Systems with off-diagonal disorder on a lattice -2

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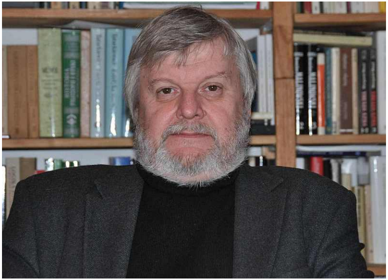
Luchon, March 2015

What is to come

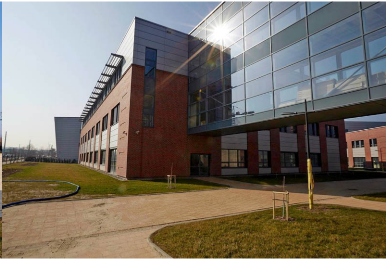
- Supplement to Karol Zyczkowski talk
- Optical lattice and Bose-Hubbard model
- BH with Gavish-Castin disorder and density dependent tunnelings
- Periodically modulated interactions
- ?

Supplement to Karol's talk

Marek Kuś 60th birthday
Organized by Centre for Theoretical Physics, together with Institute of Physics and Department of Physics, University of Warsaw
The Symposium will be held on 24-25 April in Warsaw.
Detailed info on the website:
http://www.cft.edu.pl/SymposiumMarek/

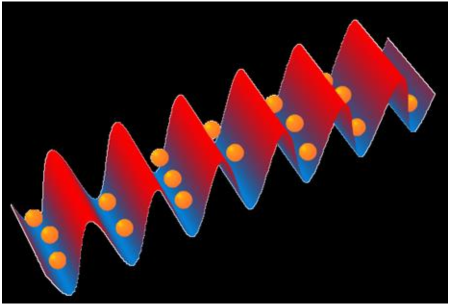


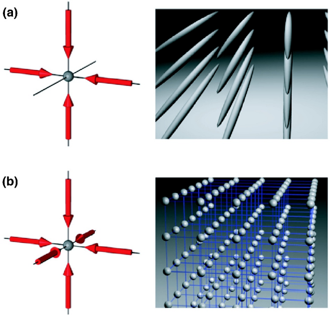
Supplement to Karol’s talk-2 - IF UJ



Optical lattice

$$V(x) = -\vec{d} \cdot \vec{E} = -\alpha |E(x)|^2 \propto \frac{I(x)}{\delta} \qquad V(x) = V_0 \cos^2(kx)$$

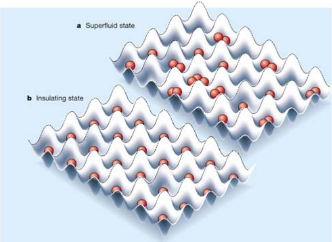


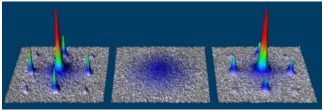


Bose-Hubbard model[1]

$$\hat{H} = -J\sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$

J - tuneling, hopping U - on-site interaction,





(after Greiner et al. (2002)

$J >> U$ -
superfluid, gapless

$J << U$
Mott insulator
gap $<= U$

[1] H.A. Gersch and G. C. Knollman, Phys. Rev. **129**, 959 (1963).

Typical Bose-Hubbard with disorder

$$\hat{H} = -J\sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \sum_i \mu_i \hat{n}_i$$

with μ_i random

- uniform disorder Fisher et al 1989, Schulz-Giamarchi 1988 gapless insulator Bose glass phase (BG)
- BG separates MI from SF always (no direct MI-SF transition -“theorem of inclusions” - Pollet 2009)
- Gavish-Castin disorder $V\hat{n}_i\hat{M}_i$. M particles heavy and immobile $\rightarrow V\mu_i\hat{n}_i$ with binary μ_i.
- MI with non integer filling (due to impurities)
- MI survives for arbitrary disorder strength

Bose-Hubbard with off-diagonal binary disorder[2]

For diagonal disorder

$$\hat{H} = -J\sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \sum_i (\mu - \gamma\omega_i)\hat{n}_i$$

But interaction induced tunnelings yield

$$\hat{H}_2 = -J\sum_{\langle i,j\rangle} \left[1 + \alpha(\omega_i + \omega_j)\right] \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \sum_i (\mu - \gamma\omega_i)\hat{n}_i$$

Origin: two-body interactions. Typically $|\gamma| > \alpha$.

Off diagonal disorder

- Mean field approach 1D, 2D: local mean field $\psi_i = \langle \hat{b}_i \rangle$

[2] J. Stasinska et al Phys. Rev. A 2014

Bose-Hubbard with off-diagonal binary disorder-2

Denoting $\bar{J} := J/U$ etc. standard perturbative in $\bar{J}$ approach:

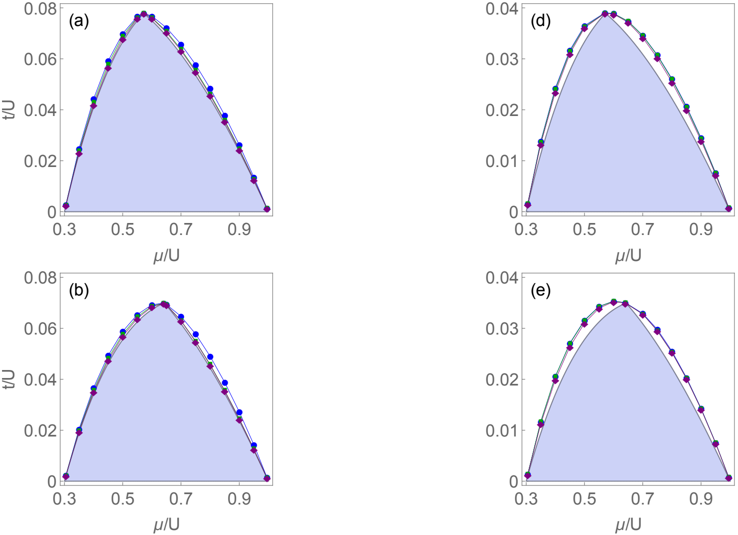
$$\psi_i = \sum_{\langle j \rangle_i} \bar{J} \mathcal{R}_{ij} \psi_j,$$

with random matrix

$$\mathcal{R}_{ij} = [1 + \alpha(\omega_i + \omega_j)] \left(\frac{\bar{n}_i + 1}{\bar{n}_i - \bar{\mu} + \bar{\gamma}\omega_i} - \frac{\bar{n}_i}{(\bar{n}_i - 1) - \bar{\mu} + \bar{\gamma}\omega_i} \right),$$

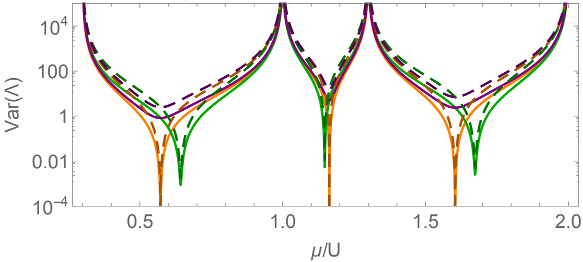
for $\bar{n}_i - (1 - \bar{\gamma}\omega_i) < \bar{\mu} < \bar{n}_i + \bar{\gamma}\omega_i$.
If $\det(\bar{J}\mathcal{R} - \mathbf{1}) \neq 0$ then $\psi_i = 0$ MI border: $\bar{J} \max[\lambda(\mathcal{R})] = 1$. In thermodynamic limit MI border from assuming all ω_i identical (Mering-Fleishhauer). In 1D spectrum analytic - Toeplitz matrix $2\bar{J} \max \left[\sqrt{X_i^+ X_i^-} \right] = 1$ with $X_i^{\pm} := \mathcal{R}_{i,i\pm1}$

Bose-Hubbard with off-diagonal binary disorder-3



Bose-Hubbard with off-diagonal binary disorder-4

Variance of eigenvalues:



- for a given *p* exists a single μ for which the matrix non random – this givces a tip of MI lobe
- direct MI-SF transition not violating "'inclusion theorem" for diagonal binary disorder
- off-diagonal disorder - only via Bose glass phase, BG more prominient
- still BG regions small for moderate off-diagonal disorder
- Can we produce strong off-diagonal disorder?

Periodically modulated parameters

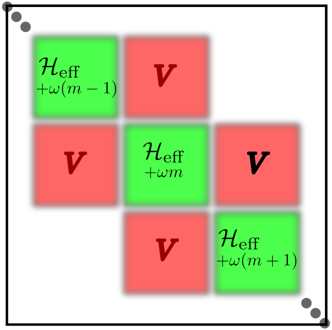
Gaston Floquet theorem (1883). In modern formulation similar to (younger) Bloch theorem. Consider Hamiltonian with periodic time dependence: $\mathcal{H}(t) = \mathcal{H}(t+T)$. Stationary states

$$|\psi_i(t)\rangle = \sum_i e^{-ie_i t}|u_i(t)\rangle \tag{1}$$

Where e_i are called quasienergies and $|u_i(t)\rangle = |u_i(t+T)\rangle$. $|u_i\rangle\rangle$ are eigenstates to eigenenergy e_i in extended phase space for $H - \partial_t$. The corresponding scalar product involves integration over the period.

Effective Hamiltonian

Hamiltonian in the extended space has a block structure:



- $\mathcal{H}_{eff}$ is time-averaged Hamiltonian in F. basis
- V for big ω is negligible
- As blocks differ only by energy shift we can consider one only.

Side remark: With resonant coupling of Bloch bands:

A. Przysiezna, O. Dutta, and JZ, Rice-Mele model with topological solitons in an optical lattice New J. Phys. **17**, 013018(2015)

O. Dutta, A. Przysiezna, and JZ, Spontaneous magnetization and anomalous Hall effect in an emergent Dice lattice arXiv:1405.2565

Bose-Hubbard model with time-modulated energies

Bose-Hubbard Hamiltonian (no interactions) first:

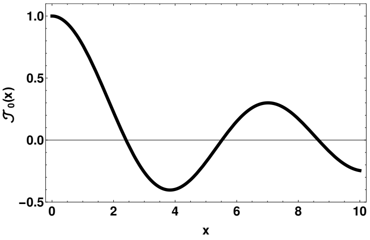
$$H = \sum_i \varepsilon_i(t) n_i - J(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i) \tag{2}$$

On-site energies are time dependent: $\varepsilon_i(t) = \epsilon_i(1 + \delta \sin(\omega t))$.
If $\omega \gg J$ we can find effective time-independent Hamiltonianian:

$$\mathcal{H}_{eff} = \sum_i \epsilon_i n_i - J_i^{eff}(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i) \tag{3}$$

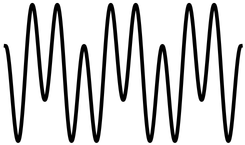
Where:

$$J_i^{eff} = J \mathcal{J}_0 \left(\frac{\delta}{\omega} (\epsilon_{i+1} - \epsilon_i) \right)$$



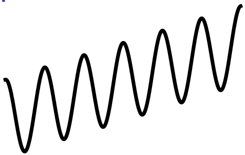
When it is interesting?

Superlattice



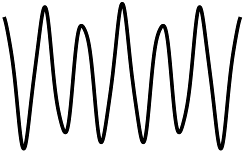
Change of tunneling amplitude between types od sites. (Morais Smith)

External potential



”Classical” shaken lattice is effectively of this type. (Eckardt, Weiss, Holthaus)

Disorder

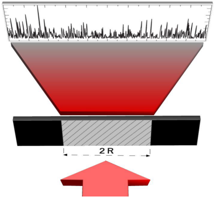


Creation of off-diagonal disorder

Disorder in optical lattice

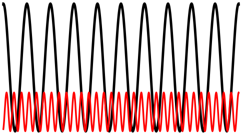
Everything can be simulated with cold atoms
– how can we create a disordered potential?

Speckle potential



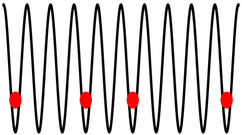
- Controllable
- Big setup

Incommensurate lat.



- Easy to create
- Not really a disorder! Aubry-André not Anderson

Frozen particles

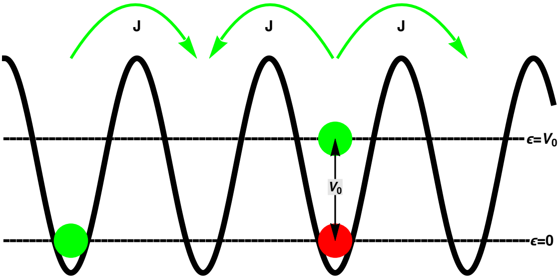


- uncontrollable (or by Y. Castin only)
- Binary disorder

Binary disorder

Recipe as before:

- Put some fermions/hardcore bosons into a lattice
- Let them evolve
- ”Freeze” them to create disorder pattern
- Put another atoms interacting with the *frozen* ones



On-site energies:

$$\epsilon_i = V n_i^f$$

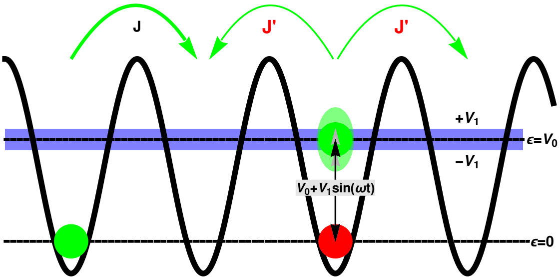
For $n_i^f \in \{0, 1\}$
$\Rightarrow \epsilon_i \in \{0, V\}$

Periodically modulated (interspecies) interaction[3]

If we modulate periodically the interspecies interaction strength:

$$V \rightarrow V_0 + V_1 \sin(\omega t), \tag{4}$$

effectively we modulate on-site energies.
$\Rightarrow$ We get renormalized tunneling (for $\omega \gg J$).



[3] A. Kosior et al. arXiv 1502.02453

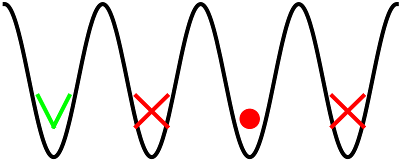
Effective time independent Hamiltonian

Effective Hamiltonian:

$$\mathcal{H}_{eff} = \sum_i (V_0 n_i^f) n_i - J\mathcal{J}_0 \left(\frac{V_1(n_{i+1}^f - n_i^f)}{\omega} \right) (b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i)$$

$$\epsilon_i = \begin{cases} 0, & \text{if } n_i^f = 0 \\ V_0, & \text{if } n_i^f = 1 \end{cases}, \quad J_i^{eff} = \begin{cases} J, & \text{if } n_i^f = n_{i+1}^f \\ J' = J\mathcal{J}_0(V_1/\omega), & \text{if } n_i^f \neq n_{i+1}^f \end{cases}$$

For uncorrelated disorder states localization at all energies (1D). So DRDM (Dual Random Dimer Model) – no two adjacent sites can be occupied by frozen particles



Delocalized resonance modes (Schaff, Akdeniz, Vignolo 2010)

Approximation of the localization length

$$J_i\psi_{i+1} + J_{i-1}\psi_{i-1} + (\epsilon_i - E)\psi_i = 0.$$

Let: $\psi_i = \phi_i\eta_i$, where $\eta_i = 1/(J_i\eta_{i-1})$, then:

$$\phi_{i+1} + \phi_{i-1} + \tilde{V}_i\phi_i = 0,$$

where $\tilde{V}_i = |\eta_i|^2(\epsilon_i + E)$ is a new effective diagonal disorder. For DRDM: $\eta_i = 1/J_i^{eff}$. Mapping onto kicked oscillator[4] yields perturbative localization length

$$\begin{aligned} \lambda^{-1} &= \frac{\rho}{(1+\rho)^2}\frac{(V_0 + 2(1-J'^2)\cos(k))^2}{8\sin^2(k)} \\ &\times \left(1 - 2\frac{\rho(\rho+\cos(2k))}{1+\rho^2+2\rho\cos(2k)}\right), \end{aligned}$$

where $\rho = \tilde{\rho}/(1-\tilde{\rho})$ and $\tilde{\rho}$ – the density of frozen particles.

[4] L. Tessieri and F. Izrailev Physica E**9**, 405 (2001).

Transparent modes

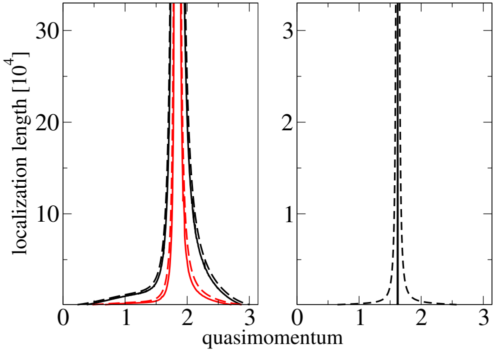
Delocalized mode exists for:

$$\cos(k_t) = \frac{V_0}{2(1-J')^2}$$

if condition:

$$V_0 < 2(1-J')^2$$

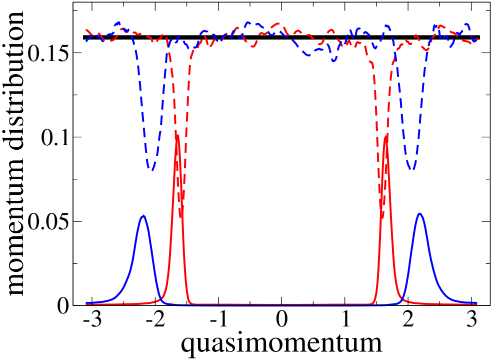
is fulfilled.



Localization length. Left panel: $\{V_0 = 0.05, J' = 0.95\}$ (black) and $\{V_0 = 0.1, J' = 0.9\}$ (red). Right panel: $\{V_0 = 0.1, J' = 0.1\}$

Band pass filter?

Transparent mode position $\cos(k_t) = \frac{V_0}{2(1-J'(V_1,\omega))^2}$ can be chosen by changing interaction (V_0, V_1) or modulation frequency ω.

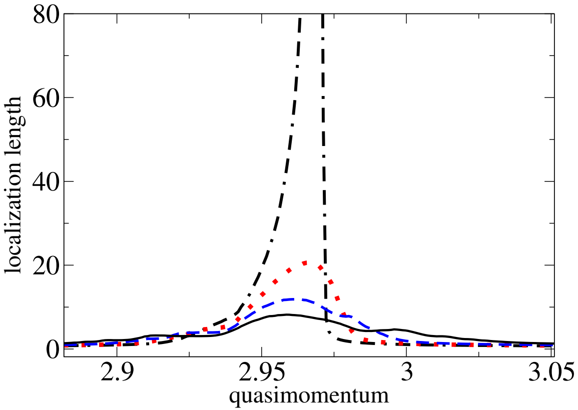


Only particles with chosen quasimomentum leave the disordered region.

Long-range hoppings

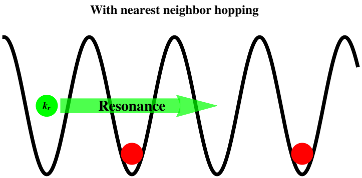
What if we add next-nearest neighbor hoppings to our Hamiltonian?

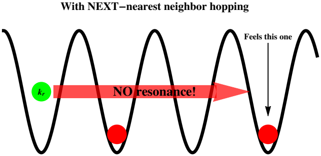
$$\mathcal{H}_{eff} = \sum_i \epsilon_i n_i - J_i^{eff}(b_i^\dagger b_{i+1} + h.c.) + \bar{J}_i^{eff}(b_i^\dagger b_{i+2} + h.c.)$$



Solid line for $\bar{J} = 0.01J$.

Mechanism





If particle resonantly passes one obstacle it will pass all of them.

If *frozen* particles are separated by at least two sites, resonance reappears.

Summary

- Off-diagonal disorder may be interesting
- It may be controlled using periodically modified interactions
- Future plans: Realization of random tunneling phases – random gauge fields

Recent review on Hubbard: O. Dutta et al. arXiv:1406.0181, Rep. Prog. Phys. in press

Financed: MAESTRO ST-2 DEC-2012/04/A/ST2/00088
http://chaos.if.uj.edu.pl/AOD/maestro/

NARODOWE CENTRUM NAUKI

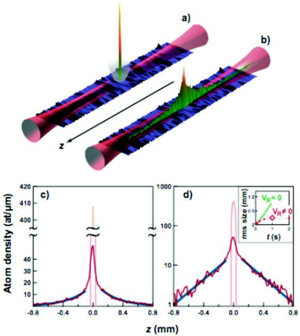
Many-body Anderson localization in 1D

D. Delande, C. Mueller, K. Sacha, M. Płodzień, S. Avazbaev, JZ

Effects for BEC with interactions

- Effective one body (EOB) approach: Phys. Rev. Lett. **103** 210402 (2009)
- Full many body solution: New J. Phys. **15** 045021 (2013)

$$\left[-\frac{1}{2}\partial_z^2 + V(z) + g|\phi(z)|^2\right]\phi(z) = \mu\,\phi(z), \qquad \langle\phi|\phi\rangle = N$$



Experiments in the non-interacting limit...

J. Billy et al., Nature **453**, 891 (2008)

G. Roati et al., Nature **453**, 895 (2008)

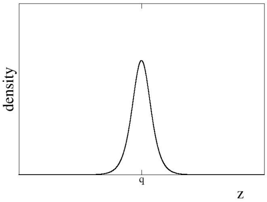
Bright solitons in a BEC

Gross-Pitaevskii equation:

$$\left[-\frac{1}{2}\partial_z^2 + g|\phi_0|^2\right]\phi_0 = \mu\phi_0, \qquad g < 0 \qquad \langle\phi|\phi\rangle = N$$

$$\phi_0(z-q) = \sqrt{\frac{N}{2\xi}}\,\frac{\exp(-i\theta)}{\cosh\left(\frac{z-q}{\xi}\right)}, \tag{5}$$

$$\xi = \frac{-2}{Ng} \qquad \mu = -\frac{N^2 g^2}{8} \tag{6}$$



Yet full exact many-body solution - uniform

Bright solitons in a BEC

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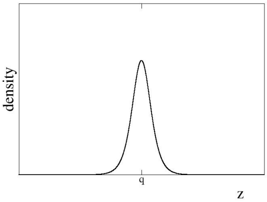


Yet full exact many-body solution - uniform
The position of the center of mass should be treated quantum mechanically.
q - position operator for N particle soliton.

Bright soliton in a BEC

The effective Hamiltonian

If we add **weak disorder** potential $V(z)$, the effective Hamiltonian describing center of mass motion reads:

$$\hat{H}_{\mathrm{eff}} \approx \frac{p^2}{2N} + \int dz\ V(z)\ |\phi_0(z-q)|^2 \,.$$

- Simple arguments:

Substitution of a time-dependent bright soliton solution $\sim e^{ipz}\phi_0(z-q)$ to the energy functional

$$\begin{aligned} E &= \int dz \left[\frac{1}{2}|\partial_z\phi|^2 + \frac{g}{2}|\phi|^4 - \mu|\phi|^2 + V(z)|\phi|^2\right] \\ &= \frac{p^2}{2N} + \int dz V|\phi_0(z-q)|^2 \end{aligned}$$

Bright solitons in a BEC

The effective Hamiltonian

- Bogoliubov theory:
  (J. Dziarmaga, (2004) for $V = 0$)

$$\hat{H} \approx \sum_{n, E_n > 0} E_n \, \hat{b}_n^\dagger \, \hat{b}_n + \frac{\hat{p}^2}{2N} + \int dz \; V(z) \; |\phi_0(z - q)|^2 .$$

$$N \; V_0 \;\ll\; E_1 = |\mu| = \frac{N^2 g^2}{8},$$

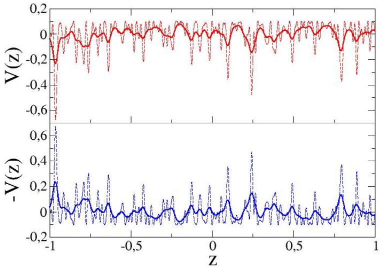
Weak perturbation cannot populate internal excited states of the soliton.
Shape preserved

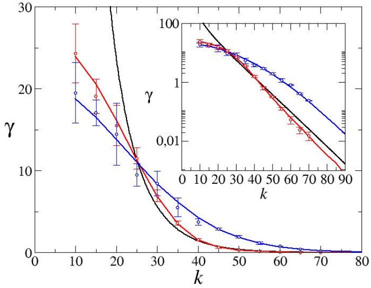
Bright solitons in a BEC

Anderson localization

- $V(z)$ is optical speckle potential with correlation length $\sigma_0 \ll \xi$.
- To the second order (Born approximation) in the potential strength V_0, inverse localization length, valid for $\gamma(k) \ll k$,

$$\gamma(k) \approx \frac{N^2}{k^2}\pi\sigma_0 V_0^2 \left(\frac{N\pi k\xi}{\sinh(\pi k\xi)}\right)^2$$





$NV_0 = 10^{-2}|\mu|$, $\sigma_0 = 0.3\xi$, $N = 100$

Many-body approach to Anderson localization
Motivation

Any many body effect omitted in the former effective CM quantization approach could destroy the phase coherence and the wavefunction.
Full many body test required

$$\hat{H} = \int dz\, \hat{\psi}^\dagger(z) \left[-\frac{1}{2} \frac{\partial^2}{\partial z^2} + V(z) \right] \hat{\psi}(z) + \frac{g}{2} \int dz\, \hat{\psi}^\dagger(z)\, \hat{\psi}^\dagger(z)\, \hat{\psi}(z)\, \hat{\psi}(z)$$

Discretization (3-point kinetic energy) gives Bose-Hubbard Hamiltonian:

$$H = \sum_l \left[-J(a_l^\dagger a_{l+1} + h.c.) + \frac{U}{2} a_l^\dagger a_l^\dagger a_l a_l + V_l\, a_l^\dagger a_l \right]$$

$J = \frac{1}{2\delta^2}$, $U = \frac{g}{\delta}$ and $V_l = V(z_l)$, $z_l = l\delta$

B. Schmidt and M. Fleischhauer, Phys. Rev. A75 (2007)

Many-body approach to Anderson localization

Technicalities

- space restricted to $2K + 1 = 1921$ points $[-K\delta, K\delta]$
- Matrix product state (MPS) cvariational representation

$$|\psi\rangle = \sum_{\alpha_1,\ldots,\alpha_M;\ i_1,\ldots,i_M} \Gamma^{[1],i_1}_{1\alpha_1} \lambda^{[1]}_{\alpha_1} \Gamma^{[2],i_2}_{\alpha_1\alpha_2} \ldots \Gamma^{[M],i_M}_{\alpha_{n-1}1} |i_1, \ldots, i_M\rangle$$

  $\Gamma^{[l],i_l}$ - site dependent tensors, $\lambda^{[l]}$ - bond vectors
- We find a quasi-exact many body ground state – bright soliton in a shallow trap (imaginary time propagation TEBD)
- Trap is removed, disorder turned on, real time propagation with TEBD
- Reliable calculations for $N = 25$ particles

Many-body approach to Anderson localization

Technicalities

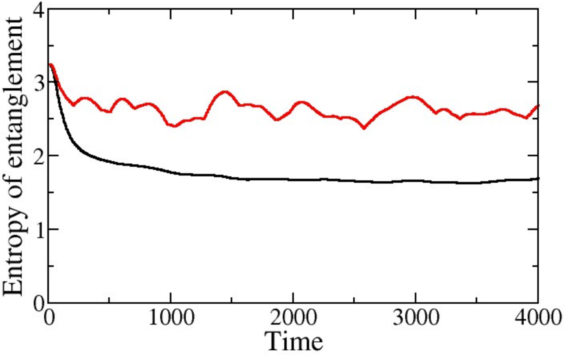
- Unit of length – soliton size ξ – Unit of time ξ^2
- Initial harmonic oscillator $\omega = 0.025/\xi^2$ (not to disturb the soliton shape yet to confine CM to a distance slightly larger than ξ)
- strength of the random potential comparable to soliton energy $\omega/4$ i.e. $V_0 = 2.5 \times 10^{-4}$
- correlation of the disorder $\sigma_0 = 0.4\xi$
- discretization $\delta = \xi/5$ tests on smaller...
- time step (Trotter errors!) $dt = 0.008\xi^2$
- $N_{max} = 14$ (needed!) despite $N\delta/2\xi = 2.5$ for $N = 25$
- $\chi = 30$ (small possible) dimension per site 450 (1921 sites)

Many-body approach to Anderson localization

Tests

- χ, N_{max}, dt, ...
- entropy of entanglement growth

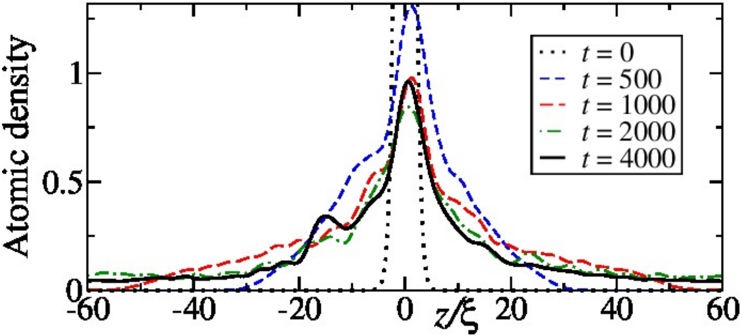
$$S = \sup_l S_l = \sup_l \left[-\sum (\lambda_\alpha^{[l]})^2 \ln(\lambda_\alpha^{[l]})^2 \right]$$



Many-body approach to Anderson localization

Results

- Atomic density in time

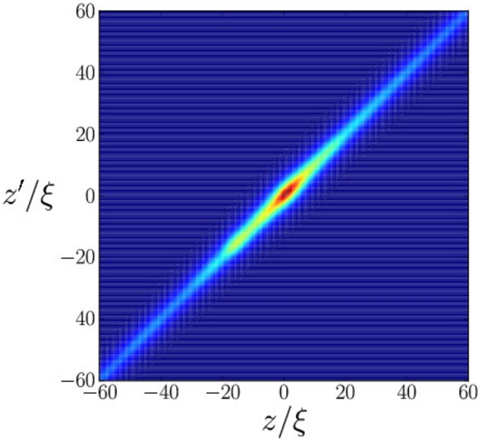


96 realizations of disorder

Many-body approach to Anderson localization

Results

- One body density matrix $\langle\psi^\dagger(z)\psi(z')\rangle$

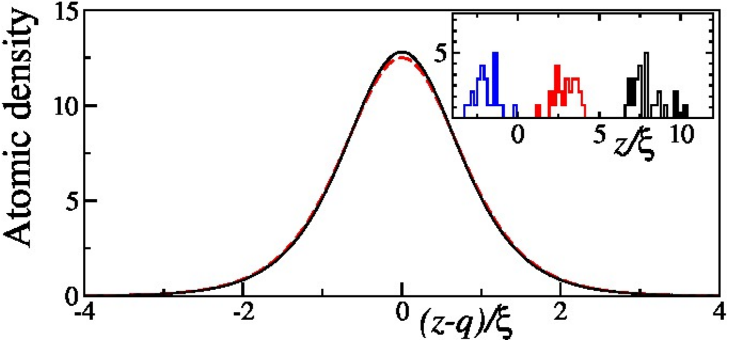


Transverse width $\approx \xi$. Largest eigenvalue = condensate fraction =0.14!

Many-body approach to Anderson localization

Simulation of the measurement

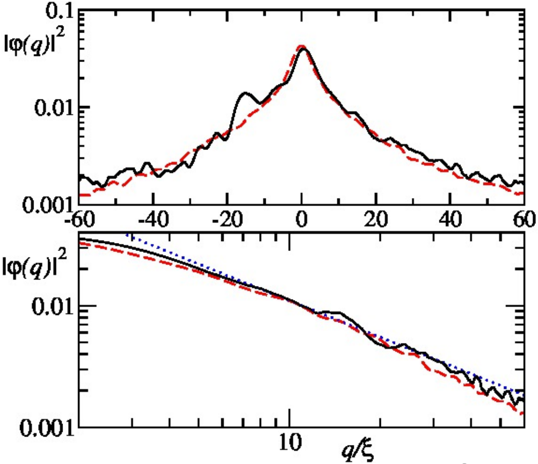
- From MPS repesentation $\rightarrow \rho^{[l]}$ by contraction of tensors..
- Choose n_l according to the statistical distribution
- Project MPS on subspace with that n_l on l site and normalize
- Repeat scanning other sites till reaching N



Many-body approach to Anderson localization

Comparison with effective one body of 2009

- Let us compare CM densities coming from both approaches



96 versus 10 000 realizations of disorder for EOB. Dotted 1/q.

Conclusions:

- AL for attractive interactions in 1D disorder
- Excellent agreement between full many body and EOB description
- Full simulation of the experiment including the measurements possible.