Transmission phase of a quantum dot and statistical fluctuations of partial-width amplitudes

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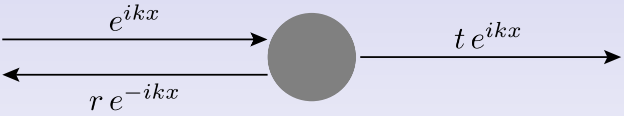
Luchon Superbagnères, March 2015

Transmission phase

Quantum scatterer connected to monochannel leads



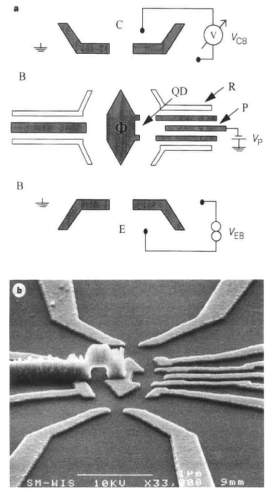
$$G = \frac{I}{V} = \frac{2e^2}{h}|t|^2$$

Transmission amplitude

$$t = |t|e^{i\alpha}$$

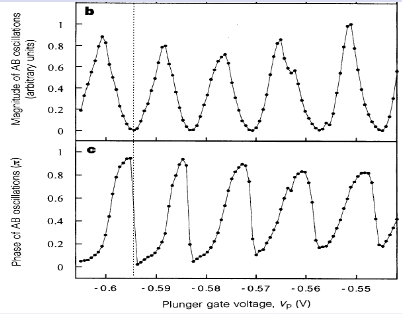
α transmission phase

AB interferometer containing a quantum dot



Schuster *et al.*, Nature '97

$\sim$ 200 electrons

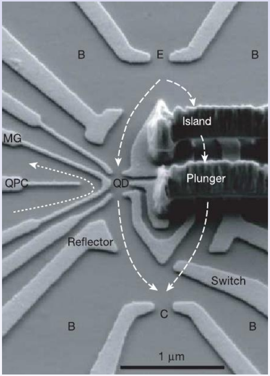


continuous phase evolution in resonances (Friedel sum rule) $\propto \delta\alpha$

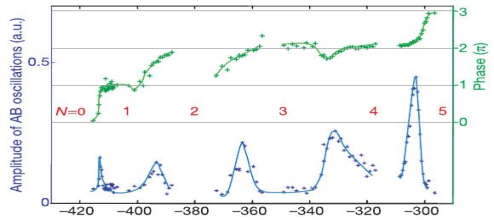
abrupt drops of π in valleys

subsequent peaks in phase

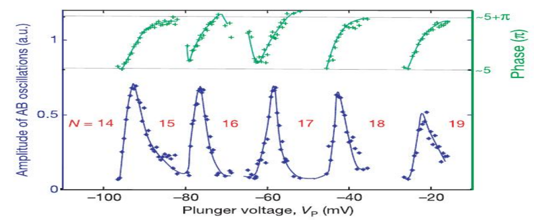
Crossover mesoscopic $\leftrightarrow$ universal



Avinum-Kalish *et al.*, Nature '05

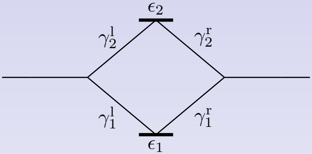


"Mesoscopic": $N \lesssim 10$; irregular phase evolution



"Universal": $N > 14$; subsequent peaks in phase

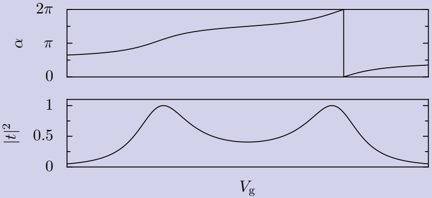
Between resonances: Two-level model



$$t(\epsilon) \sim \frac{\gamma_1^{\mathrm{l}}\gamma_1^{\mathrm{r}}}{\epsilon - \epsilon_1 + i\Gamma_1/2} + \frac{\gamma_2^{\mathrm{l}}\gamma_2^{\mathrm{r}}}{\epsilon - \epsilon_2 + i\Gamma_2/2}$$

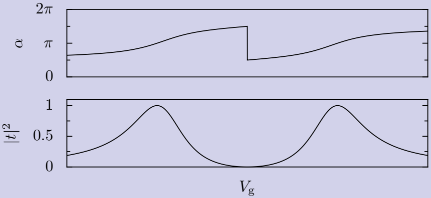
$$\Gamma_n = |\gamma_n^{\mathrm{l}}|^2 + |\gamma_n^{\mathrm{r}}|^2 \qquad \epsilon_n = \epsilon_n^{(0)} - V_{\mathrm{g}}$$

$\gamma_1^{\mathrm{l}}\gamma_1^{\mathrm{r}} = -\gamma_2^{\mathrm{l}}\gamma_2^{\mathrm{r}}$ opposite parity



$|t| > 0 \leftrightarrow$ smooth phase increase

$\gamma_1^{\mathrm{l}}\gamma_1^{\mathrm{r}} = \gamma_2^{\mathrm{l}}\gamma_2^{\mathrm{r}}$ same parity

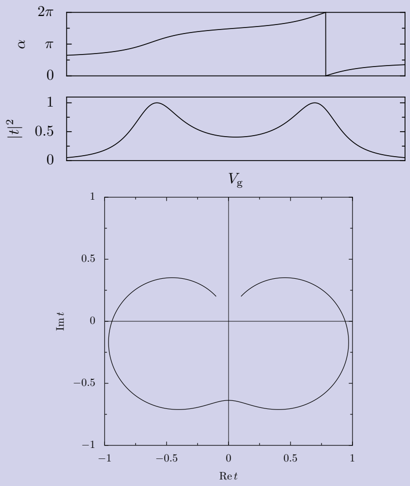


$|t| = 0 \leftrightarrow$ phase jump

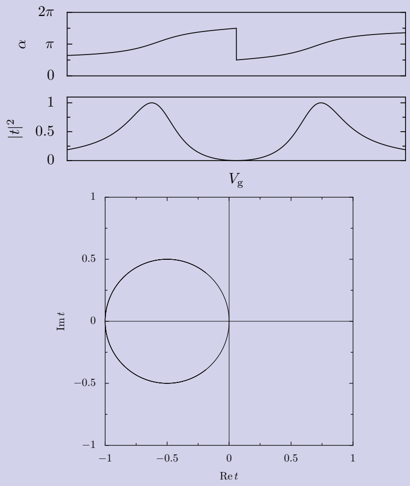
[Lee PRL ’99; Taniguchi & Büttiker PRB ’99, Levy-Yeyati & Büttiker PRB ’00, Aharony *et al.* PRB ’02]

Continuous evolution of t

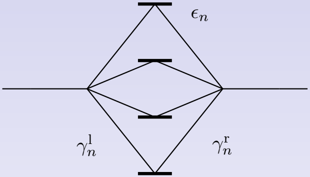
$\gamma_1^l \gamma_1^r = -\gamma_2^l \gamma_2^r$ opposite parity



$\gamma_1^l \gamma_1^r = \gamma_2^l \gamma_2^r$ same parity



Parity rule



$$D_n = \gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}}\gamma_{n+1}^{\mathrm{l}}\gamma_{n+1}^{\mathrm{r}}$$

$$\mathrm{sgn}(\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}}) = \pm\mathrm{sgn}(\gamma_{n+1}^{\mathrm{l}}\gamma_{n+1}^{\mathrm{r}}) \rightarrow D_n \gtrless 0$$

$D_n < 0$
$\rightarrow$ no transmission zero
no phase lapse

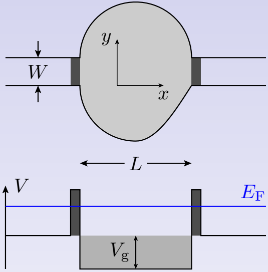
$D_n > 0$
$\rightarrow |t| = 0$
phase lapse

[Lee PRL '99; Taniguchi & Büttiker PRB '99, Levy-Yeyati & Büttiker PRB '00, Aharony *et al.* PRB '02]

Disordered dots: $\mathcal{P}(D_n < 0) = 1/2$ $\rightsquigarrow$ irregular phase evolution

Experiment: $\mathcal{P}(D_n < 0) = 0$ $\rightsquigarrow$ correlations between γ_n and γ_{n+1}?

Wave-function correlations in chaotic dots



$$\gamma_n^{\mathrm{l(r)}} \propto \int_0^W \mathrm{d}y\, \Phi_{\mathrm{l(r)}}(y)\, \psi_n(x^{\mathrm{l(r)}}, y) \sim \psi_n(x^{\mathrm{l(r)}}, 0)$$

$$\begin{aligned} D_n &= \gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}}\gamma_{n+1}^{\mathrm{l}}\gamma_{n+1}^{\mathrm{r}} \\ &= \psi_n(x^{\mathrm{l}},0)\psi_n(x^{\mathrm{r}},0)\psi_{n+1}(x^{\mathrm{l}},0)\psi_{n+1}(x^{\mathrm{r}},0) \end{aligned}$$

Random wave model: [M.V. Berry, J. Phys. A ’77]

$$\psi_n^{\mathrm{RWM}}(\mathbf{r}) = \frac{1}{\mathcal{N}} \sum_{j=1}^{\mathcal{N}} \cos[\mathbf{k}_j \mathbf{r} + \delta_j]$$

random δ_j $\qquad E_n = \frac{\hbar^2 k_n^2}{2m} \rightsquigarrow$ randomly oriented $\mathbf{k}_j$ with $|\mathbf{k}_j| = k_n$

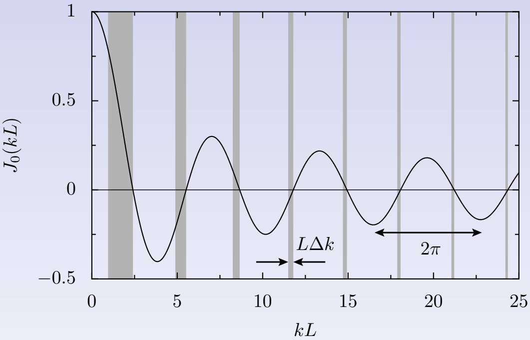
Correlations over a distance $L = x^{\mathrm{r}} - x^{\mathrm{l}}$

$$\langle \psi_n(\mathbf{r})\psi_n(\mathbf{r}')\rangle \simeq \frac{1}{\mathcal{A}} J_0(k|\mathbf{r} - \mathbf{r}'|) \qquad \rightsquigarrow \langle D_n \rangle \sim J_0(k_n L) J_0(k_{n+1} L)$$

Universal behavior in the semiclassical limit

$$\langle D_n \rangle \sim J_0(k_n L) J_0(k_{n+1} L)$$

$$\Delta k L = k_{n+1} L - k_n L \simeq \frac{\pi}{kL}$$



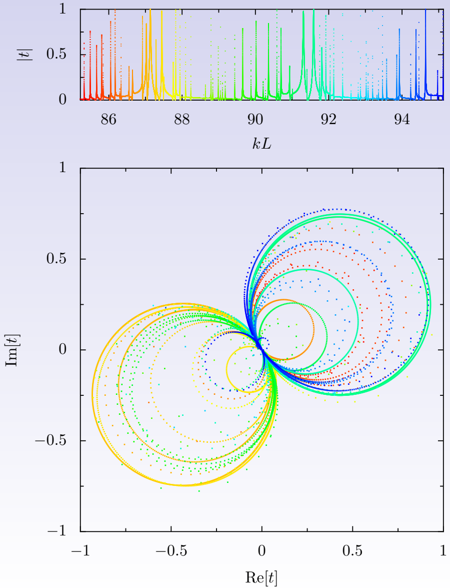
Average-based probability of having no phase lapse ($\langle D_n \rangle < 0$)

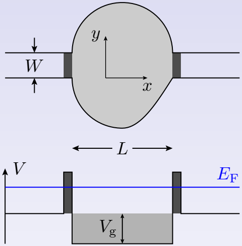
$$\mathcal{P}(\langle D_n \rangle < 0) \sim \frac{1}{kL}$$

Tendency towards the universal regime at large kL

R.A. Molina, R.A. Jalabert, DW, Ph. Jacquod, PRL **108**, 076803 (2012)

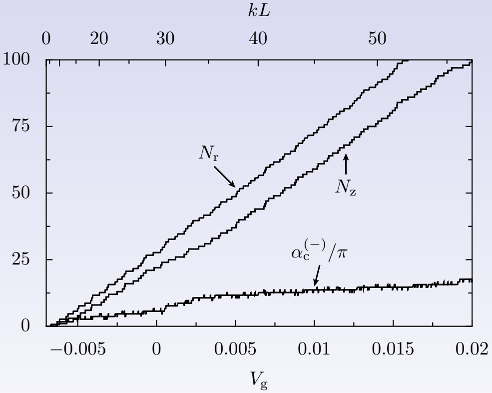
Numerics for the transmission amplitude



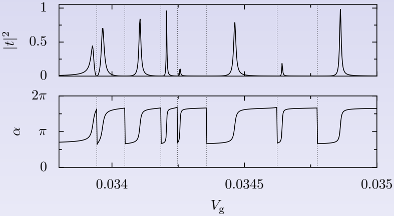


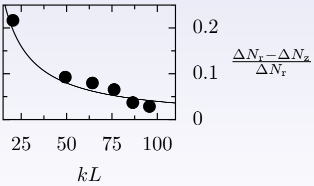
circles ↔ Breit-Wigner peaks

Counting zeros and resonances



some long universal sequences





“universal” regime at very large *kL*

Are averaged correlations sufficient?

Conclusions were drawn from the sign of the average

$$\langle D_n \rangle = \langle \gamma_n^{\mathrm{l}} \gamma_n^{\mathrm{r}} \gamma_{n+1}^{\mathrm{l}} \gamma_{n+1}^{\mathrm{r}} \rangle$$

and assuming narrow leads

$$\gamma_n^{\mathrm{l/r}} \sim \psi_n(x^{\mathrm{l/r}}, 0)$$

Questions:

- What happens in the case of wider leads?
- Do statistical fluctuations of the $\gamma_n^{\mathrm{l/r}}$ change the results?

Gaussian fluctuations of the partial-width amplitudes

Dots with chaotic classical dynamics $\rightsquigarrow$ Gaussian distribution of ψ_n

[Voros '76, Berry '77, Srednicki '96]

$$\gamma_n^{\mathrm{l(r)}} \propto \int_0^W \mathrm{d}y\, \Phi_{\mathrm{l(r)}}(y)\, \psi_n(x^{\mathrm{l(r)}}, y)$$

$\rightsquigarrow$ Gaussian distribution of the PWAs $\gamma_n^{l/r}$ with joint density

$$p(\gamma_n^{\mathrm{l}}, \gamma_n^{\mathrm{r}}) = \frac{1}{2\pi\sigma_n^2\sqrt{1-\rho_n^2}} \exp\left(-\frac{(\gamma_n^{\mathrm{l}})^2+(\gamma_n^{\mathrm{r}})^2-2\rho_n\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}}}{2\sigma_n^2(1-\rho_n^2)}\right)$$

Variance and correlator with LR symmetry:

$$\sigma_n^2 = \langle\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{l}}\rangle = \langle\gamma_n^{\mathrm{r}}\gamma_n^{\mathrm{r}}\rangle \qquad \rho_n = \frac{1}{\sigma_n^2}\langle\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}}\rangle$$

$\rightsquigarrow$ Probability for positive parity

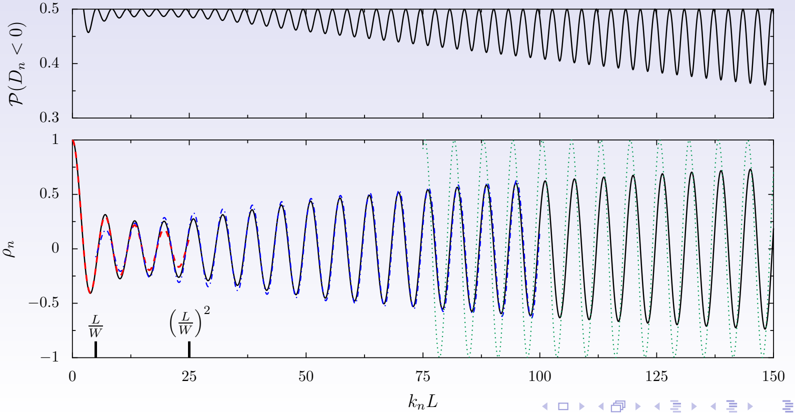
$$\mathcal{P}(\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}} > 0) = \frac{1}{2} + \frac{1}{\pi}\arcsin(\rho_n)$$

Probability for no phase lapse

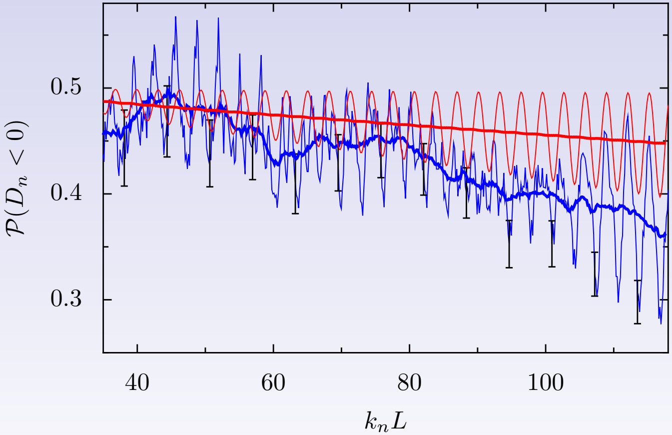
$$\mathcal{P}(D_n < 0) = \mathcal{P}(\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}} > 0)\left[1 - \mathcal{P}(\gamma_{n+1}^{\mathrm{l}}\gamma_{n+1}^{\mathrm{r}} > 0)\right] + \mathcal{P}(\gamma_{n+1}^{\mathrm{l}}\gamma_{n+1}^{\mathrm{r}} > 0)\left[1 - \mathcal{P}(\gamma_n^{\mathrm{l}}\gamma_n^{\mathrm{r}} > 0)\right]$$

With mean wave-number spacing $\Delta k_n = \pi / k_n L^2$:

$$\mathcal{P}(D_n < 0) \simeq 2f(k_n) + \Delta k_n f'(k_n) \quad \text{with} \quad f(k) = \frac{1}{4} - \frac{1}{\pi^2}\arcsin^2\left(\rho(k)\right)$$



Numerics for $\mathcal{P}(D_n < 0)$ averaging over 14 cavities



Blue: $\mathcal{P}(D_n < 0)$; smoothing k_n interval of δ/L with $\delta = \pi/4$ and π

Red: from the statistical model; same smoothing

Conclusions

- Wave-function correlations: probability for non-universal evolution $\sim 1/kL$ in chaotic dots
  $\rightsquigarrow$ Tendency towards universal behavior at large N
- Gaussian fluctuations of partial-width amplitudes
  $\rightsquigarrow$ Reduced tendency towards universal behavior
- Numerics for ballistic cavities
  $\rightsquigarrow$ Intermediate tendency towards universal behavior

Outlook: More realistic dot models? Beyond Gaussian fluctuations?
Correlations $n \leftrightarrow n+1$?

R.A. Molina *et al.*, PRL **108**, 076803 (2012); PRB **88**, 045419 (2013)
R.A. Jalabert *et al.*, PRE **89**, 052911 (2014)