Symplectic map description of Halley's comet dynamics

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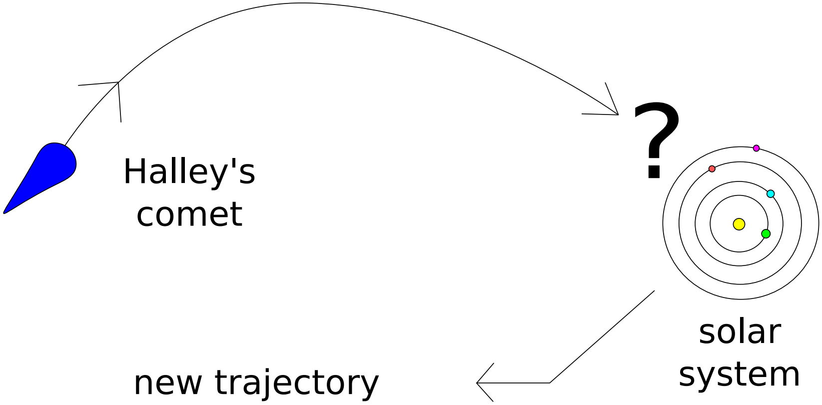
Overall view

1. Melnikov's method
2. Halley's comet
3. Poincaré section

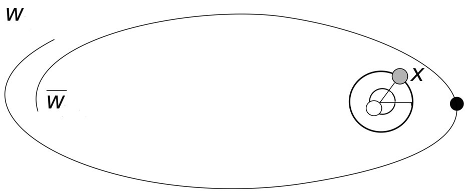
Overall view

The comet gets or looses energy on going through the solar system



Definitions

- We redefine the energy as $w = -\frac{2E}{m} \Longrightarrow a = \frac{1}{w}$
- We define x as the mean anomaly, it is related to time by $\frac{x}{2\pi} = \frac{t}{P}$ with P the planet's period
- The planet's orbit is circular, its position is marked by x
- We define the kick as the increase of energy $F(x)$ of the comet when it passes at the perihelion



Melnikov's method

- Starting from the orbital elements of the comet, we choose an osculating orbit (reference orbit)

$$F(x) = \frac{2}{m} \oint_{\text{orb. osc.}} \overrightarrow{\nabla} \left(\Phi(\overrightarrow{r}, x) - \Phi_0(r) \right) \cdot \overrightarrow{dr}$$

- $\Phi(\overrightarrow{r}, x)$ is the potential energy of the restricted three-body problem (Sun, planet, comet)
- $\Phi_0(r)$ is the potential energy of the two-body problem (Sun, comet) for which the osculating orbit is solution.

Overall view

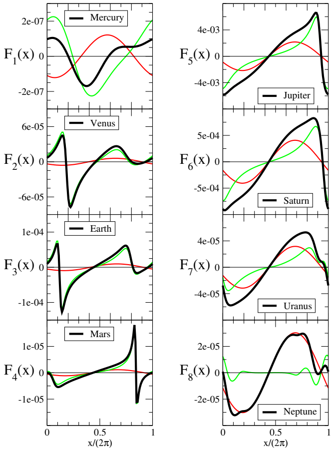
1 Melnikov's method

2 Halley's comet

3 Poincaré section

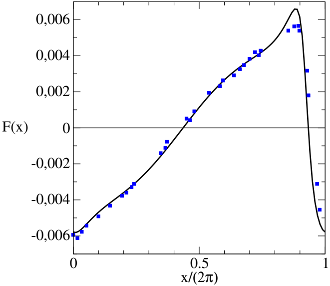
Contribution of each planet

- We determine the osculating orbit from Halley's actual orbital elements
- We determine the kick which would be caused by one planet only (and the Sun) with a mean anomaly x_i
- Considering the eight planets of the solar system, we obtain eight kicks : $F_1(x_1)$, $F_2(x_2)$, etc

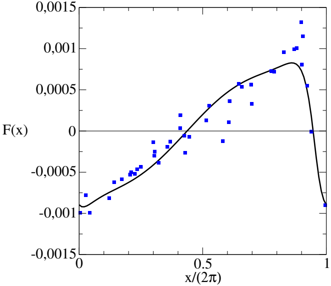


Major contributions : Jupiter and Saturn

Jupiter's contribution



Saturn's contribution



Reference : R. V. Chirikov, V. V. Vecheslavov, Chaotic dynamics of Comet Halley, Astronomy and Astrophysics, vol. 221, 1989, p. 146-154. (figure 2)

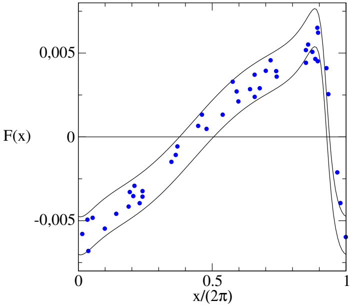
Total kick : addition of the contributions

- We define contribution

$$F_{tot}(x = x_5) = \sum_{i=1}^{8} F_i(x_i)$$

- The total kick may be bounded as below :
  - We trace the kick produced by Jupiter only
  - We add the kicks of the other planets so as to minimize or maximize this kick

Total kick compared with the observations



Reference : R. V. Chirikov, V. V. Vecheslavov, Chaotic dynamics of Comet Halley, Astronomy and Astrophysics, vol. 221, 1989, p. 146-154. (figure 1)

Overall view

Halley's symplectic application

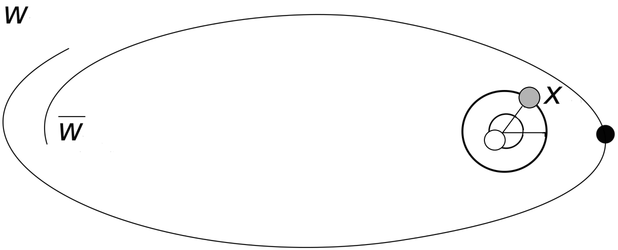
- We can define an application which gives the comet's energy after each passage and the position of a planet at the next passage

$$\begin{array}{rcccl} \overline{w} & = & w & + & F(x) \\ \overline{x} & = & x & + & 2\pi\left(\overline{w}\right)^{-3/2} \end{array}$$

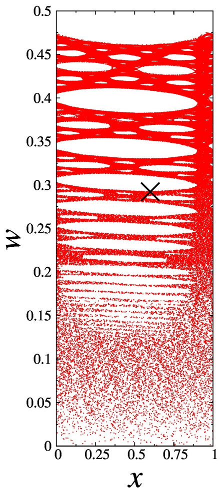
- The application gives $(\overline{x}, \overline{w})$ from (x, w)

Poincaré section

- We only consider the influence of Jupiter (and of the Sun)
- We trace a series of points (x, w), $(\overline{x}, \overline{w})$, $(\overline{\overline{x}}, \overline{\overline{w}})$ ...
- We get a Poincaré section

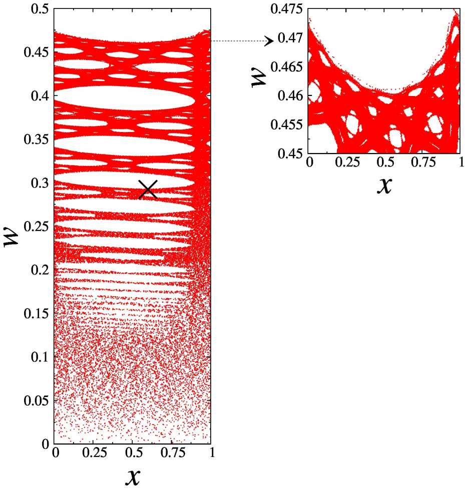


Chaos and comet's position



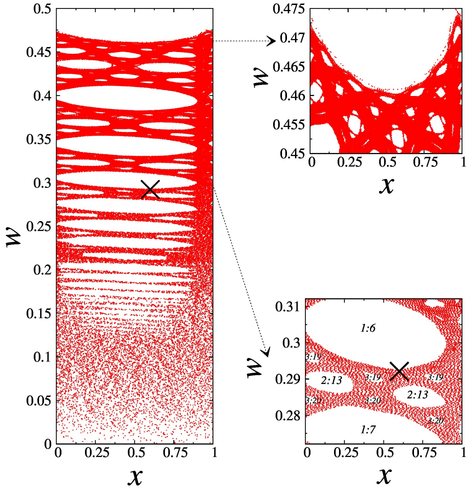
- The cross represents the actual position of Halley's comet (outside the islets)
- Presence of a chaotic component for $w \lesssim 0.15$ which co-exists with stability islets for $0.15 \lesssim w \lesssim 0.475$
- Around $w \simeq 0.475$, a limit defined by Kam's invariant curve stops the chaotic diffusion.

KAM's invariant curve



- There is self-similarity around the stability islets (fractal structure)

Stability islets and resonances with Jupiter



- Resonances $p : n$ are determined by w and the number of islets a line contains

$$p\left(\frac{\bar{x}-x}{2\pi}\right) = n$$

$$w_{p:n} = \left(\frac{n}{p}\right)^{-2/3}$$

$$n, p \in \mathbb{N}^*$$

- The comet makes p tours while Jupiter makes n

Ejection/residence time

- Use of Poincaré section with the influence of all the planets ($F_{tot}(x)$)
- The number of passages in Solar System before ejection is around 50 000
- It is the number of passages since the comet's capture too
- Chirikov & Vecheslavov get 100 000 passages ($\sim$ 10 millions of years)

Conclusion

- Our results are similar to Chirikov & Vecheslavov (1989)
  - the main contributions to the total kick, *i.e.* those of Jupiter and Saturn, are the same as C&V (1989)
  - moreover, we have determined the contribution of the other planets of Solar System and constructed the total kick $F_{tot}(x)$
  - the Halley's symplectic application incorporating $F_{tot}(x)$ gives residence/ejection times equivalent to C&V (1989)
- We confirm the comet has been captured, and this a long time after the formation of Solar System (origin : Oort's cloud ?)
- Perspectives
  - Consider the elliptical orbits of the Solar System planets in order to refine the kick functions
  - Check the robustness of Halley's symplectic application : we shall have to compare it to its real dynamics