : normal derivative}$$

$$T\left(p\right)=\sqrt{1-p^{2}}=\cos\alpha:\ \text{normal component}$$

Solution of Helmholtz Eqn.

$$\psi\left(p,z\right)=e^{ikzT(p)}\psi\left(p,0\right)$$

$$\hat{\Gamma}_{z}\left(p_{1},p_{2}\right)=e^{ikz\left(T(p_{1})-T^{*}(p_{2})\right)}\hat{\Gamma}_{0}\left(p_{1},p_{2}\right)$$

Radiation into free space: Wigner function

Using Wigner Transform (in plane z = const):

$$W_z(x,p) = \left(\frac{k}{2\pi}\right)^d \int e^{ikxq}\, \Gamma_z\left(p+\frac{q}{2}, p-\frac{q}{2}\right) dq$$

… and back-transformation:

$$\Gamma_z\left(p+\frac{q}{2}, p-\frac{q}{2}\right) = \int e^{-ikx'q}\, W_z(x',p)\, dx'$$

Note – spatial correlation function can be recovered:

$$W_z(x,p) = \left(\frac{k}{2\pi}\right)^d \int e^{ikps}\, \Gamma_z\left(x+\frac{s}{2}, x-\frac{s}{2}\right) ds$$

Radiation into free space: WF Propagator

The WF is propagated in phase space (*x*,*p*) according to

$$W_z(x,p) = \int\int \hat{\mathcal{G}}(x,p;x',p';z)\, W_0(x',p')\, dx'dp'$$

with propagator:

$$\hat{\mathcal{G}}(x,p,x',p';z) =$$

$$= \delta(p-p')\left(\frac{k}{2\pi}\right)^d \int e^{ik(x-x')q}\, e^{ikz\left(\sqrt{1-\left(p+\frac{q}{2}\right)^2}-\sqrt{1-\left(p-\frac{q}{2}\right)^2}^{\,*}\right)} dq$$

This propagator acts in phase space – see Dietrich et al (2006)

Radiation into free space: Forbenius Perron Operator

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Taylor expanding exponential $\Delta T\left(p,q\right) = \sqrt{1-\left(p+\frac{q}{2}\right)^2} - \sqrt{1-\left(p-\frac{q}{2}\right)^2}^{*}$ *to first order* in q:

$$\hat{\mathcal{G}}\left(x, x', z; p\right) \approx \left(\frac{k}{2\pi}\right)^d \int e^{ik\left(x-x'\right)q - ikz\frac{p}{\sqrt{1-p^2}}q}\, dq$$

$$= \delta\left(x - x' - \frac{p}{\sqrt{1-p^2}}z\right)$$

Ray-tracing / Frobenius-Perron approximation

(Ray) densities are propagated along classical rays:

$$W_z\left(x,p\right) \approx W_0\left(x - z\frac{p}{\sqrt{1-p^2}}, p\right)$$

Valid for quasi-homogeneous sources

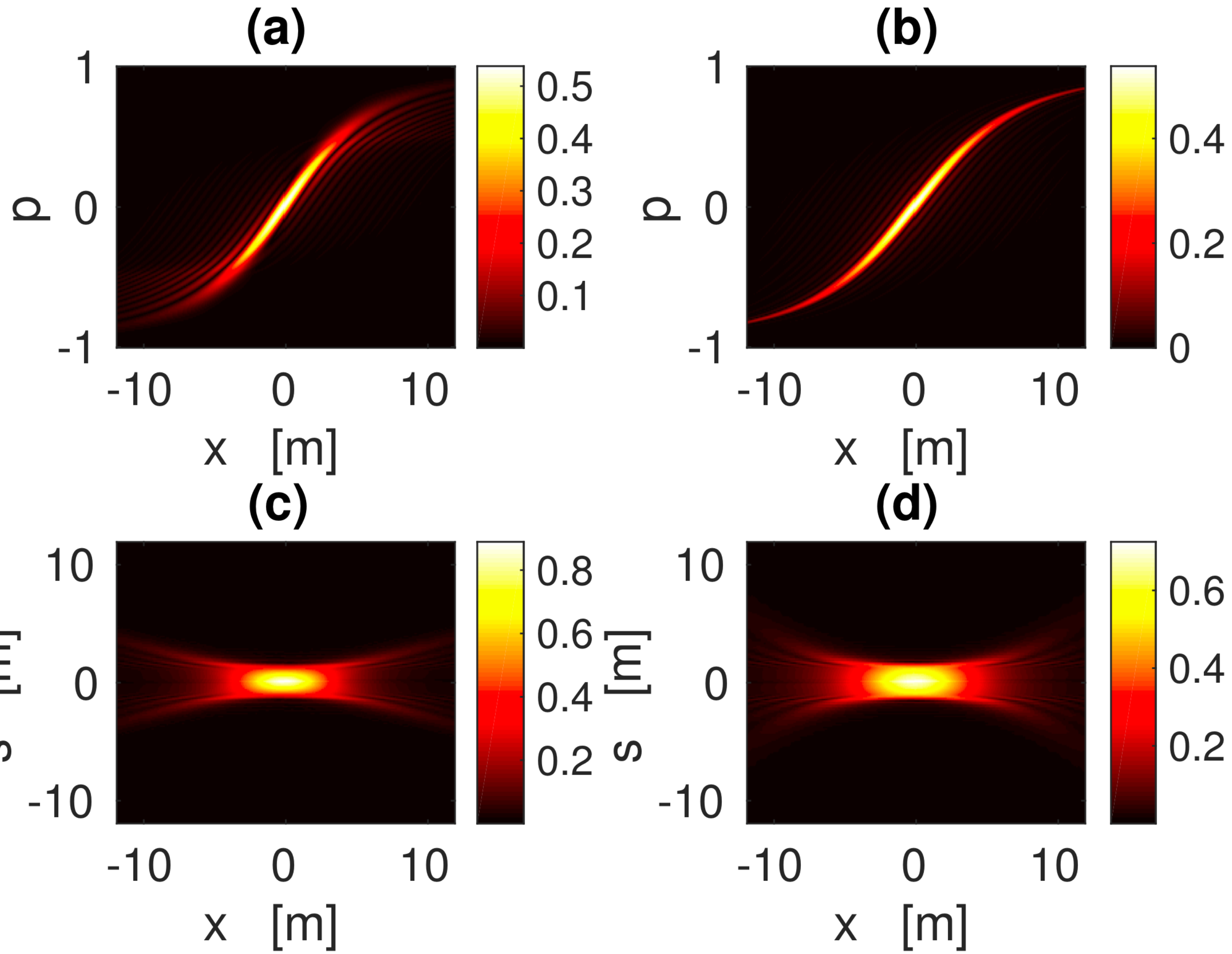
Propagation of Gaussian source in free space

Exact Wigner

Approximate Wigner

Exact Correlation Function

Approximate Correlation Function



$z = 10\,\lambda$

Radiation into free space: Forbenius Perron Operator + corrections

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Taylor expanding exponential $\Delta T\left(p,q\right)=\sqrt{1-\left(p+\frac{q}{2}\right)^2}-\sqrt{1-\left(p-\frac{q}{2}\right)^2}^{*}$ *to third order* in q:

$$\hat{\mathcal{G}}\left(x,x',z;p\right)\approx$$

$$\approx\left(\frac{k}{2\pi}\right)^d\int e^{ik\left(x-x'\right)q-ikz\left[\frac{p}{\sqrt{1-p^2}}\right]q+ikz\left[\frac{p}{4(1-p^2)^{3/2}}+\frac{p^3}{4(1-p^2)^{5/2}}\right]q^3}dq$$

(in 1D)

$$=a\ \mathrm{Ai}\left[a\left(x-x'-\frac{zp}{T(p)}\right)\right]$$

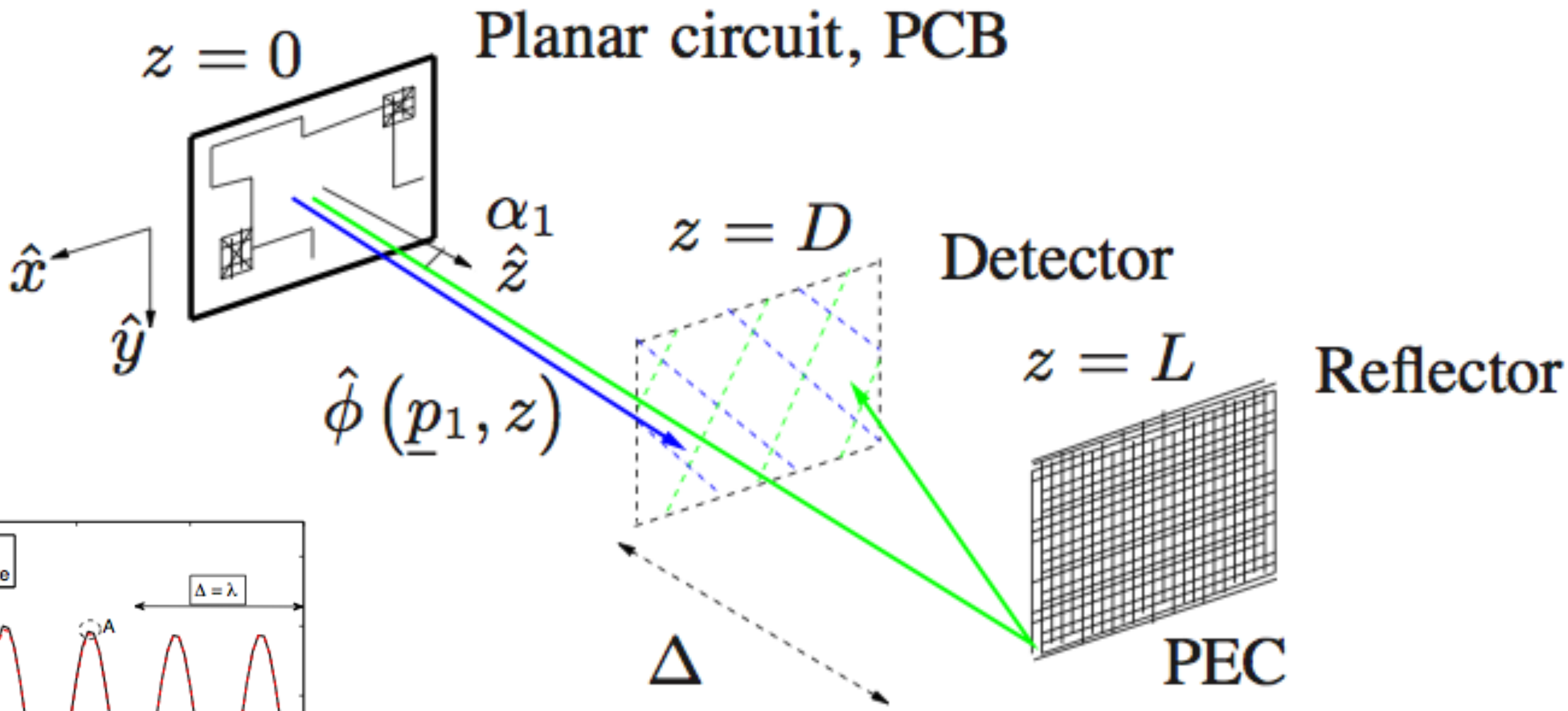
…similar to Marcuvitz (1991)

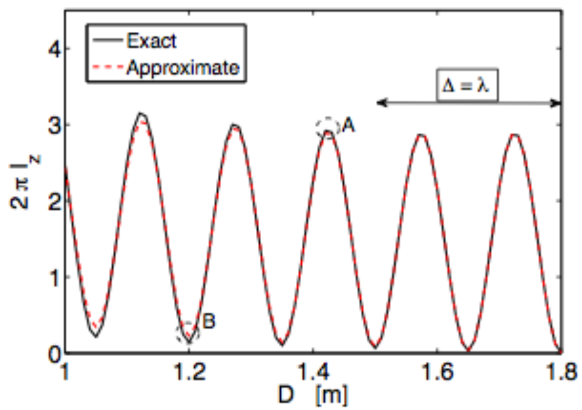
with

$$a=2k^{2/3}\left(zT'''(p)\right)^{-1/3}$$

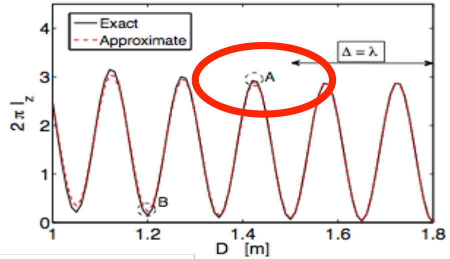
Converges to Frobenius-Perron form for k → ∞

Radiation into free space: Reflections



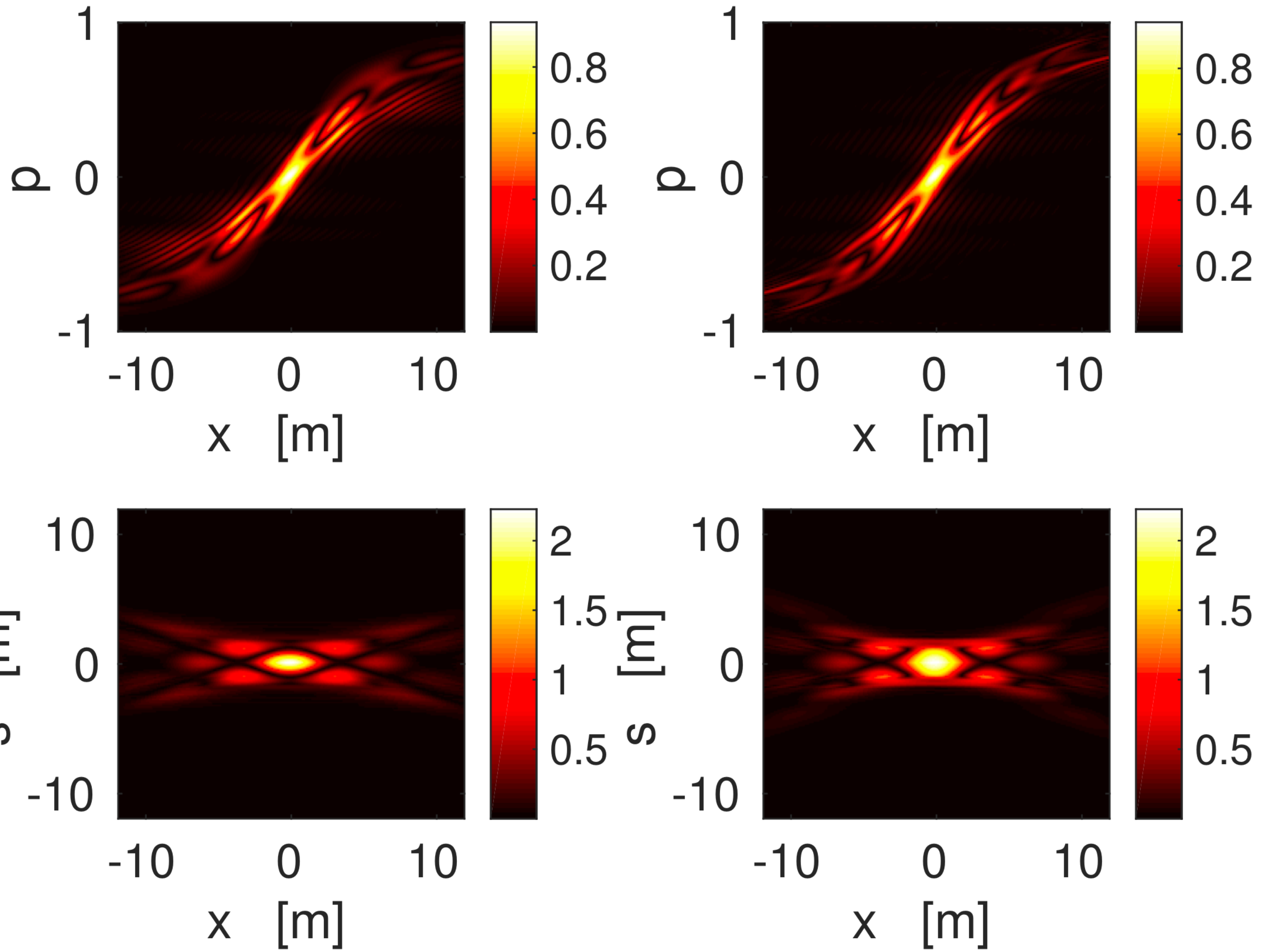


Radiation into free space: Reflections



Exact Wigner

Approximate Wigner

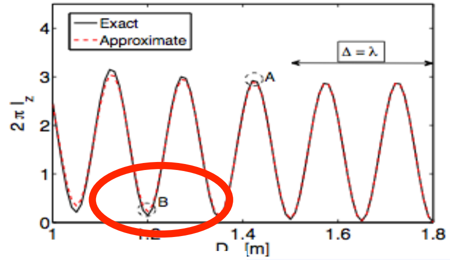


Exact Correlation Function

Approximate Correlation Function

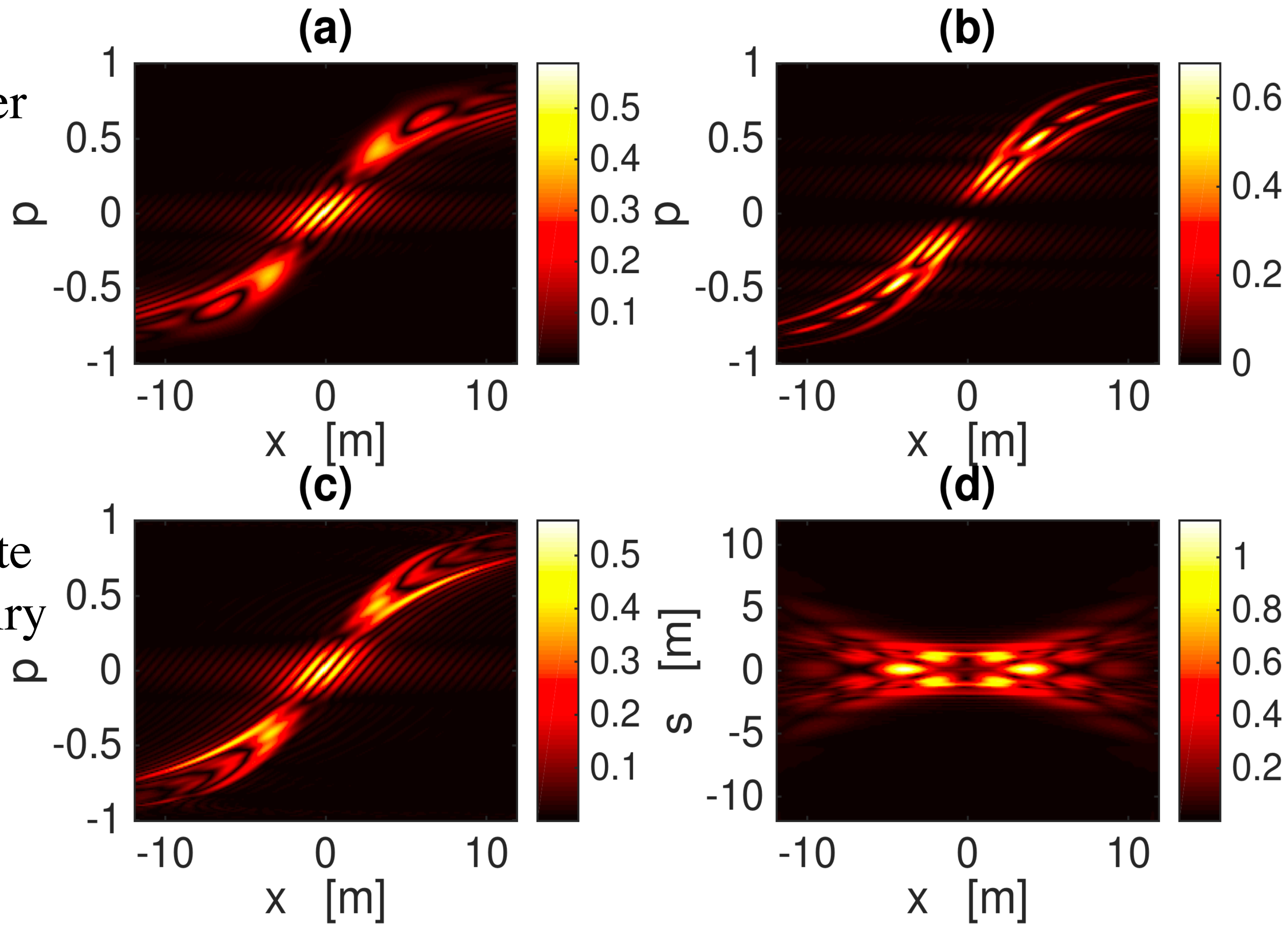
FP – approximation + interference term

Radiation into free space: Reflections



Exact Wigner

Approximate Wigner - FP



Approximate Wigner - Airy

Exact Correlation Function

FP/Airy approximation + interference term

Radiation into free space

Strategy:

Transform source correlation function into Wignerfct → Propagate Wigner Function in phase space (either exactly or using linear (3rd order) approximation→Transform $W_z(x,p)$ back to correlation fct $\Gamma_z(x,x')$

In particular for FP approximation – simplified propagation rule

$$\boxed{\Gamma_z\left(x,s\right) \approx \int\int e^{ikp'\left(s-s'\right)}\,\Gamma_0\left(x-\frac{p'}{\sqrt{1-p'^2}}z,s'\right)\,ds'\,dp'}$$

(generalised) van Cittert - Zernike theorem - *Cerbino 2007*

Correlation length: $\Delta s = z\,\lambda\,/\,L$

Radiation into free space: Van Cittert - Zernike

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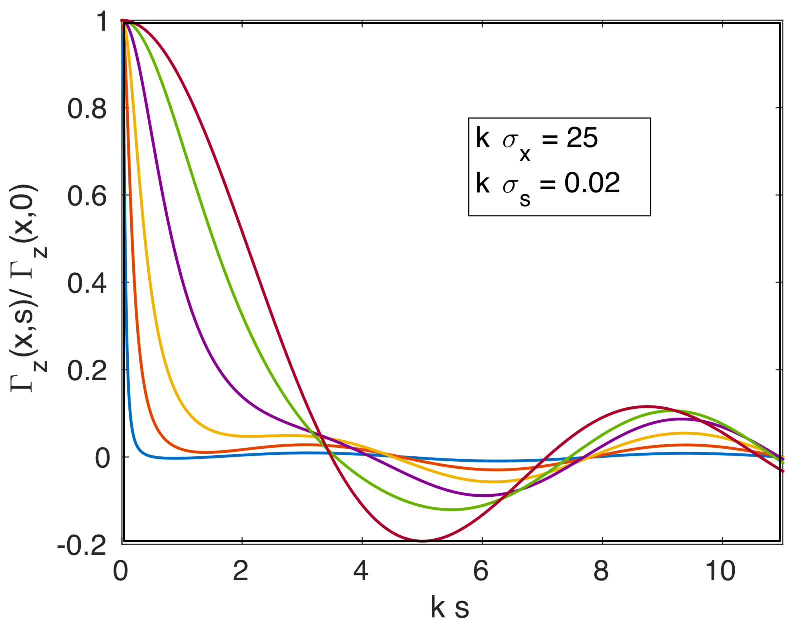
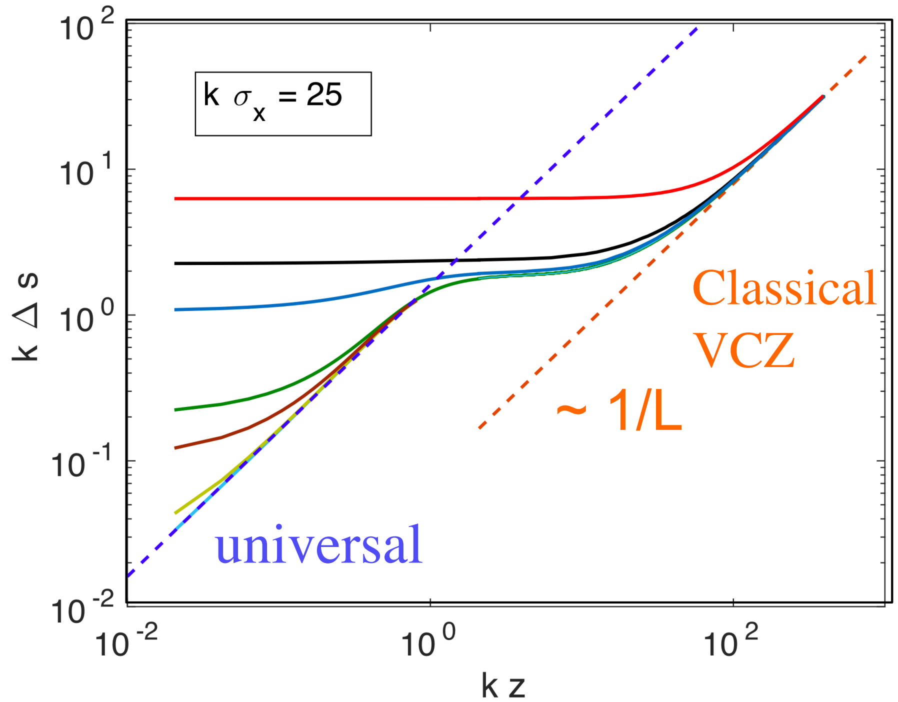
Corrections due to evanescent contribution!

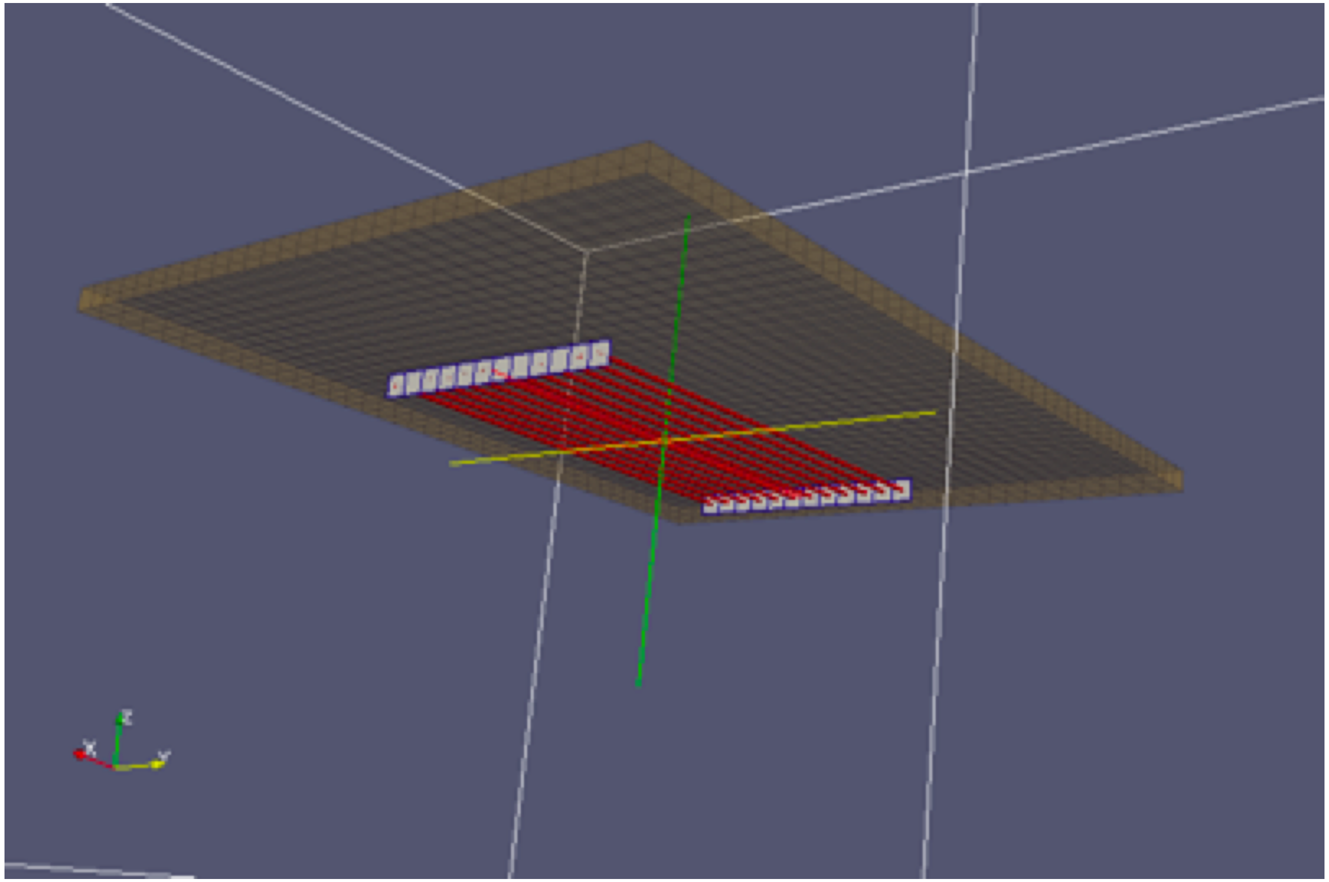
Propagation of
Correlation function



Correlation Length



Propagation of realistic signal – cable bundle driven by random voltage



Propagation of realistic signal – cable bundle driven by random voltage – near field

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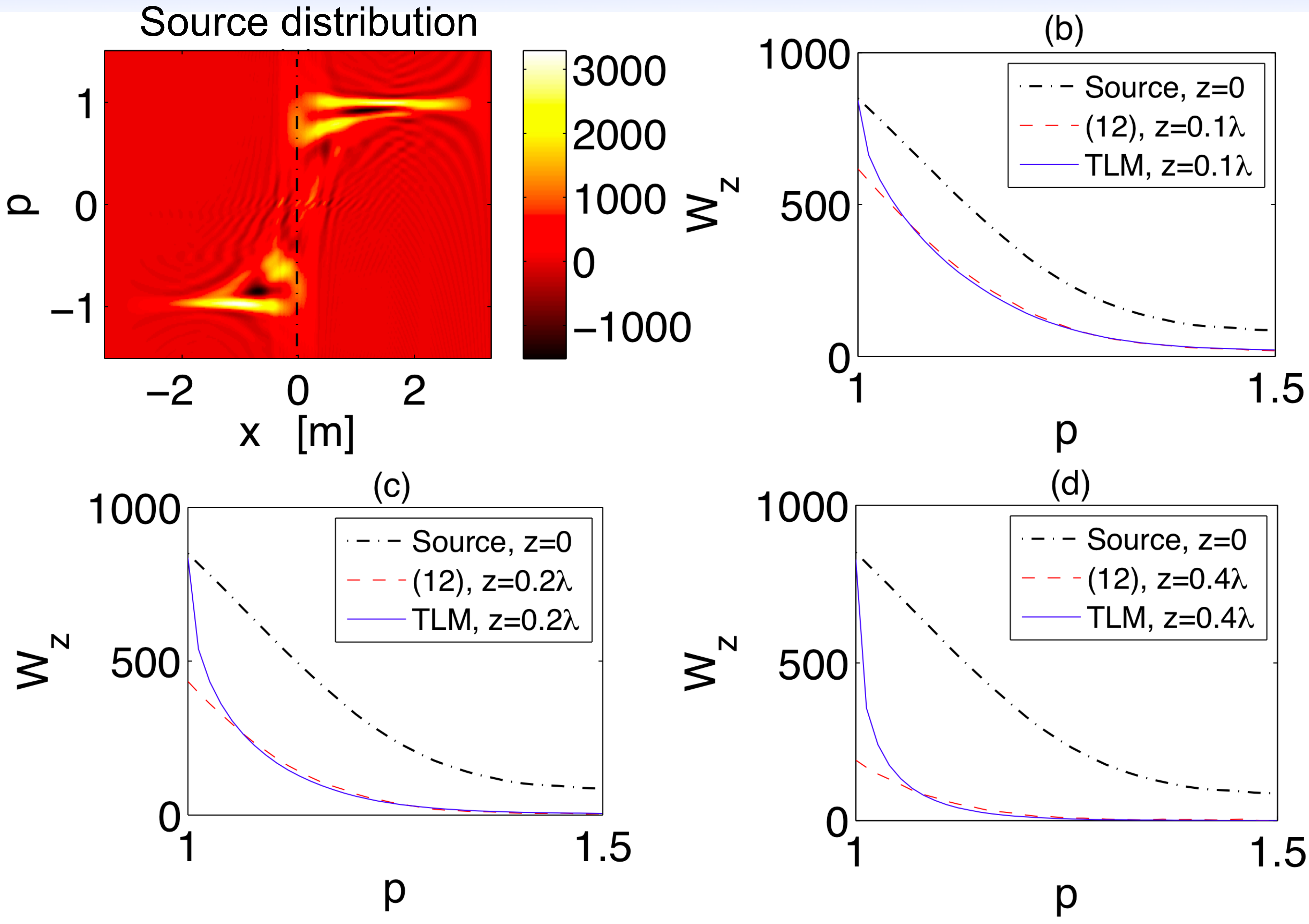

Propagation of realistic signal – cable bundle driven by random voltage – <u>far field</u>

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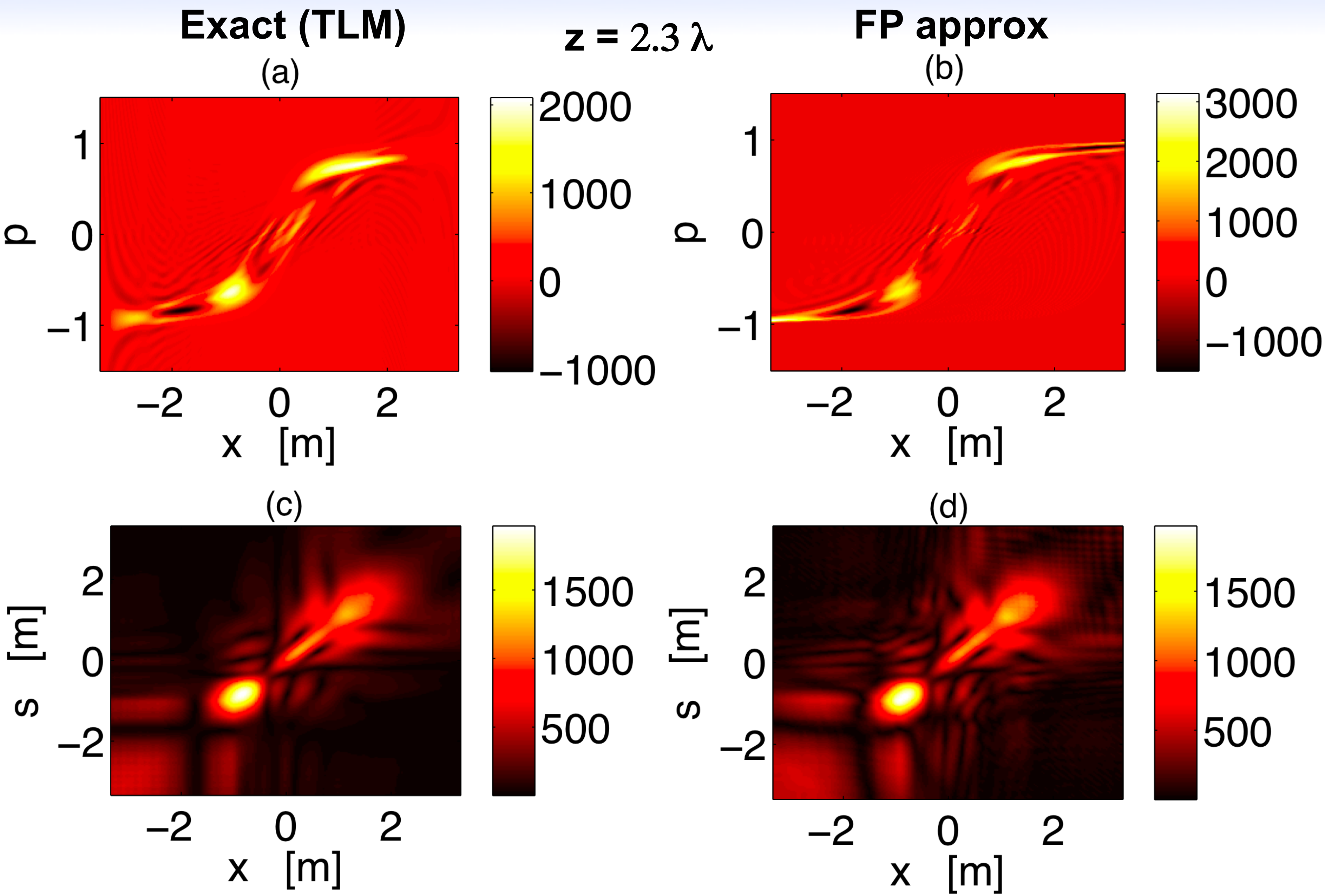

Propagation of correlation functions including multiple reflection – a semiclassical approach

Propagation including multiple reflection – a semiclassical approach

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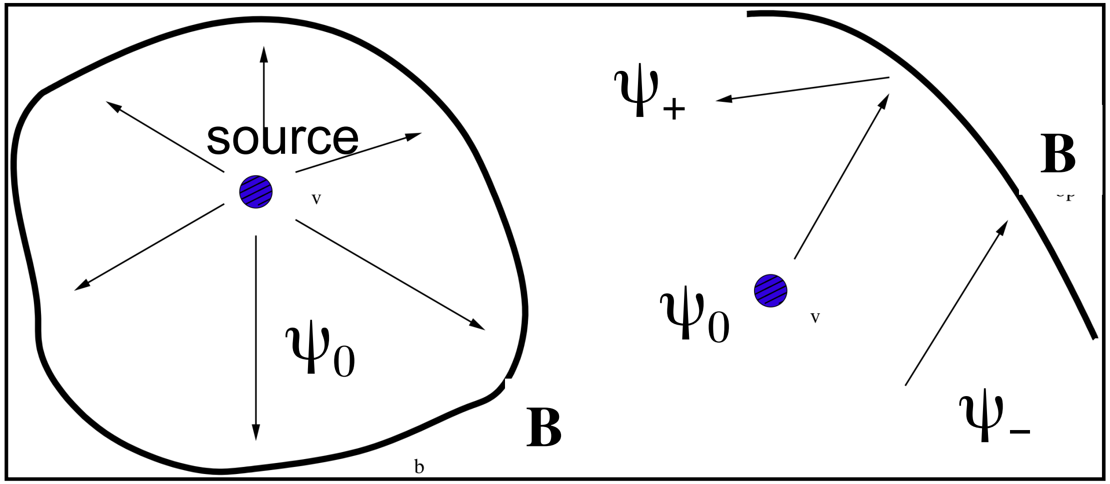
Can we propagate correlation function including multiple-reflections – open or closed?



Consider transfer operator method:

$$\boxed{(1 - \mathrm{T})\,|\psi_-\rangle = |\psi_0\rangle}$$

$\psi_{+/-}$: outgoing/incoming wave on boundary

T: Transfer operator – exact *Prozen, Smilansky, Creagh et al 2013*

– semiclassical *Bogomolny, Smilansky*

Propagation including multiple reflection – a semiclassical approach

Now rewrite

$$
\begin{aligned}
\Gamma &= |\psi_-\rangle\langle\psi_-| \\
&= (\mathrm{I}-\mathrm{T})^{-1}\overbrace{|\psi_0\rangle\langle\psi_0|}^{\Gamma_0}(\mathrm{I}-\mathrm{T})^{-1,\dagger} \\
&= \sum_{n,m=0}^{\infty}\mathrm{T}^n\Gamma_0\mathrm{T}^{m,\dagger}
\end{aligned}
$$

After reordering terms, we obtain

$$
\Gamma = \mathrm{K} + \sum_{n=1}^{\infty}\left[\mathrm{T}^n\mathrm{K} + \mathrm{K}\mathrm{T}^{n,\dagger}\right]
$$

with

$$
\mathrm{K} = \sum_{n=0}^{\infty}\mathrm{T}^n\Gamma_0\mathrm{T}^{n,\dagger}
$$

Propagation including multiple reflection – a semiclassical approach

What is $$\mathrm{K} = \sum_{n=0}^{\infty} \mathrm{T}^n \Gamma_0 \mathrm{T}^{n,\dagger} \quad ?$$

Set $$\mathrm{L}^n \Gamma_0 \equiv \mathrm{T}^n \Gamma_0 \mathrm{T}^{n,\dagger}$$

Using semiclassical expression (Bogomolny):

$$\mathrm{T}^n(x, x') = (2\pi i)^{(1-d)/2} \sum_{j:x \to x'}^{(n)} A_j e^{iS(x,x') - i\nu\frac{\pi}{2}}$$

Consider Wigner Transform: $(\mathrm{L}^n \Gamma_0)_W$

By evaluating the quadruple integral and the double sum over trajectories by stationary phase …

Propagation including multiple reflection – a semiclassical approach

In leading order in 1/k:

$$(\mathrm{L}^n\Gamma_0)_W \approx \mathcal{L}^n W_0$$

(… provided W_0 is **homogeneous** on the scale of 1/k).

W_0: Wigner transform of Γ_0

where

$$[\mathcal{L}^n W_0](X) = \int \delta(X - \varphi^n(X'))W_0(X')dX'$$

$X = (x,p)$

$\varphi(X)$: map

Frobenius – Perron operator for n-reflections

The Wigner Transform of $\mathrm{K} = \sum_{n=0}^{\infty} \mathrm{T}^n\Gamma_0\mathrm{T}^{n,\dagger}$ is then:

$$\boxed{\mathrm{K}_W \approx \sum_{n=0}^{\infty} \mathcal{L}^n W_0 = (1-\mathcal{L})^{-1}W_0 = \rho}$$

Stationary phase space density from source W_0 including reflections

Propagation including multiple reflection – a semiclassical approach

$$(1-\mathcal{L})^{-1} W_0 = \rho$$

can be computed using *Dynamical Energy Analysis* (DEA) method
Tanner 2009, Chappell et al 2013

Smooth part of correlation function Γ by inverse Wigner Transform:

$$K \approx K_{cl} = \mathrm{W}^{-1}[\rho]$$

Higher order oscillatory corrections may be obtained using

$$\Gamma \approx \mathrm{K}_{cl} + \sum_{n=1}^{\infty} \left[\mathrm{T}^n \mathrm{K}_{cl} + \mathrm{K}_{cl} \mathrm{T}^{n,\dagger} \right]$$

Propagation including multiple reflection – a semiclassical approach

Note:
under relatively general conditions (low or uniform absorption, ergodicity or ‘’uniformity’’ of initial ray density W_0 ,…):

$$\rho = const \Leftrightarrow K_{cl} = const \times \mathrm{I}$$

Thus

$$\Gamma \approx const \left(\mathrm{I} + \sum_{n=1}^{\infty} \left[\mathrm{T}^n + \mathrm{T}^{n,\dagger} \right] \right)$$

Equivalent to relation between Green’s fct and correlation fct:
Hortikar & Srednicki, Weaver, Richter & Urbina

Solving the classical flow equation using Frobenius Perron operatore –
Dynamical Energy Analysis

$$\left[\mathcal{L}\rho_0\right](X) = \int \delta(X - \varphi(X'))\rho_0(X')dX'$$

Dynamical Energy Analysis - DEA:

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Idea: Propagation of ray densities in phases space (position + direction variable) along rays
→ linear map

Pros:
- Linear systems of equations;
- only short trajectories;
- Flow equation – can be solved on meshes.

Cons:
- Doubling of number of variables
  → adequate choice of basis functions
- so far only for stationary processes

Summary:

• Wigner transformation → From propagating Correlation functions to the propagation of phase space densities.

• High-frequency limit leads to ray-tracing approximation.

• Perron-Frobenius operators transport correlations efficiently in phase-space – including reflections.

• Smooth part can be obtained from DEA approximation.

• Applications in electromagnetics, vibroacoustics and quantum mechanics

Future Work

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Modelling multiple antennas in confined domains – EM field description

Recent Future Emerging Technology grant (€ 3.4 Mio):

Noisy Electromagnetic Fields - A Technological Platform for Chip-to-Chip Communication

Partners:

University of Nottingham

University Nice Sophia Antipolis

Technical University of Munich

Institut Supérieur de l'Aeronautique & de l'Espace - Toulouse

IMST GmbH – Kamp-Lintfort

NXP Semiconductors - Toulouse

CST AG - Darmstadt

We are looking for a **3-year post-doc** in Nottingham – start date 1. Sept 2015


Thank you …