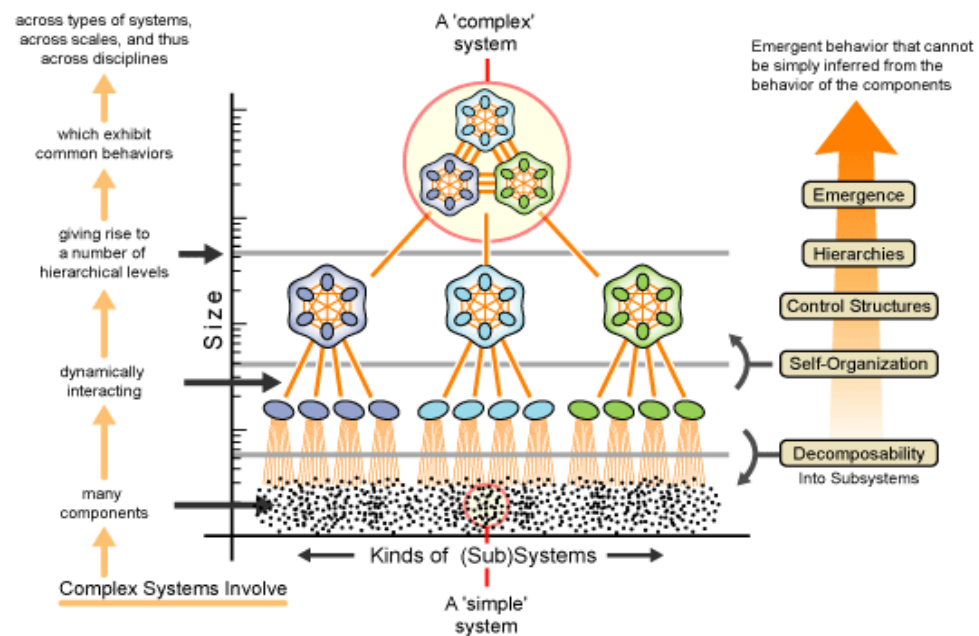


Network Inference

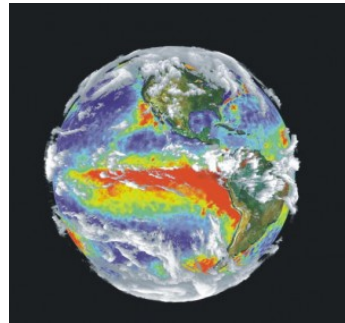
Ezequiel Bianco-Martinez
Dr. Murilo Baptista

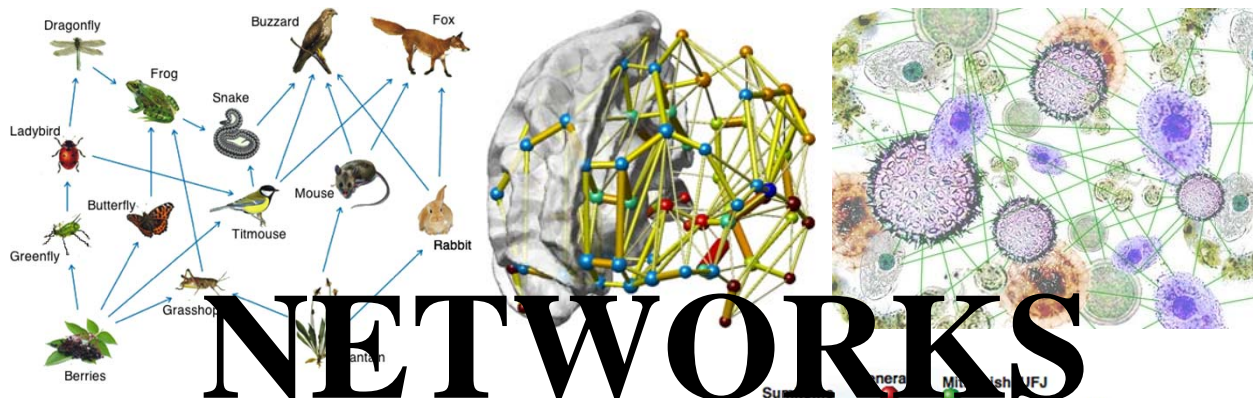


Complex Systems

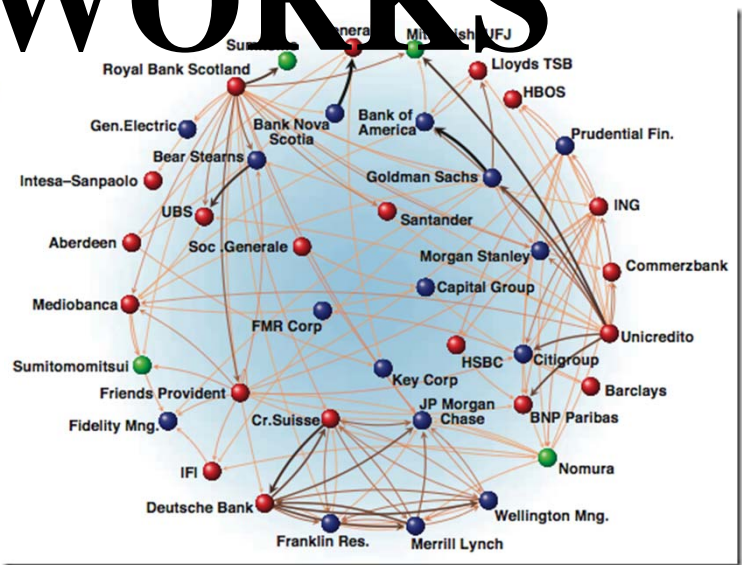


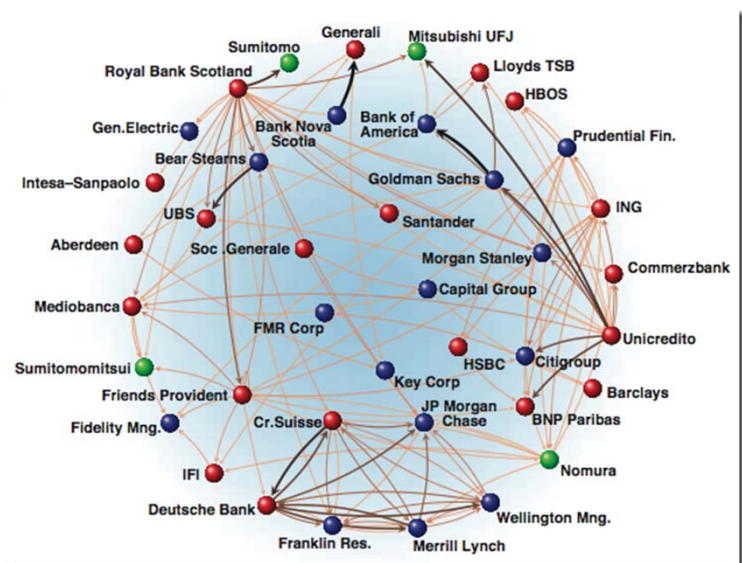
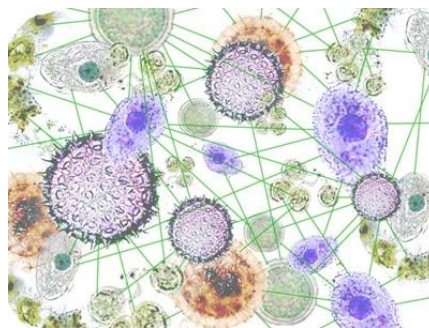
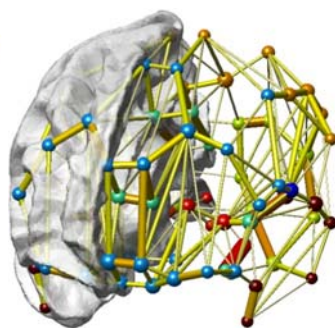
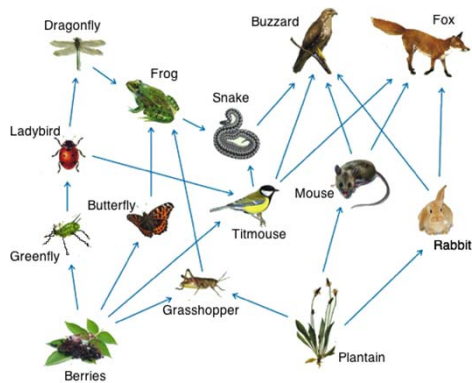
Complex Systems





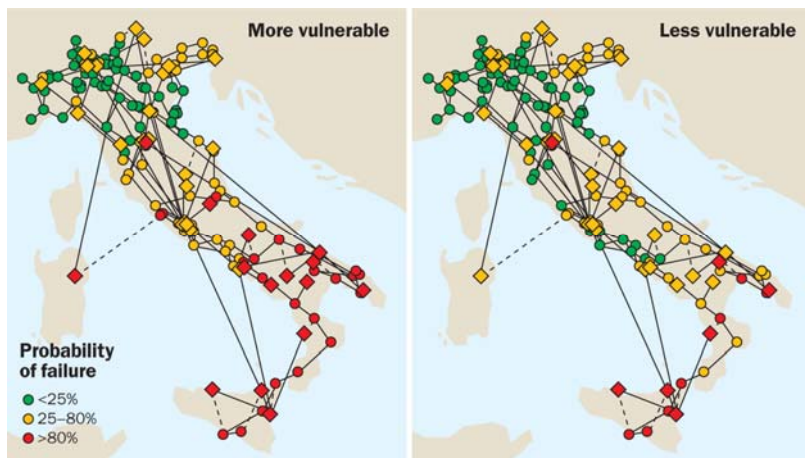
NETWORKS





Networks

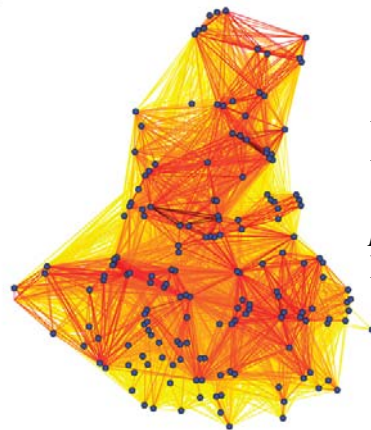
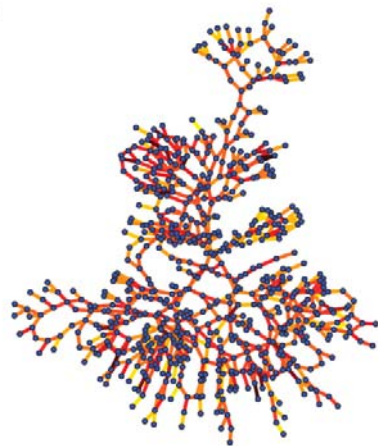
- Robustness
- Synchronizability
- Cost vs. Efficiency
- Controllability
- Observability
- ...
- Prevent cascades
- Coherence
- Improve transport
- Performance
- Predictability
- ...



a

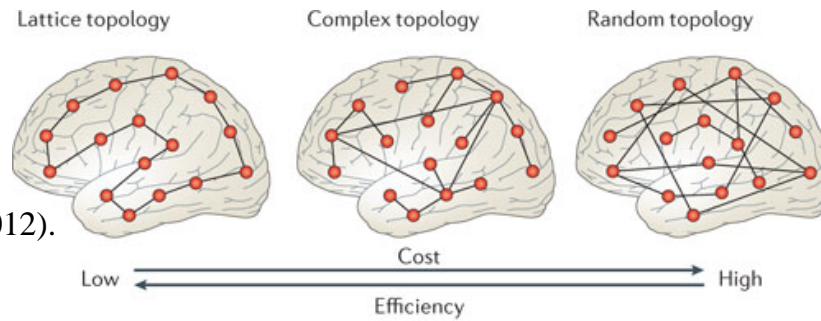
b

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S. Havlin,
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failures in interdependent
networks”,
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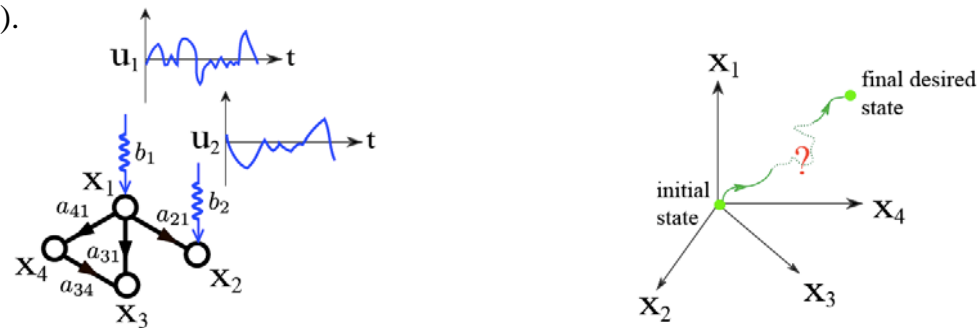
A.E. Motter, S.A. Myers, M.
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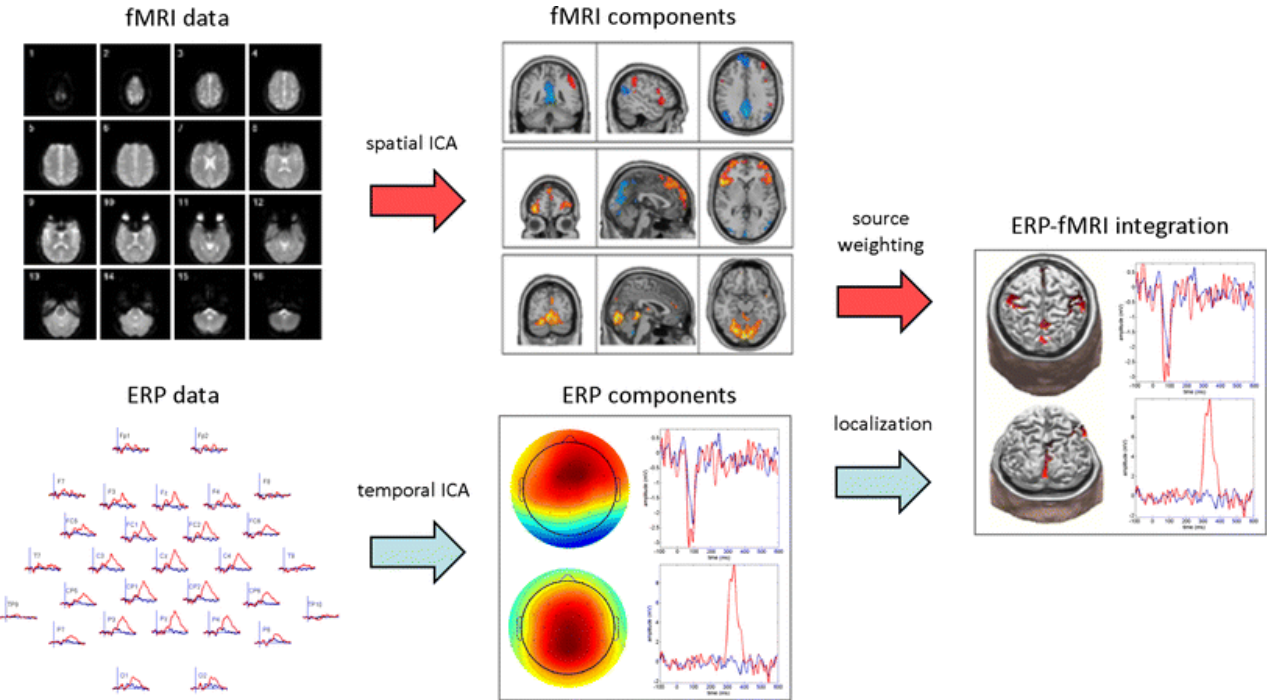
Nature Reviews | Neuroscience

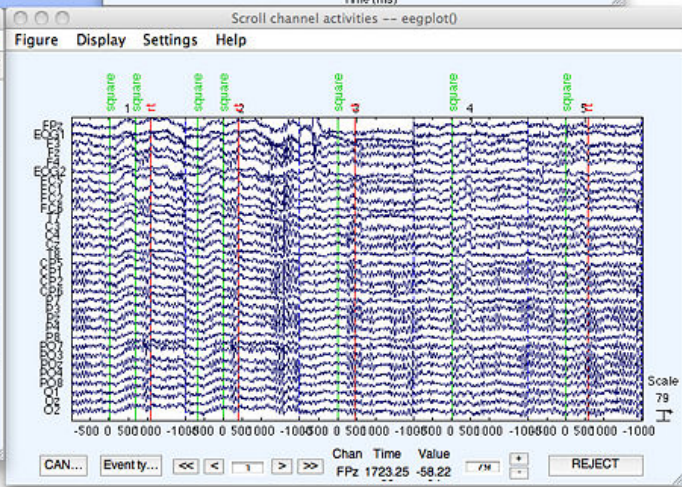
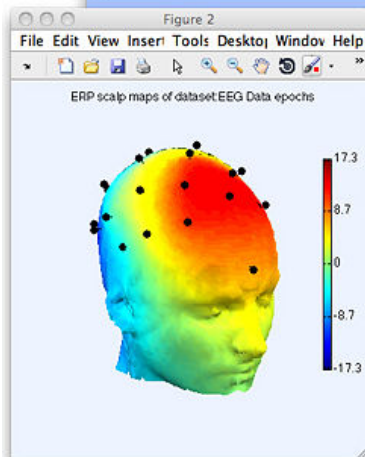
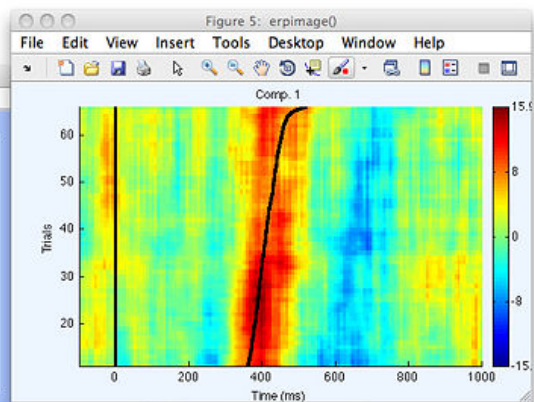
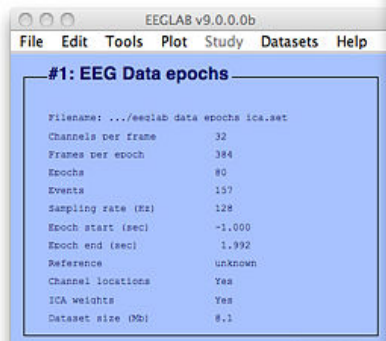


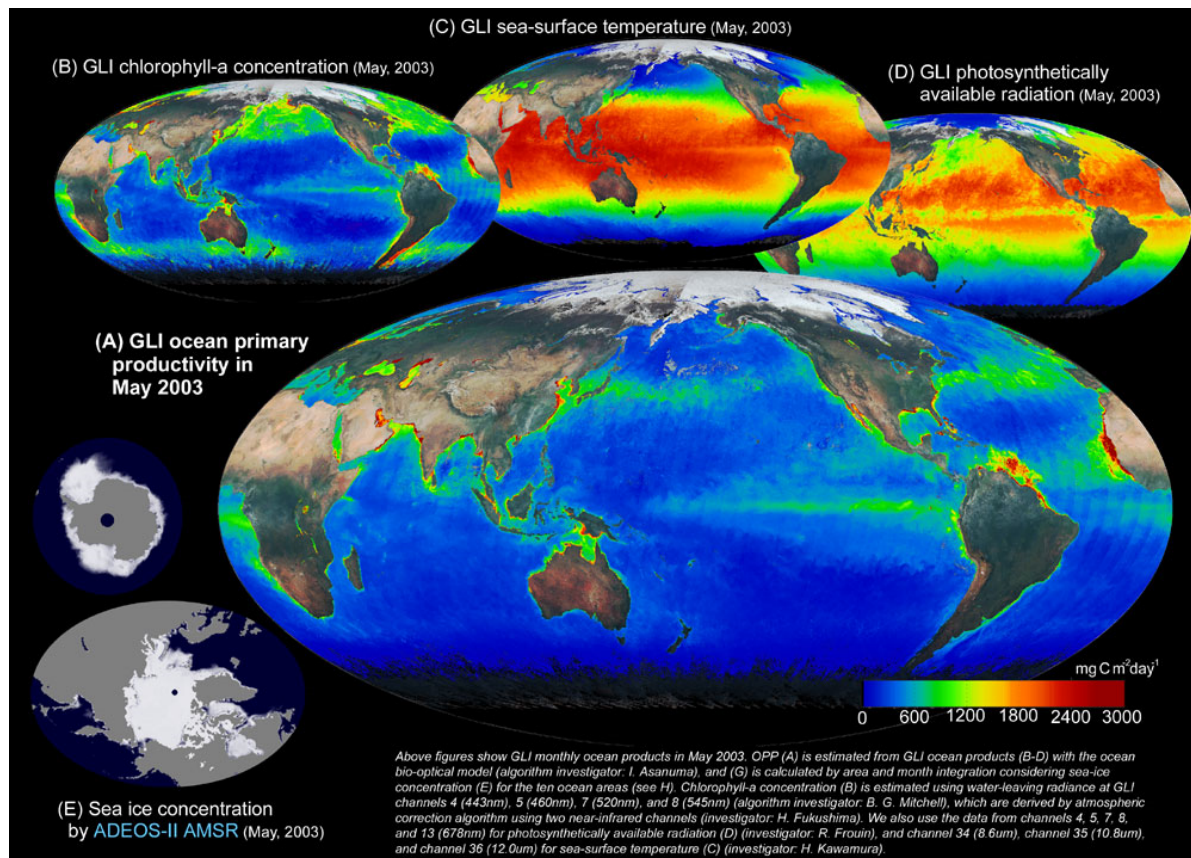
$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & 0 & 0 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}; \quad \mathbf{C} = \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & a_{21}b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{31}b_1 & 0 & a_{34}a_{41}b_1 & 0 & 0 & 0 \\ 0 & 0 & a_{41}b_1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

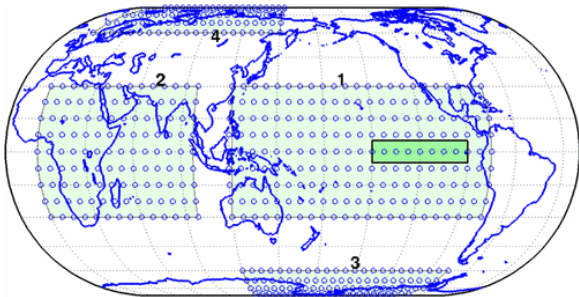
$$N = 4, M = 2, \text{rank}(\mathbf{C}) = 4 = N$$

Time-series measurements



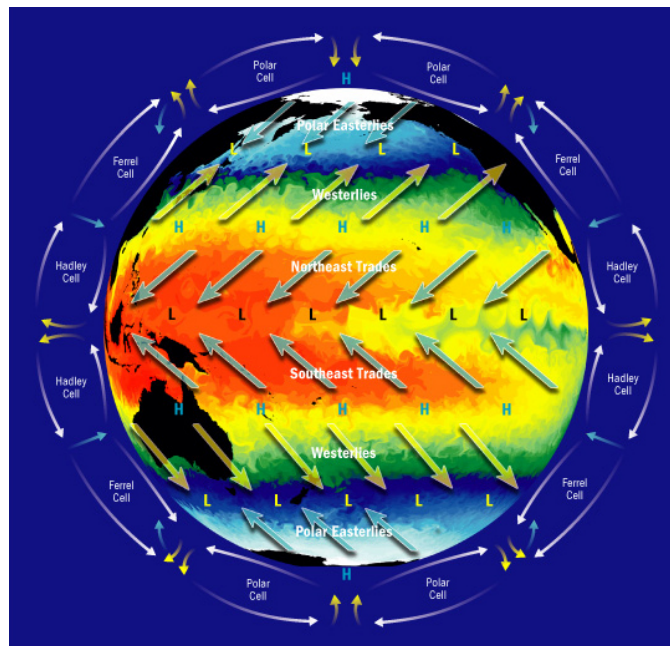
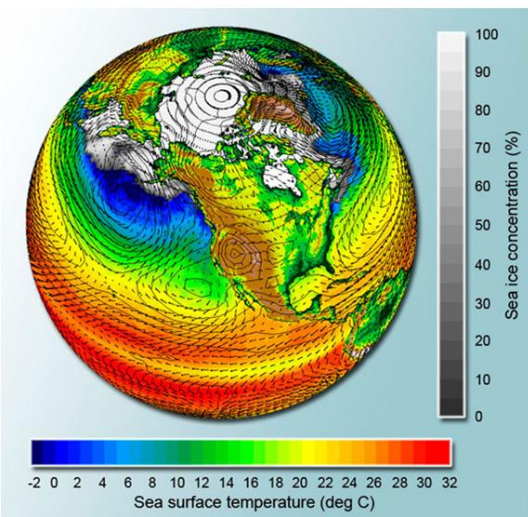




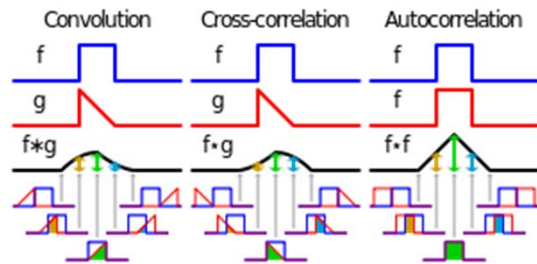


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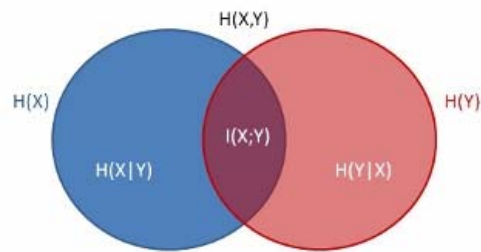


Similarity measures



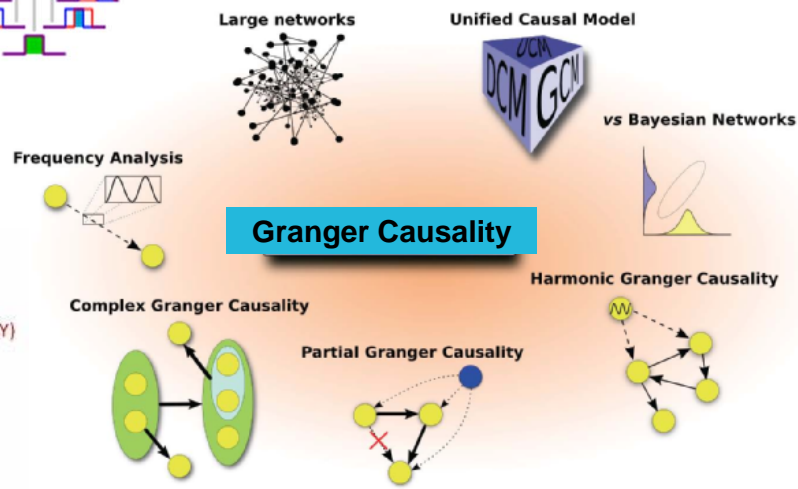
Cross-Correlation

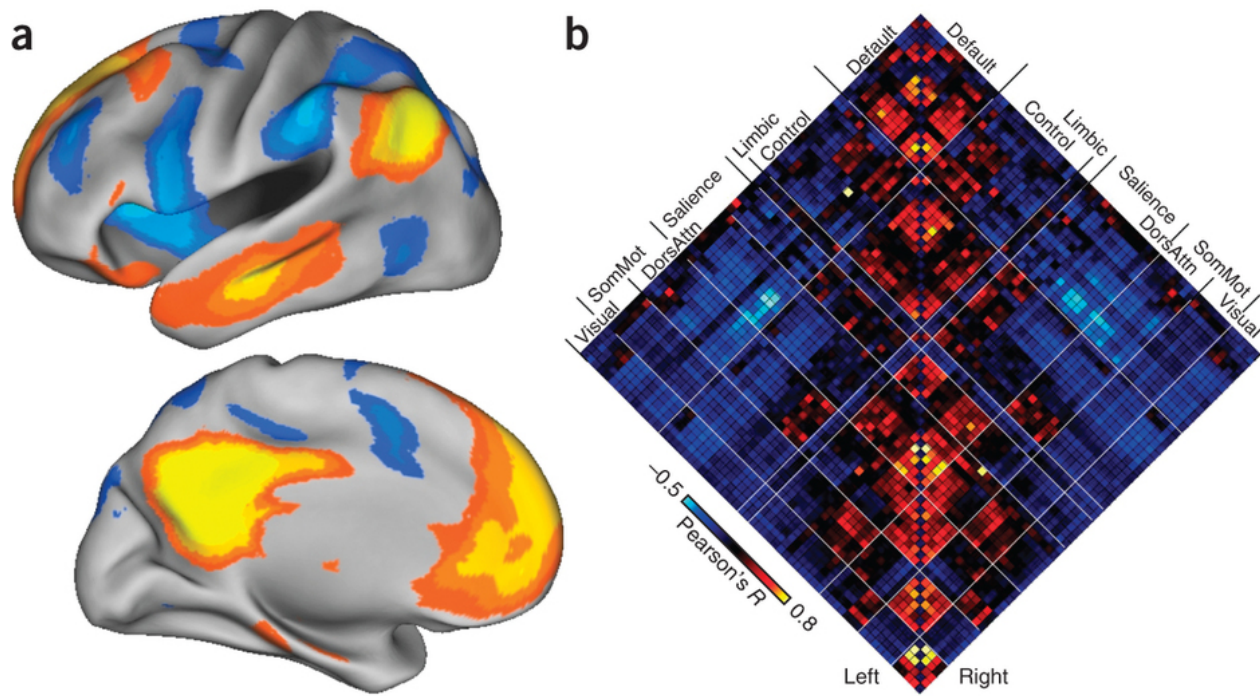
Mutual Information & Mutual Information Rate



$$I(X;Y) = H(X \cap Y)$$

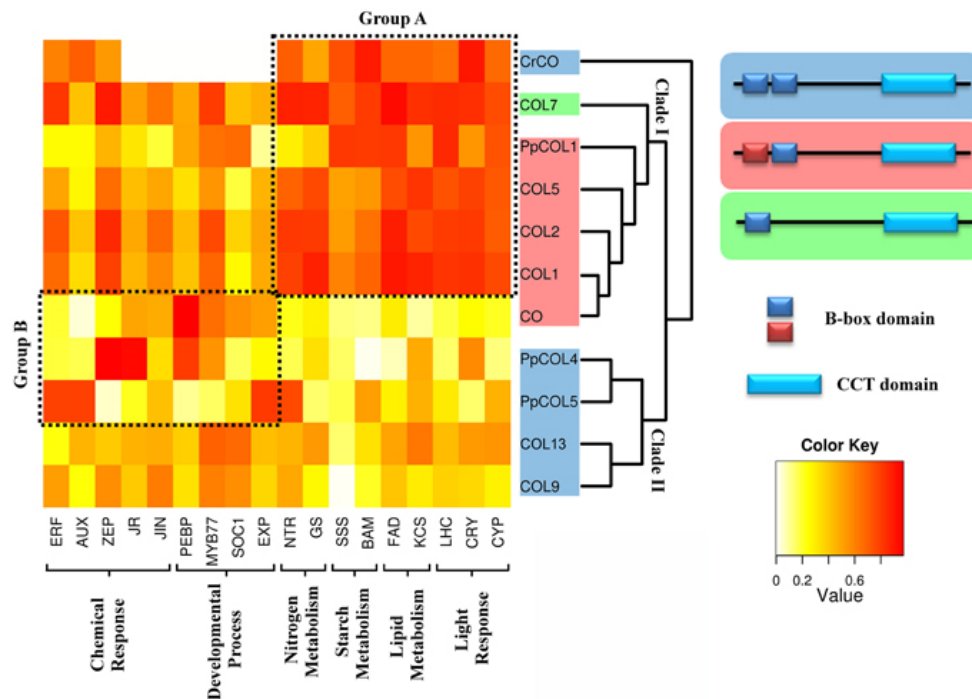
$$H(X;Y) = H(X) + H(Y) - H(X \cap Y)$$



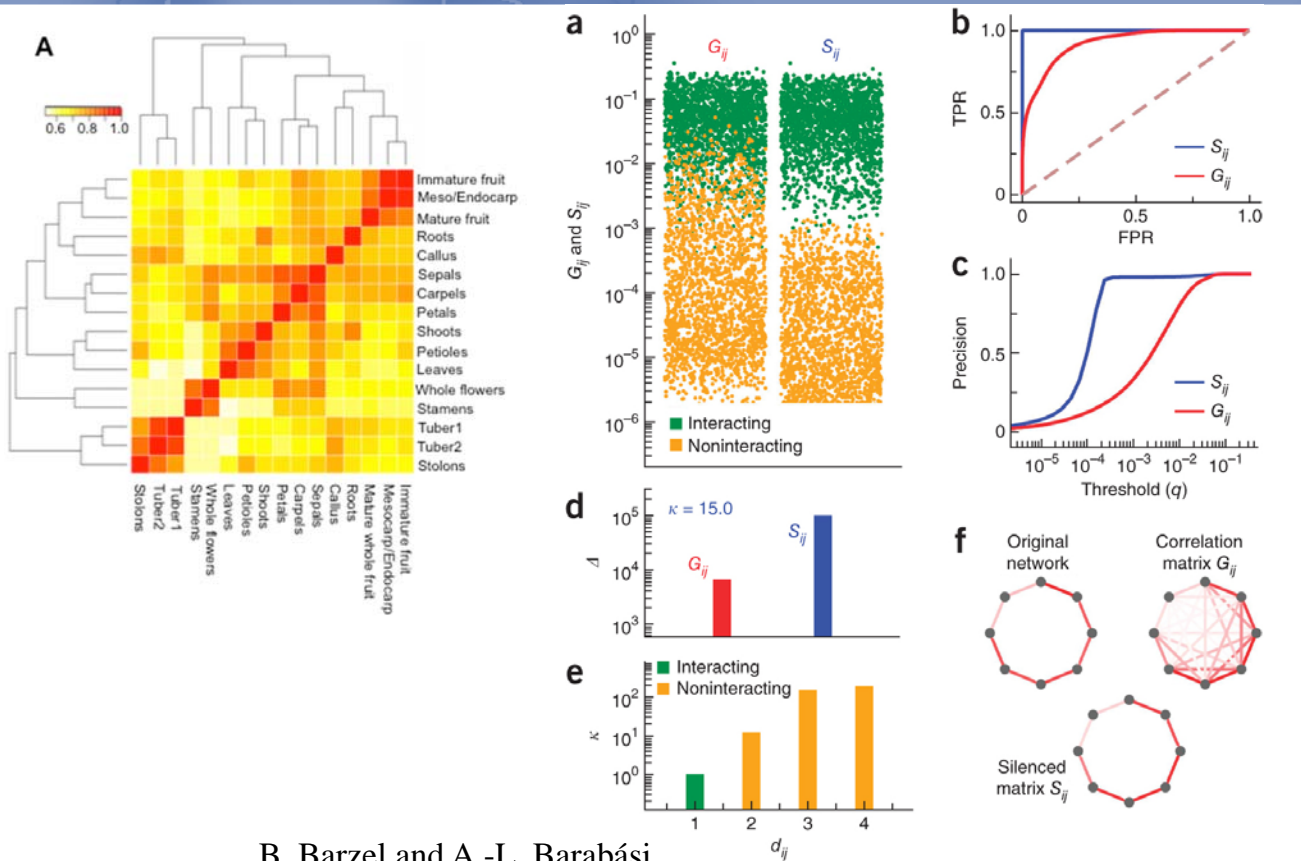


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 “Opportunities and limitations of intrinsic functional connectivity MRI”,
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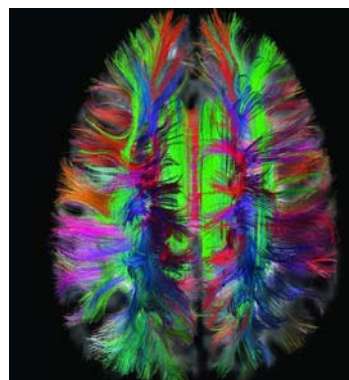
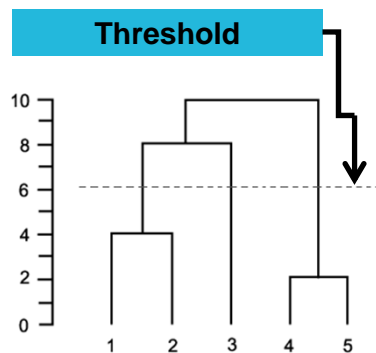
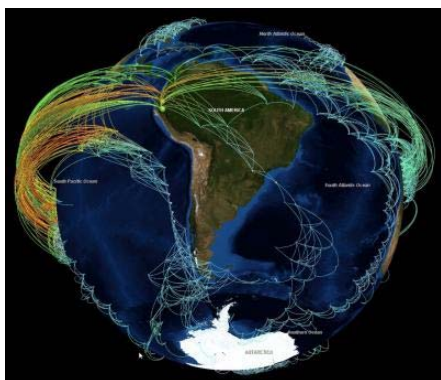
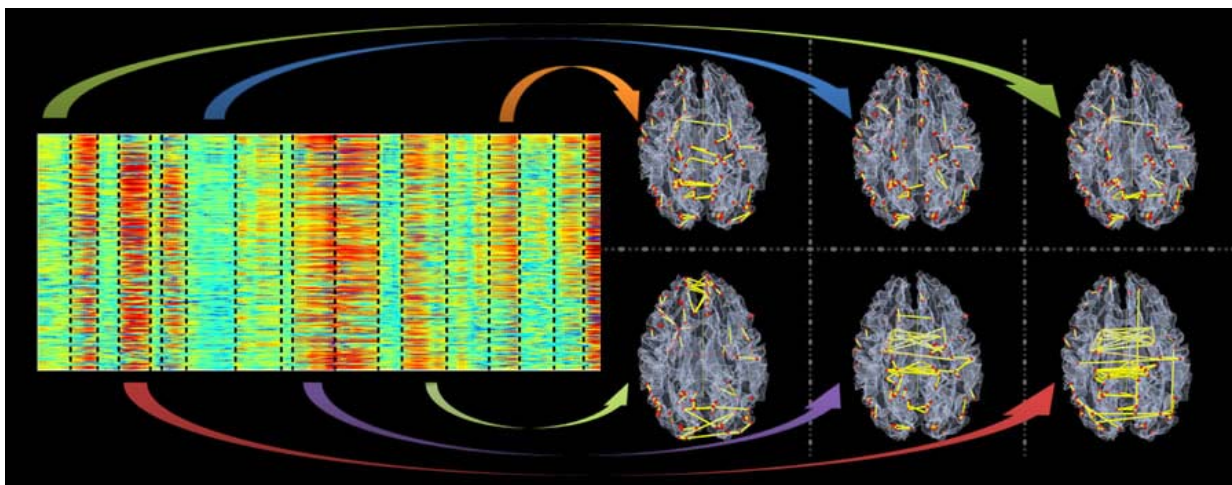
Network inference



F.J. Romero-Campero, E. Lucas-Reina, F.E. Said, J.M. Romero, and F. Valverde,
“A contribution to the study of plant development evolution based on gene co-expression networks”,
Front. Plant. Sci. **4**, 291-308 (2013).



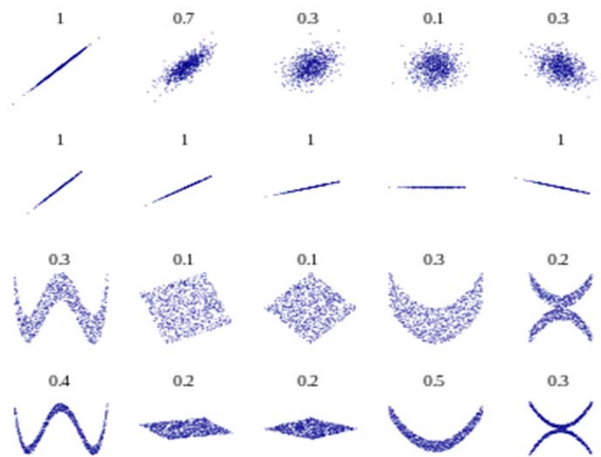
B. Barzel and A.-L. Barabási,
 “Network link prediction by global silencing of indirect correlations”,
 Nat. Biotech. **31**, 720-725 (2013).



Problems

- Which similarity measure to use
- How to choose a threshold
- How much data is available
- How to avoid the (usual) noise in the data
- How to recover coupling strengths
- Which are the directions in the interactions
- How many “units” are observed
- How many should be observed

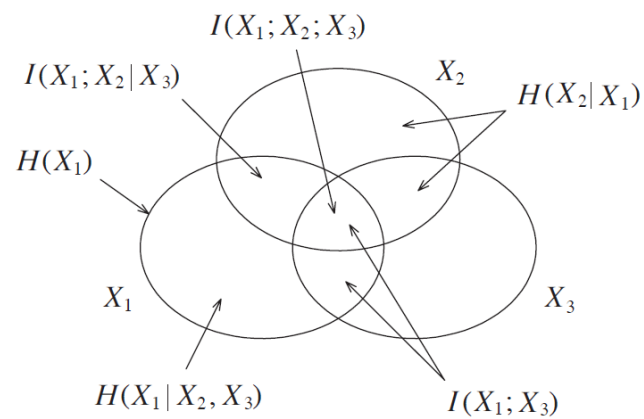
CC and MI



Cross-Correlation



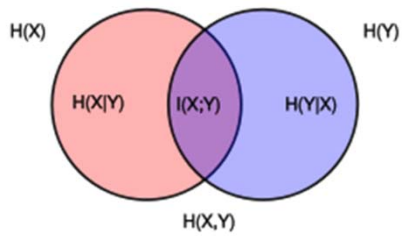
Bivariate Pearson (linear)



Mutual-Information



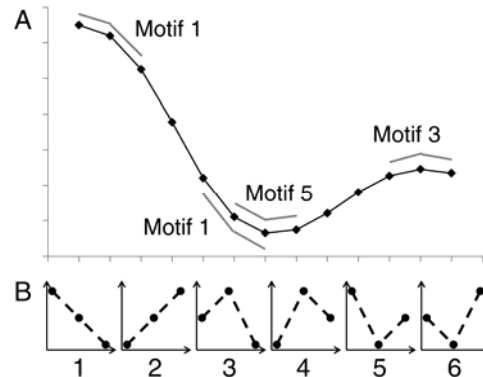
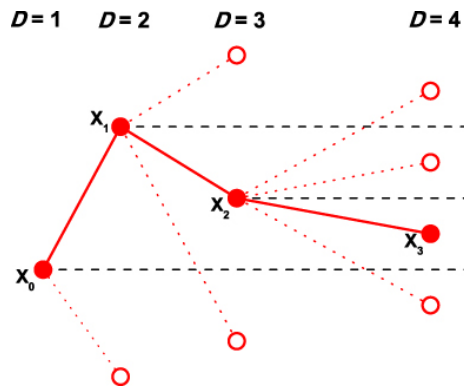
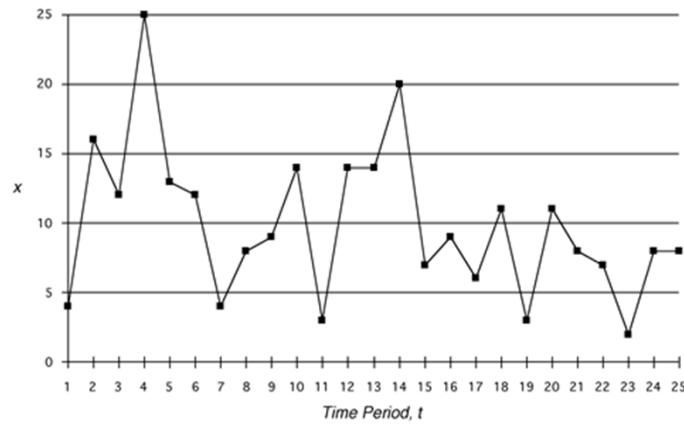
Bivariate (Ordinal Pattern)



$$I(X;Y) = H(X) - H(X|Y)$$

$$= H(X) + H(Y) - H(X,Y),$$

$$I(X;Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)},$$

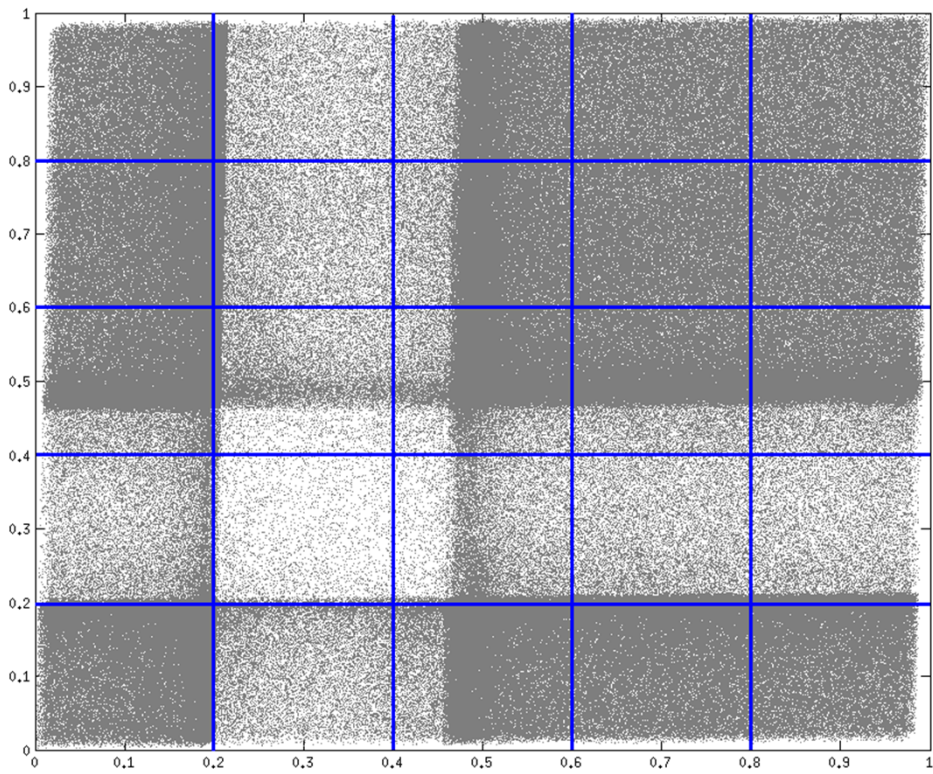


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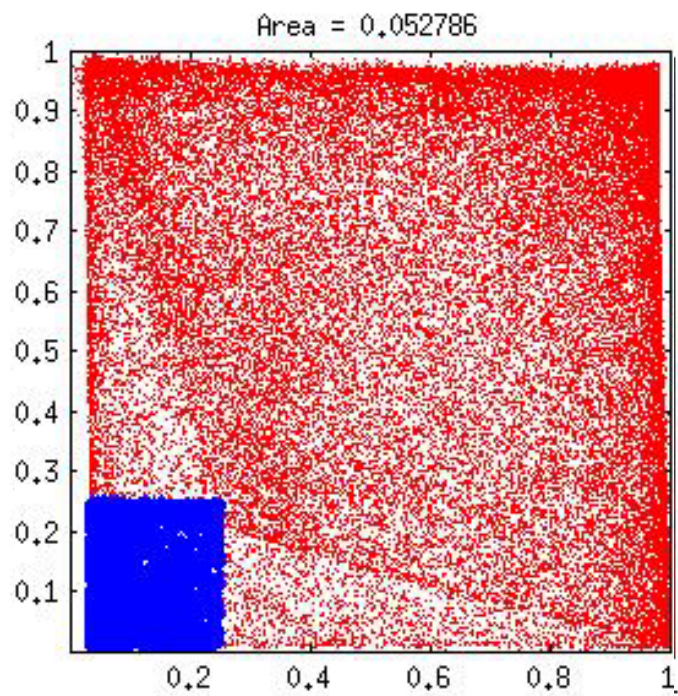
2

1

MIR



MIR

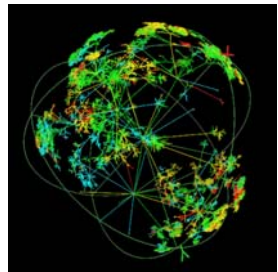


MIR

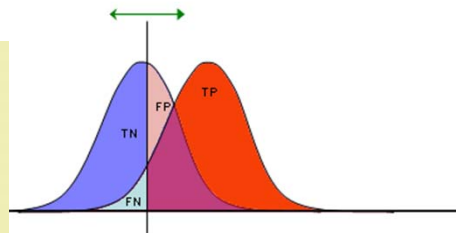
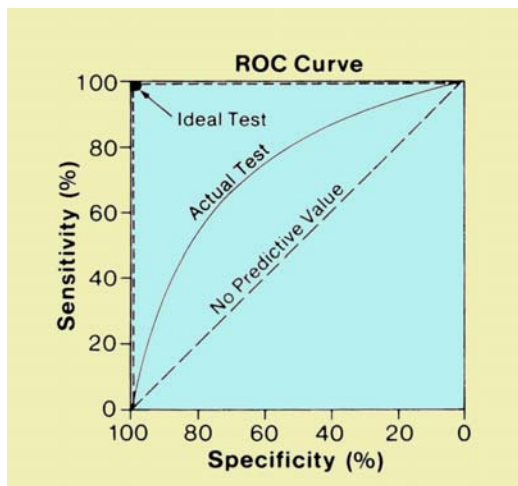
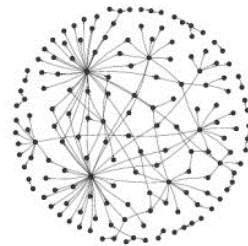
$$\text{MIR} = \frac{\text{MI}(\epsilon)}{T(\epsilon)}$$

$$T(\epsilon) \simeq \frac{1}{e_1} \log \left(\frac{1}{\epsilon} \right)$$

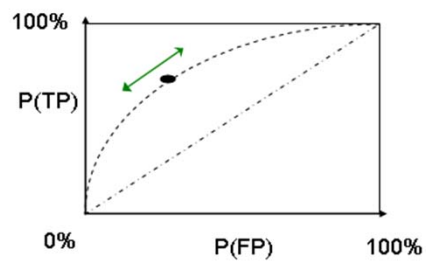
Global threshold



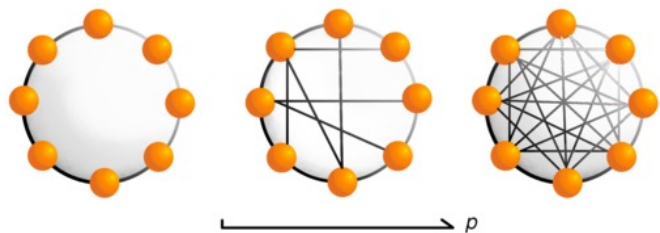
Comparison



TP	FP
FN	TN
1	1



Network models



→ p

Expected number of edges



→ p'

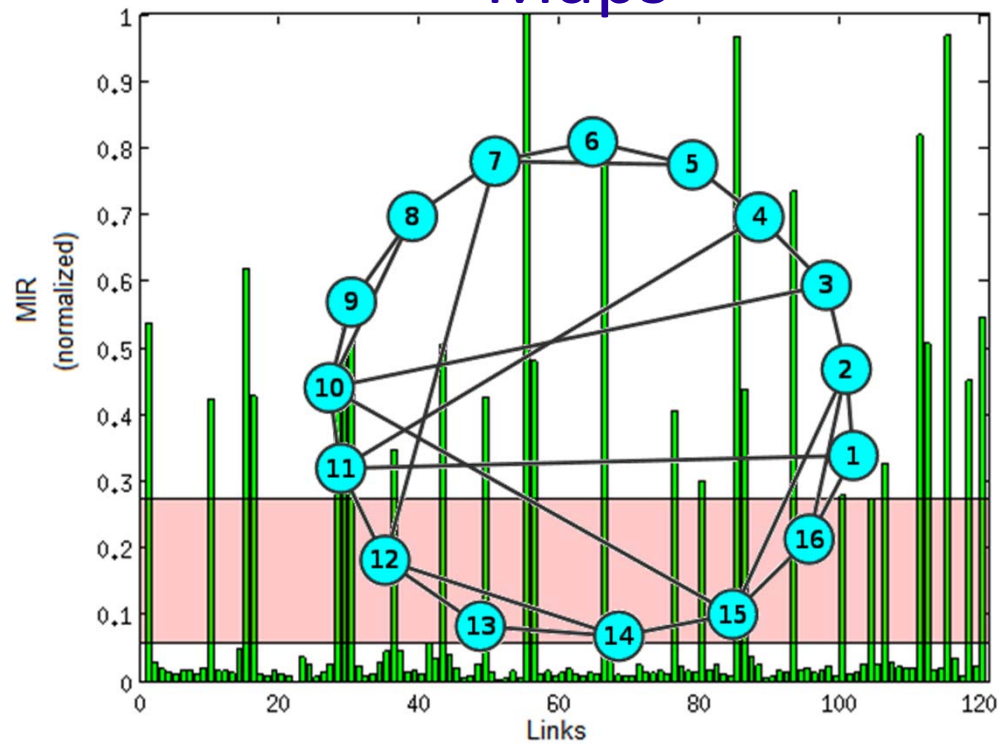
Expected number of edges

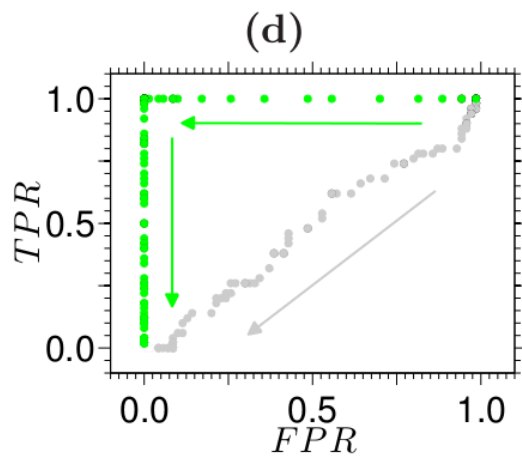
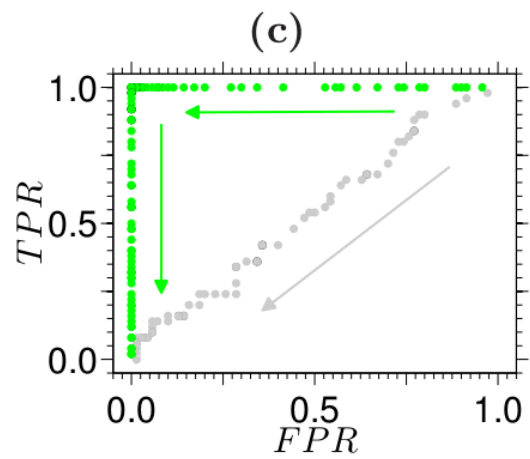
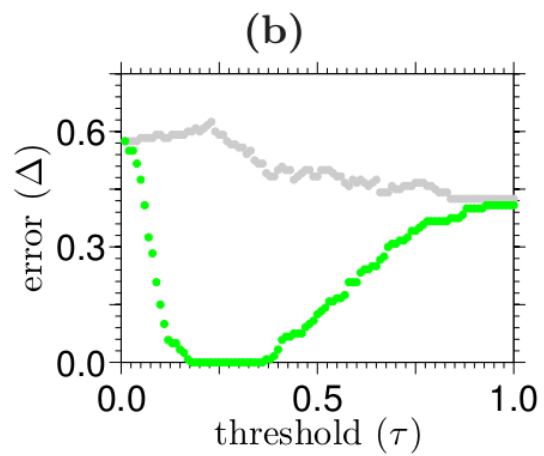
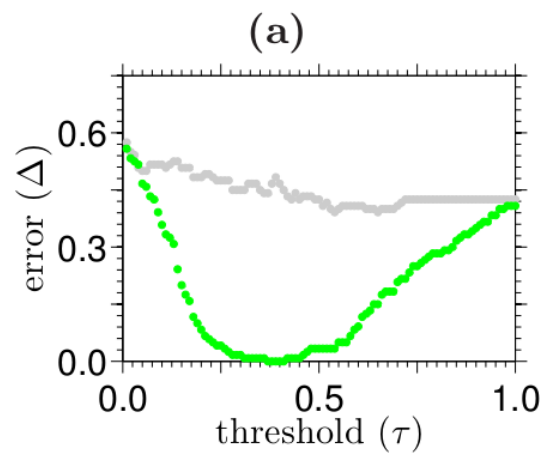
Model results

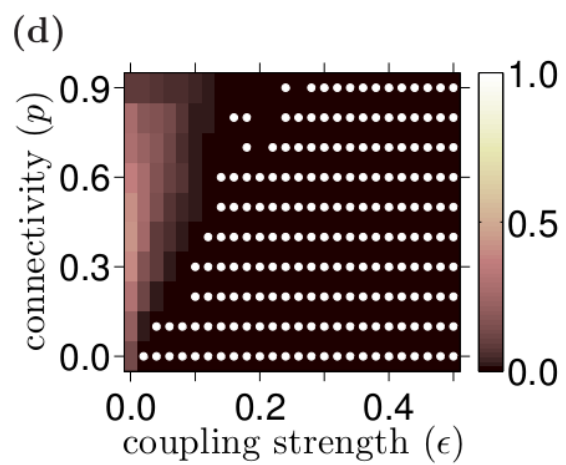
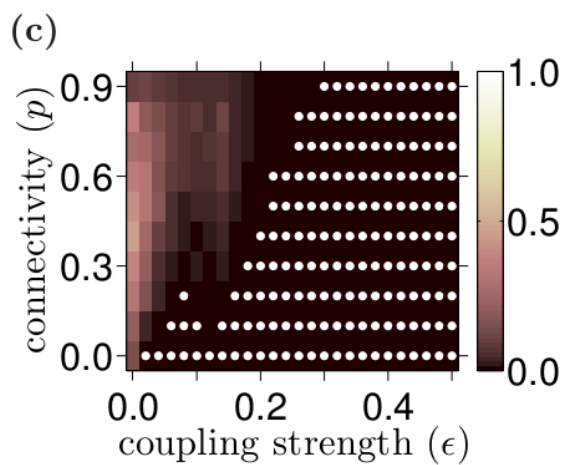
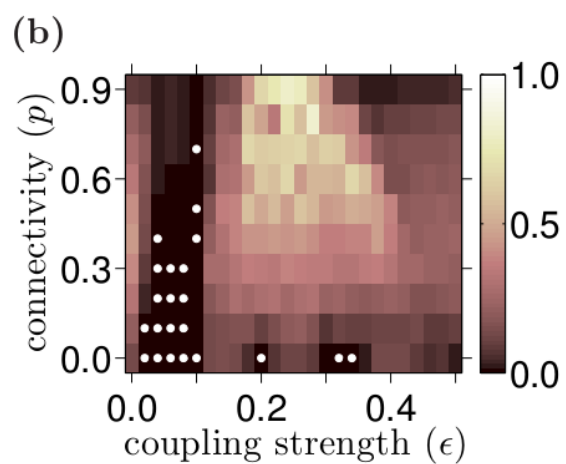
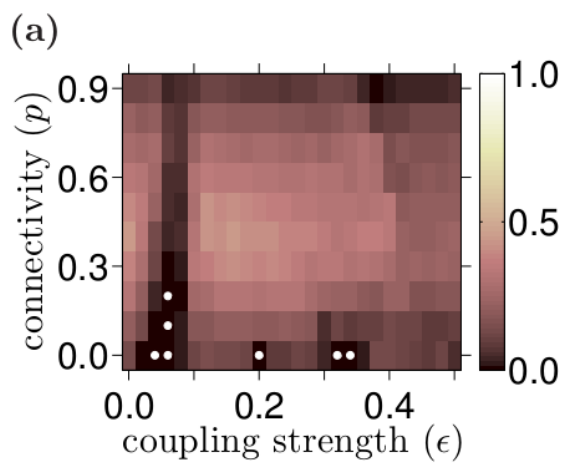
$$x_{n+1}^{(i)} = (1 - \epsilon) f_i(x_n^{(i)}) + \frac{\epsilon}{d_i} \sum_{j=1}^N W_{ij} f_j(x_n^{(j)})$$

- Logistic maps
- Circle maps
- ...
- Optical maps
- Tent maps
- ...

16 Coupled Logistic Maps







Articles:

N. Rubido, A.C. Martí, E. Bianco-Martínez, C. Grebogi, M.S. Baptista, and C. Masoller,
“*Exact detection of direct links in networks of interacting dynamical units*”, submitted (2014)
[available at: <http://arxiv.org/abs/1403.4839>].

E. Bianco-Martínez, N. Rubido, C.G. Antonopoulos, and M.S. Baptista,
“*Network Inference by Mutual Information Rates from Complex Time-series*”,
in preparation (2014).

Ongoing projects:

L'Her, P. Amil, R. García, F. Abellá, M.S. Baptista, A.C. Martí, C. Cabeza, and N. Rubido,
“*Electronic circuit implementation of a network of Logistic maps*”.
Universidad de la República (UdelaR), Montevideo, Uruguay.

N. Rubido and A.J. Pons, “*Neural circuits and transfer functions*”.
Universidad Politécnica de Barcelona (UPC), Terrassa, Spain.