Multiproduct world trade network

Leonardo Ermann
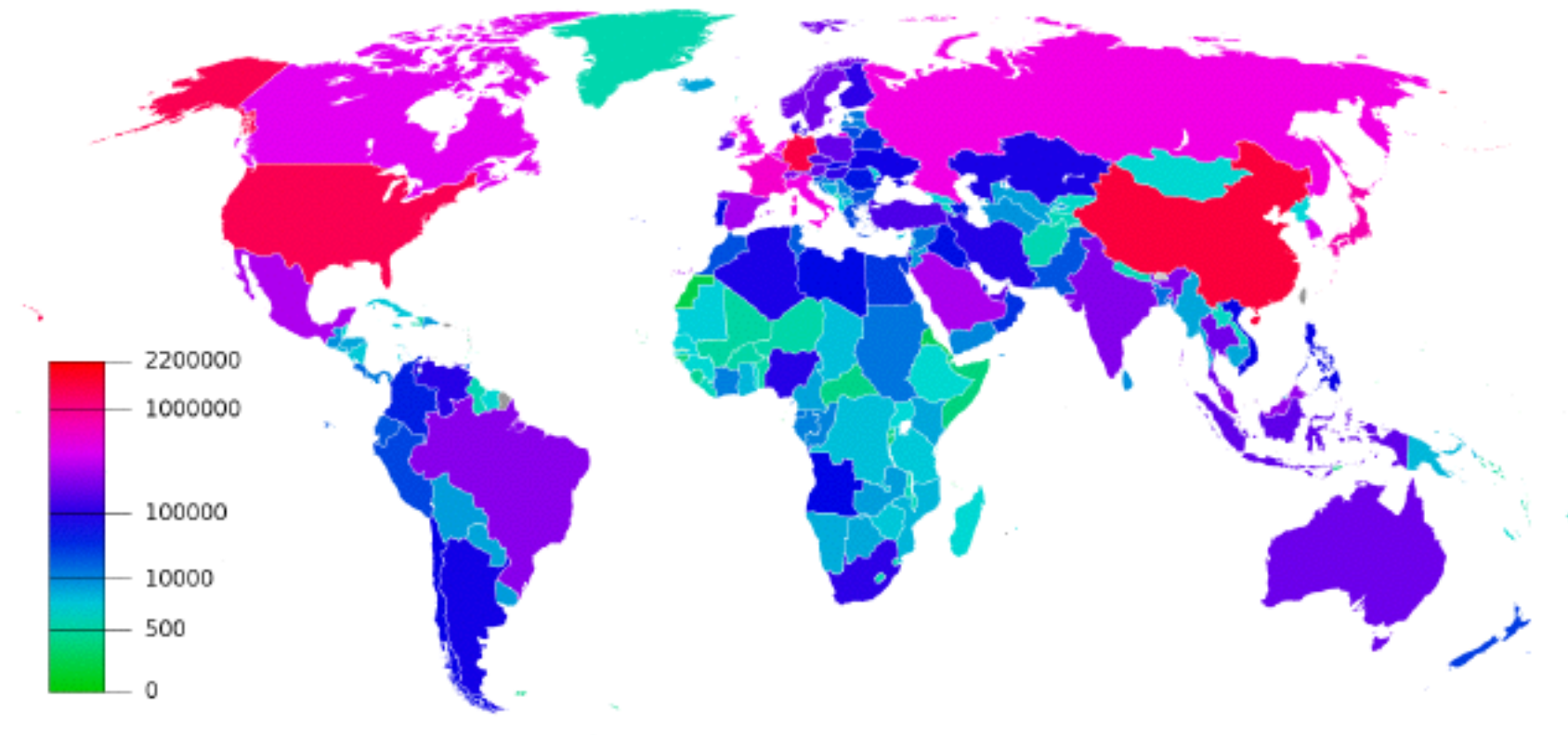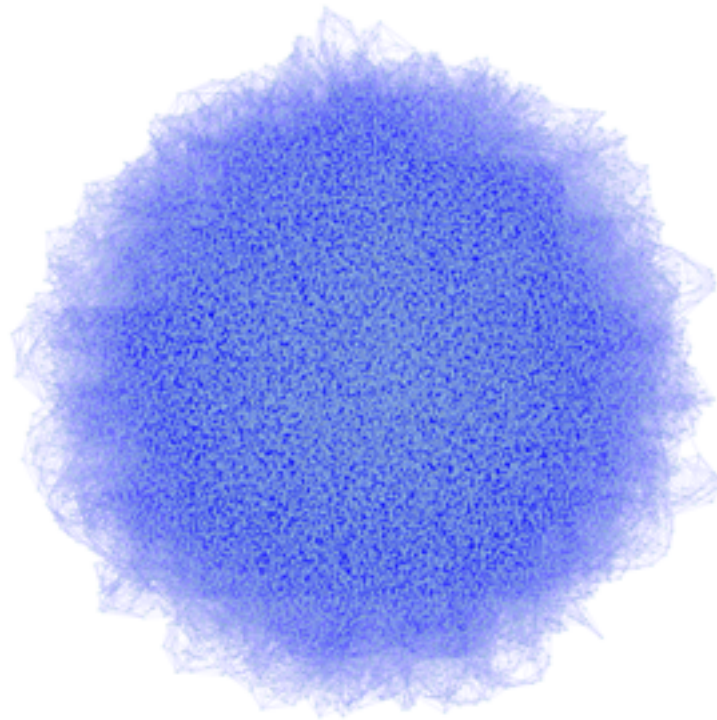### CNEA (Buenos Aires, Argentina)

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Klaus Frahm

Alexei Chepelianskii

Networks and data mining

July 1st, 2015

School for advanced sciences of Luchon

Motivations

Google approach to the World Trade Network

Outline

1) G and G* introduction

2) WTN

3) multiproduct WTN

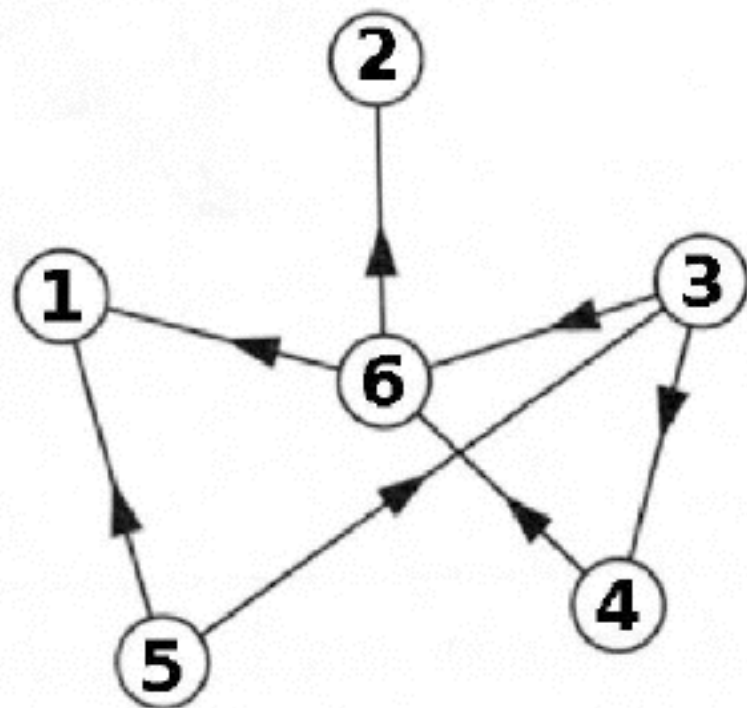**4) EcoloRank given by nestedness**

Google Matrix

thanks to Klaus introduction of G matrix

Brin and Page (1998)

centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network

adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$
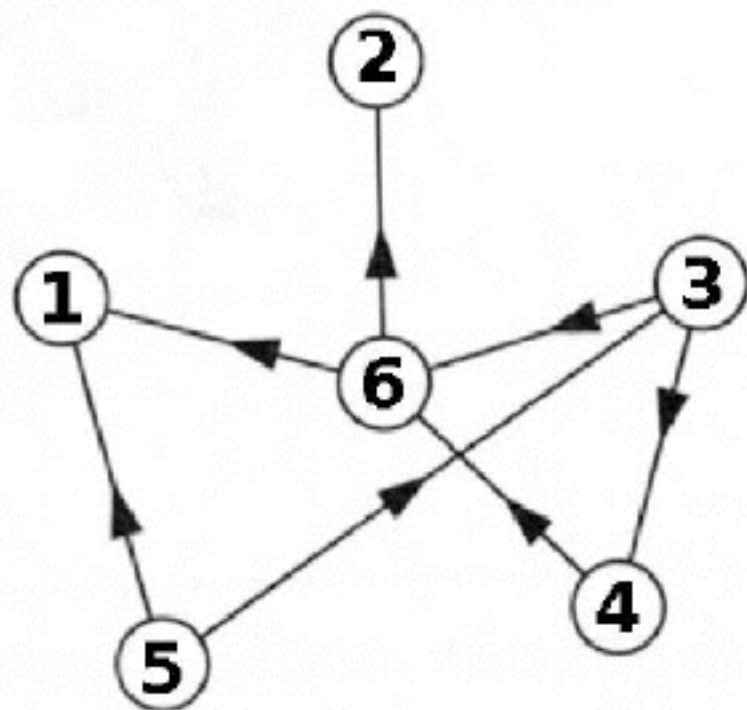
Google Matrix

thanks to Klaus introduction of G matrix

Brin and Page (1998)

centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

directed network

weighted adjacency matrix and dangling nodes

$$S = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{2} & 1 & 0 & 0 \end{pmatrix}$$

- $\sum_i S_{i,j} = 1$
- Perron-Frobenius (non-negative)
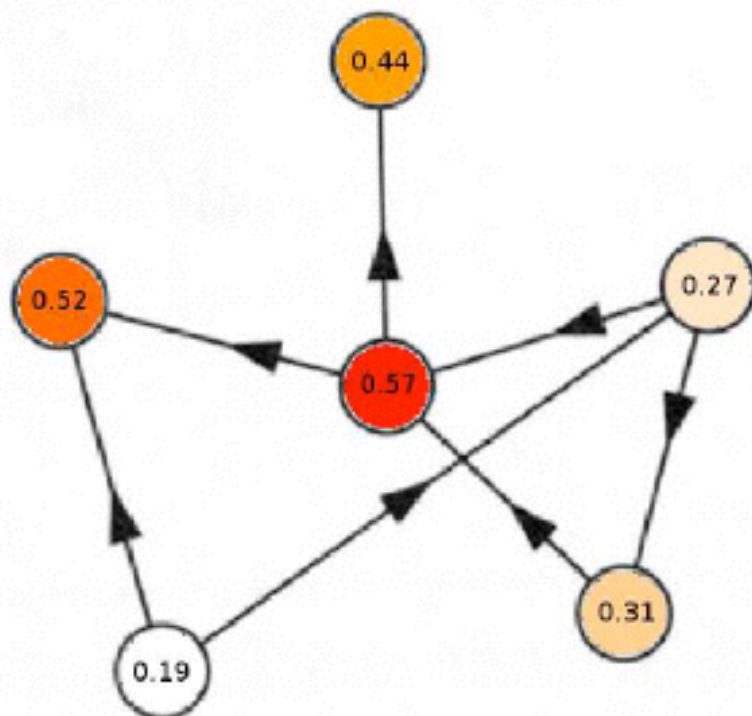- $\lambda_1 = 1$ (degeneracy)

Google Matrix

thanks to Klaus introduction of G matrix

centrality measure:

Spectral Indices

- directed networks
- easy to compute
- incoming links
- non-local properties

Brin and Page (1998)

PageRank

$$\mathbf{G}P = P$$

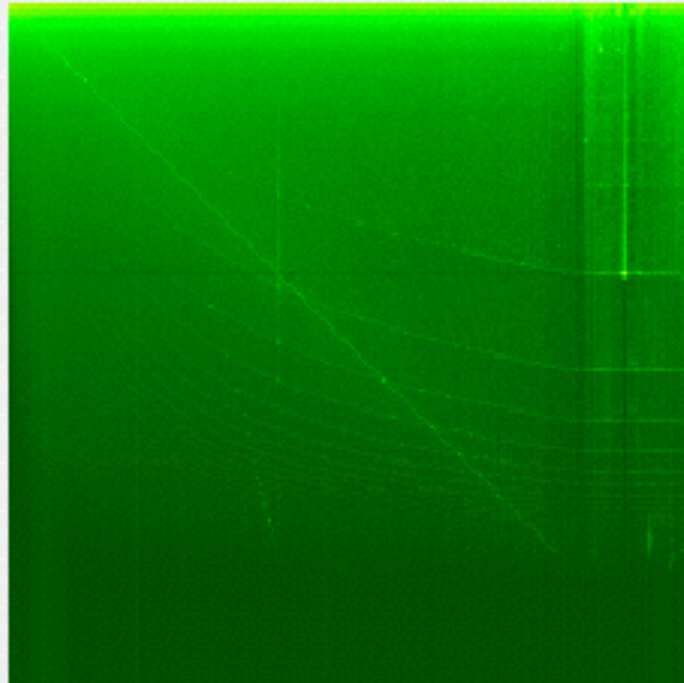$$\mathbf{G} = \alpha\mathbf{S} + (1-\alpha)\mathbf{E}/N \ (\alpha = 0.85)$$

directed network

Google Matrix

$$G = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{9}{20} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{9}{20} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{6} & \frac{1}{6} & \frac{9}{20} & \frac{7}{8} & \frac{1}{40} & \frac{1}{40} \end{pmatrix}$$
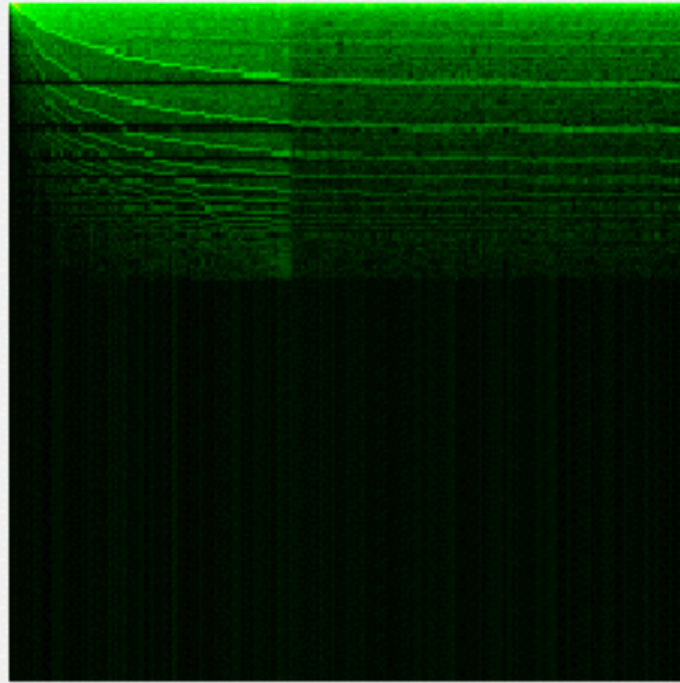
- $\alpha \to S, (1-\alpha) \to$ random node
- Perron-Frobenius (positive) $\lambda_1 = 1$
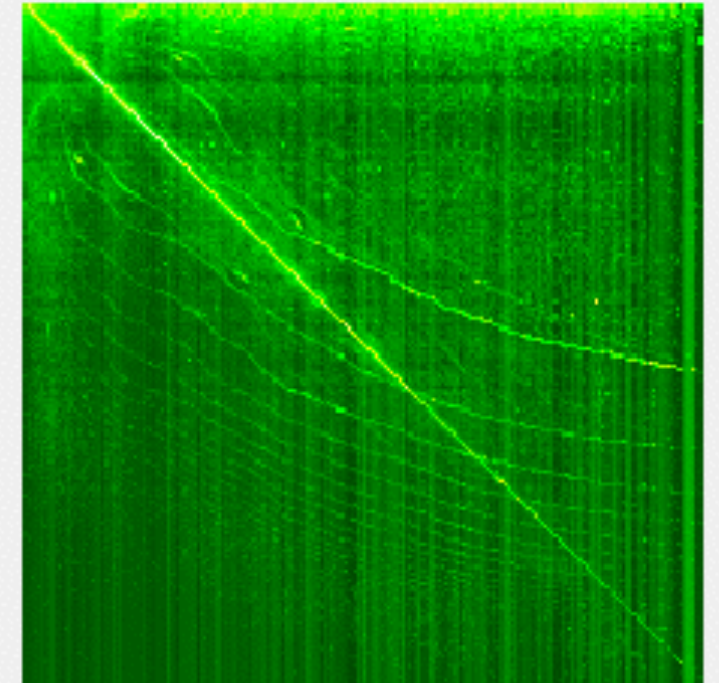- $\Delta \geq (1-\alpha)$ (global convergence)

Google Matrix
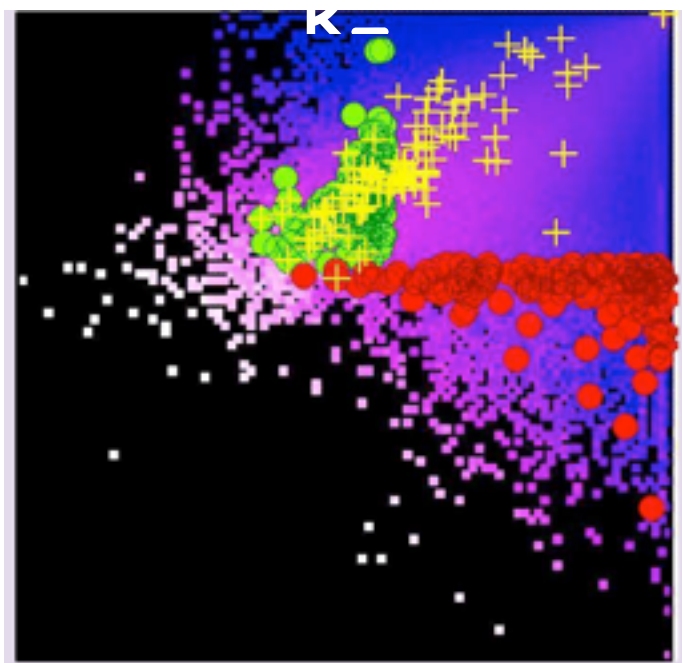

Wikipedia

Kernel Linux V2.6

Cambridge 2006

2D rankings

L.E, Chepelianskii and Shepelyansky, Jour. Phys. A 45, 275101 (2012).

wiki: K-K* plane

K_

PageRank-CheiRank 2d ranking

PageRank: 1. Napoleon I of France, 2. George W. Bush, 3. Elizabeth II of the United Kingdom, 4. William Shakespeare, 5. Carl Linnaeus, 6. Adolf Hitler, 7. Aristotle, 8. Bill Clinton, 9. Franklin D. Roosevelt, 10. Ronald Reagan.

CheiRank: 1. Kasey S. Pipes, 2. Roger Calmel, 3. Yury G. Chernavsky, 4. Josh Billings (pitcher), 5. George Lyell, 6. Landon Donovan, 7. Marilyn C. Solvay, 8. Matt Kelley, 9. Johann Georg Hagen, 10. Chikage Oogi.

2DRank: 1. Michael Jackson, 2. Frank Lloyd Wright, 3. David Bowie, 4. Hillary Rodham Clinton, 5. Charles Darwin, 6. Stephen King, 7. Richard Nixon, 8. Isaac Asimov, 9. Frank Sinatra, 10. Elvis Presley.

Chepelianskii (2010) O. Zhirov and Shepelyansky,(2010), LE, Chepeliansskii, Shepelyansky JPA(2012)

World Trade Network

Import-Export trade database:

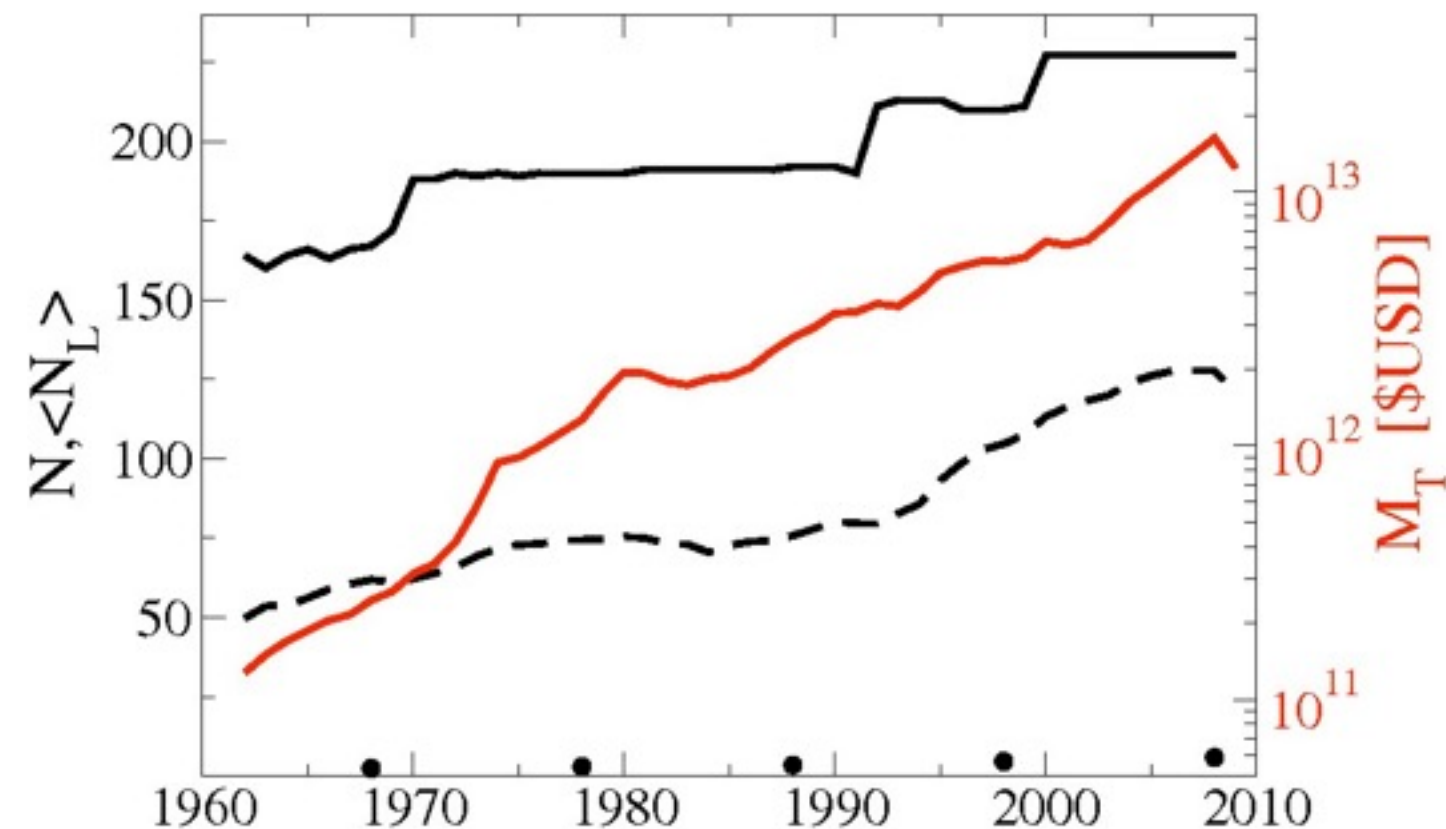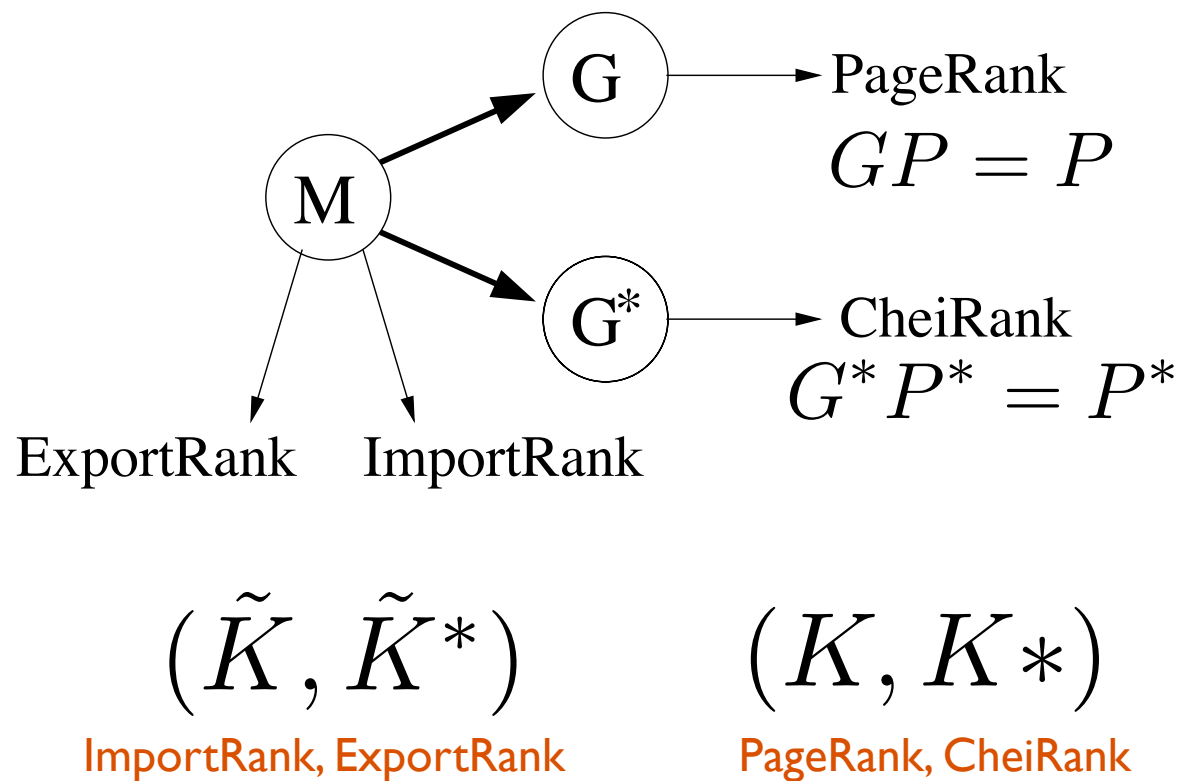United Nation Commodities Trade Network
HTTP://COMTRADE.UN.ORG/DB/

* Each year from 1962 to 2011 (2014)

* All UN countries: ~ 220 (Nc=227 in 2008)

* Product classification (SITC Rev. 1): Np=61

* Trade volume is given in USD (N=13847 x 50 years)

Money Matrices

$$M_{c,c'} = \$ \ (c' \to c)$$

$$M_{c,c'}^{p} = \$ \ (c' \to c)$$

Google matrix of the WTN

$$GP = P$$ PageRank

$$G^*P^* = P^*$$ CheiRank

$(\tilde{K}, \tilde{K}^*)$

ImportRank, ExportRank

$(K, K*)$

PageRank, CheiRank

ExportRank   ImportRank

Democracy in countries but not in products

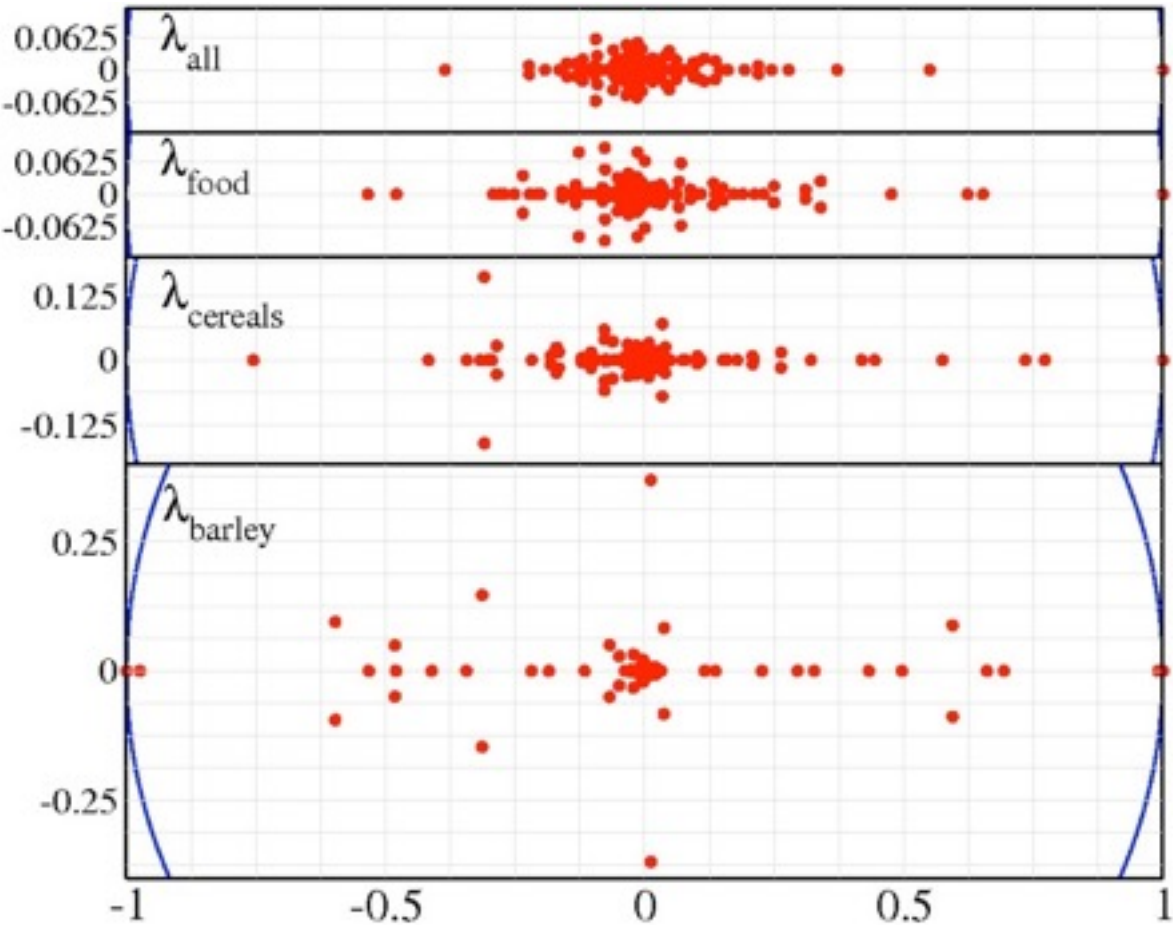# all commodities and given products (N=227)

M   G   G*

all commodities
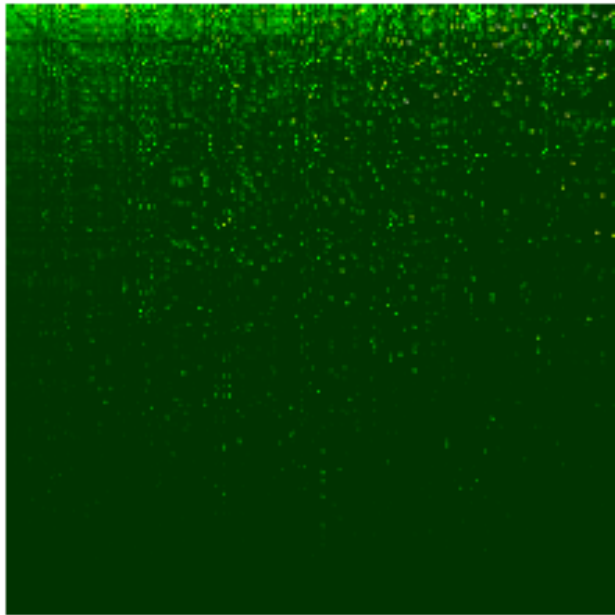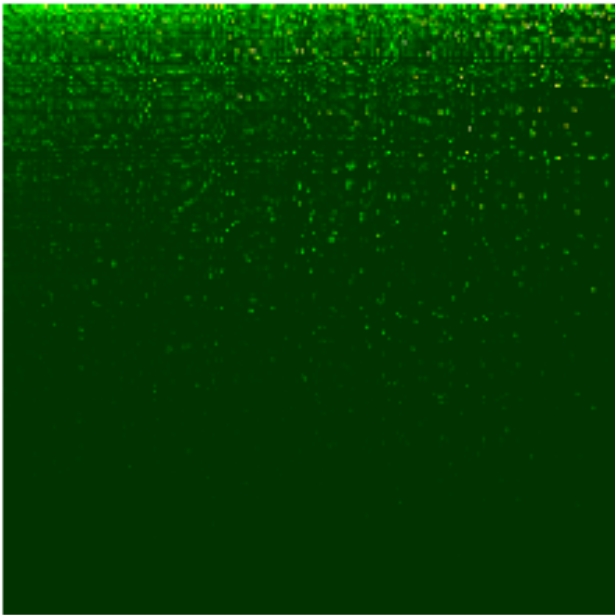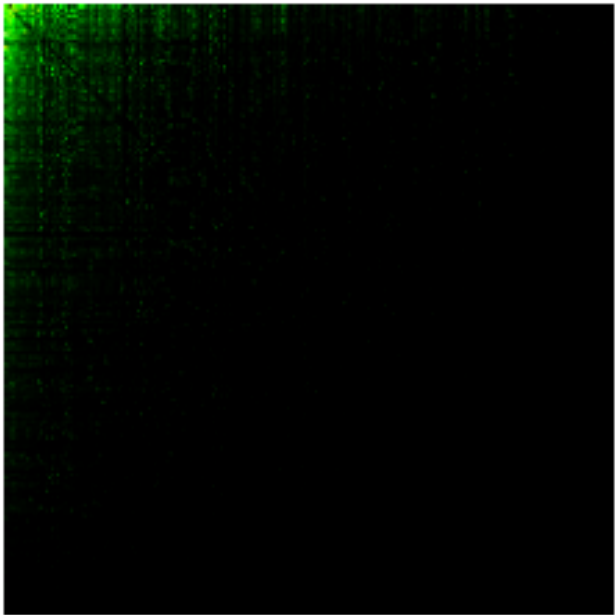
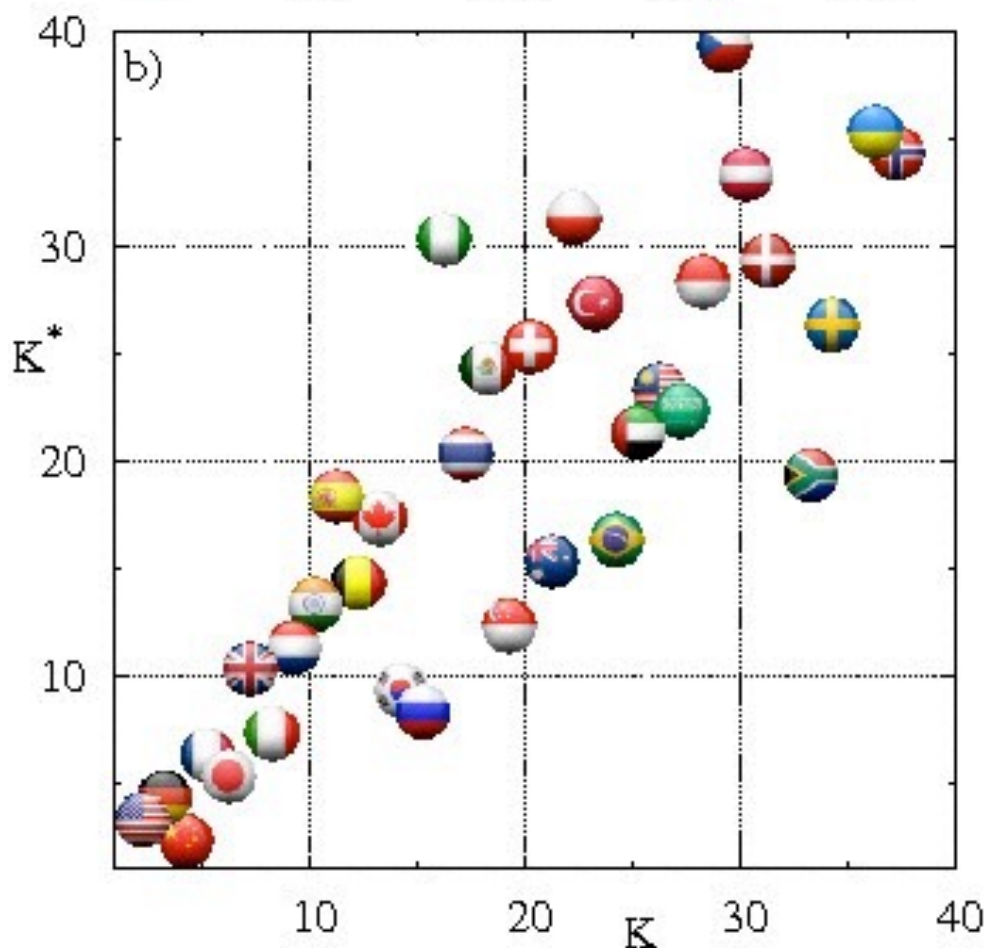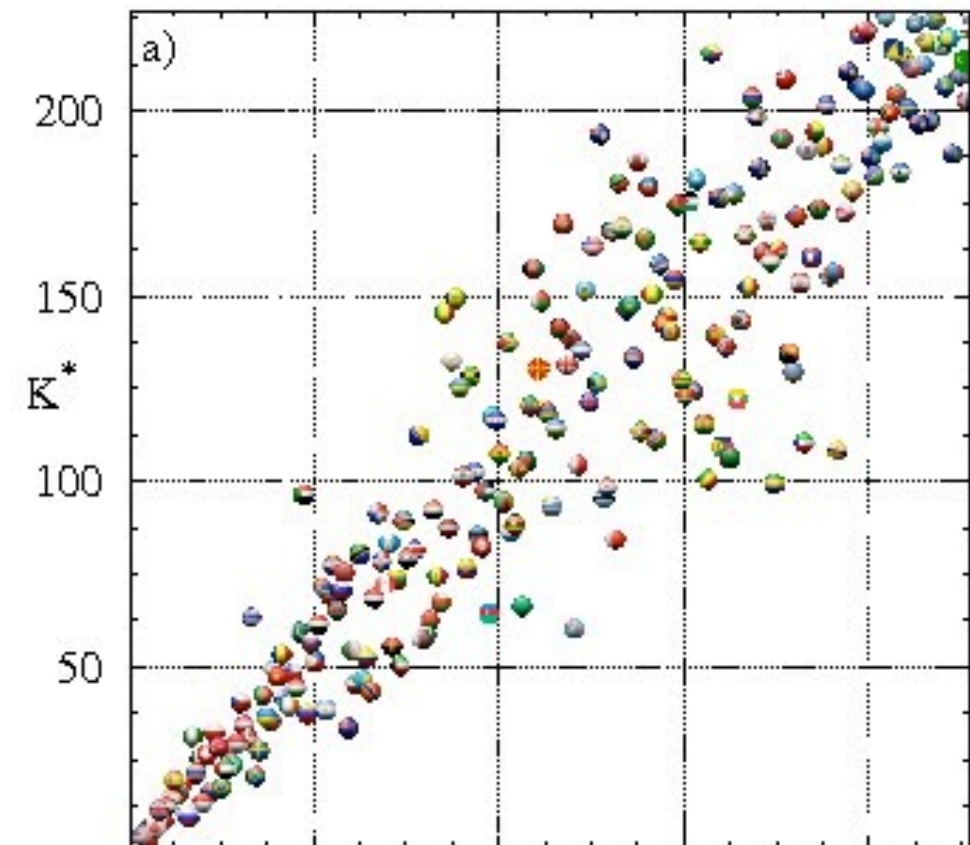PageRank, CheiRank, ImportRank, ExportRank $\alpha = 0.5$ Spectra $\alpha = 1$

Zipf law P~1/K

all comm

petroleum

less symmetric

λ_{all}
λ_{food}
$\lambda_{cereals}$
λ_{barley}
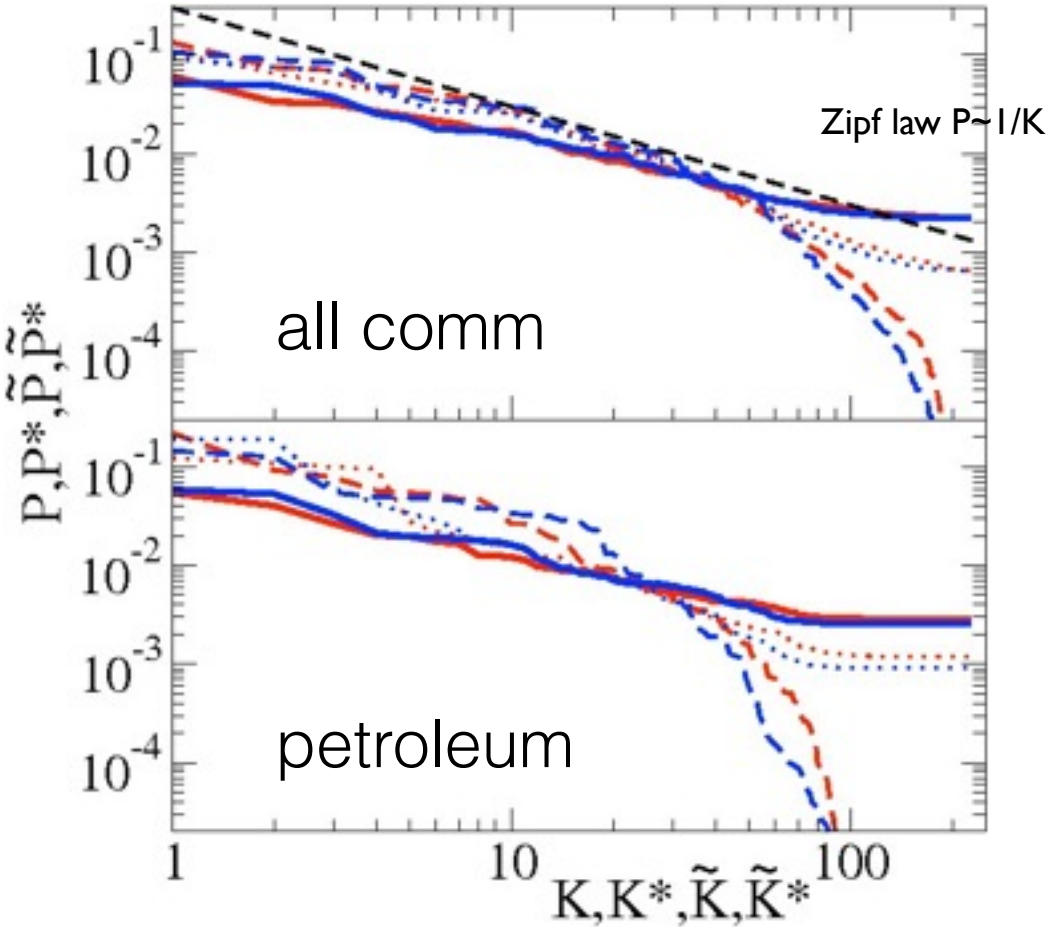
PageRank, CheiRank vs. ImportRank, ExportRank

countries are treated on equal democratic ground
G-20 ~ 74%

Ran	K	K^*	K_2	$\tilde{K}$	$\tilde{K}^*$
1	USA	China	USA	USA	China
2	Germany	USA	China	Germany	Germany
3	China	Germany	Germany	China	USA
4	France	Japan	Japan	France	Japan
5	Japan	France	France	Japan	France
6	UK	Italy	Italy	UK	Netherlands
7	Italy	Russian Fed.	UK	Netherlands	Italy
8	Netherlands	Rep. of Korea	Netherlands	Italy	Russian Fed.
9	India	UK	India	Belgium	UK
10	Spain	Netherlands	Rep. of Korea	Canada	Belgium
11	Belgium	Singapore	Belgium	Spain	Canada
12	Canada	India	Russian Fed.	Rep. of Korea	Rep. of Korea
13	Rep. of Korea	Belgium	Canada	Russian Fed.	Mexico
14	Russian Fed.	Australia	Spain	Mexico	Saudi Arabia
15	Nigeria	Brazil	Singapore	Singapore	Singapore
16	Thailand	Canada	Thailand	India	Spain
17	Mexico	Spain	Australia	Poland	Malaysia
18	Singapore	South Africa	Brazil	Switzerland	Brazil
19	Switzerland	Thailand	Mexico	Turkey	India
20	Australia	U. Arab Emir.	U. Arab Emir.	Brazil	Switzerland

$$\tilde{K}^* = 11 \longrightarrow K^* = 16$$

$$\tilde{K}^* = 13 \longrightarrow K^* > 20$$

$$\tilde{K}^* = 15 \longrightarrow K^* = 11$$

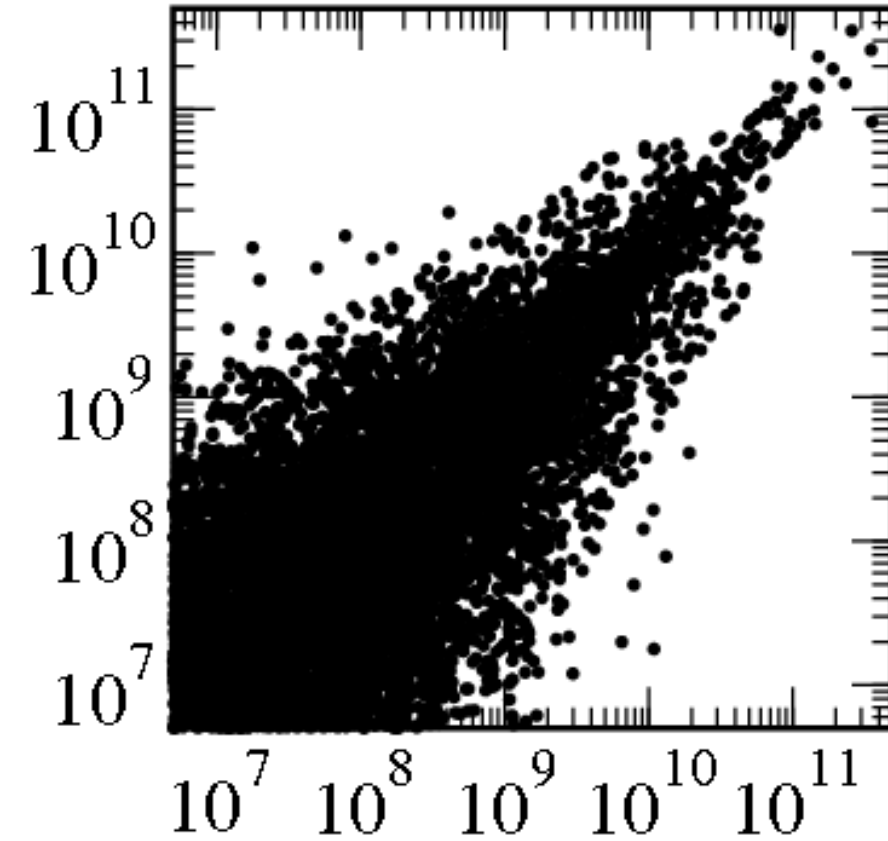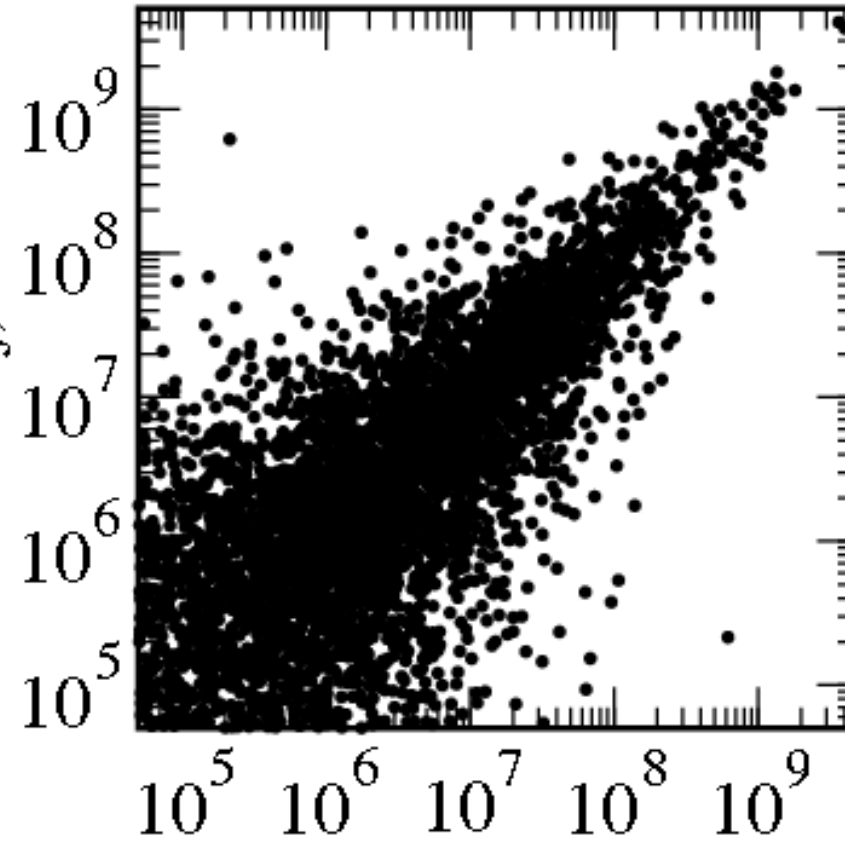$$\tilde{K}^* = 19 \longrightarrow K^* = 12$$

WTN model

- Gravity model of trade:
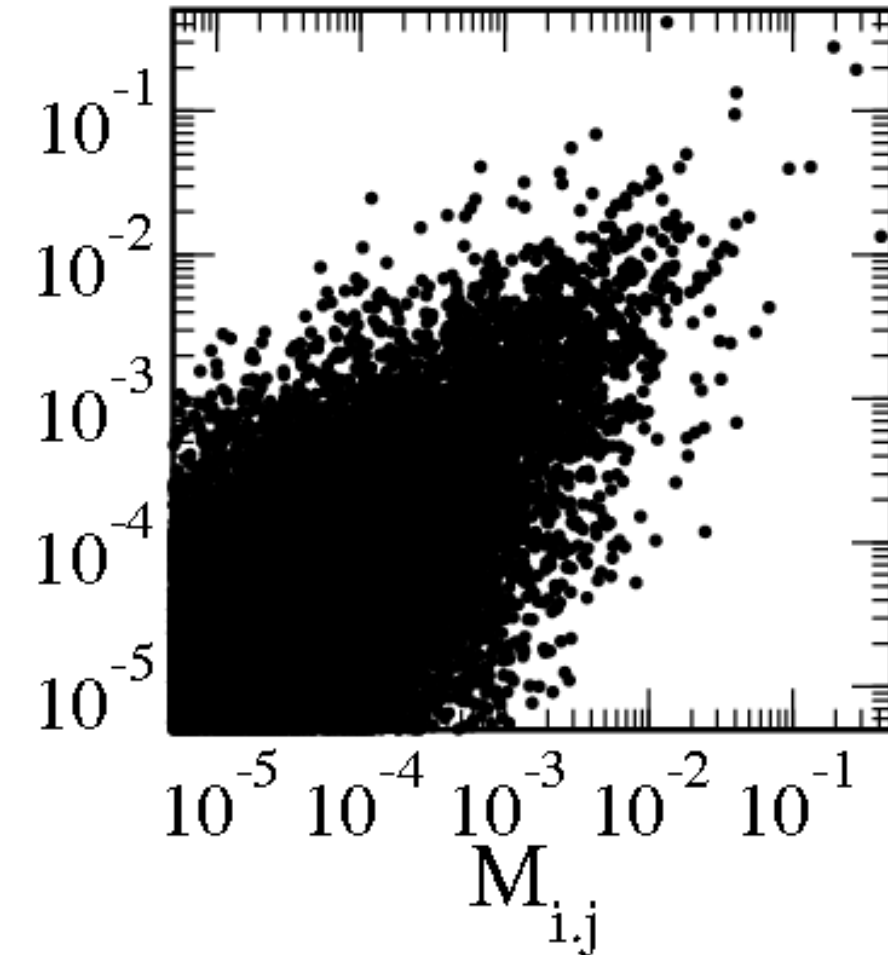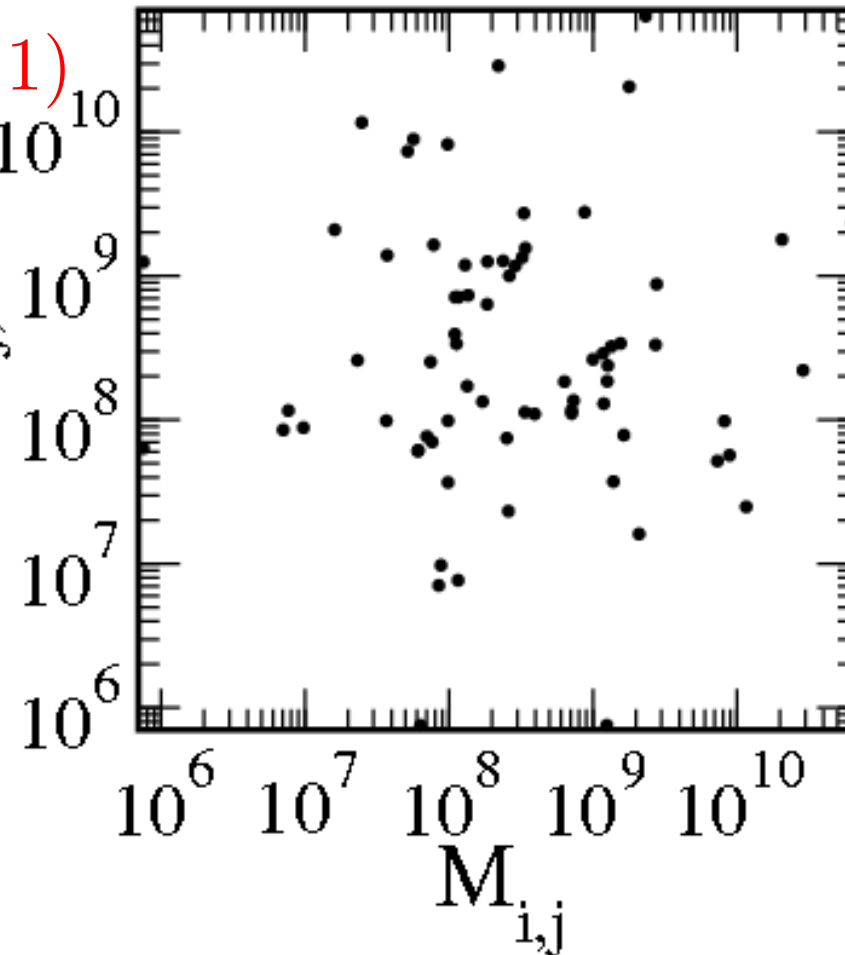
$$M_{i,j} = gm_i m_j / D_{i,j}$$

(symmetric)

- Random model

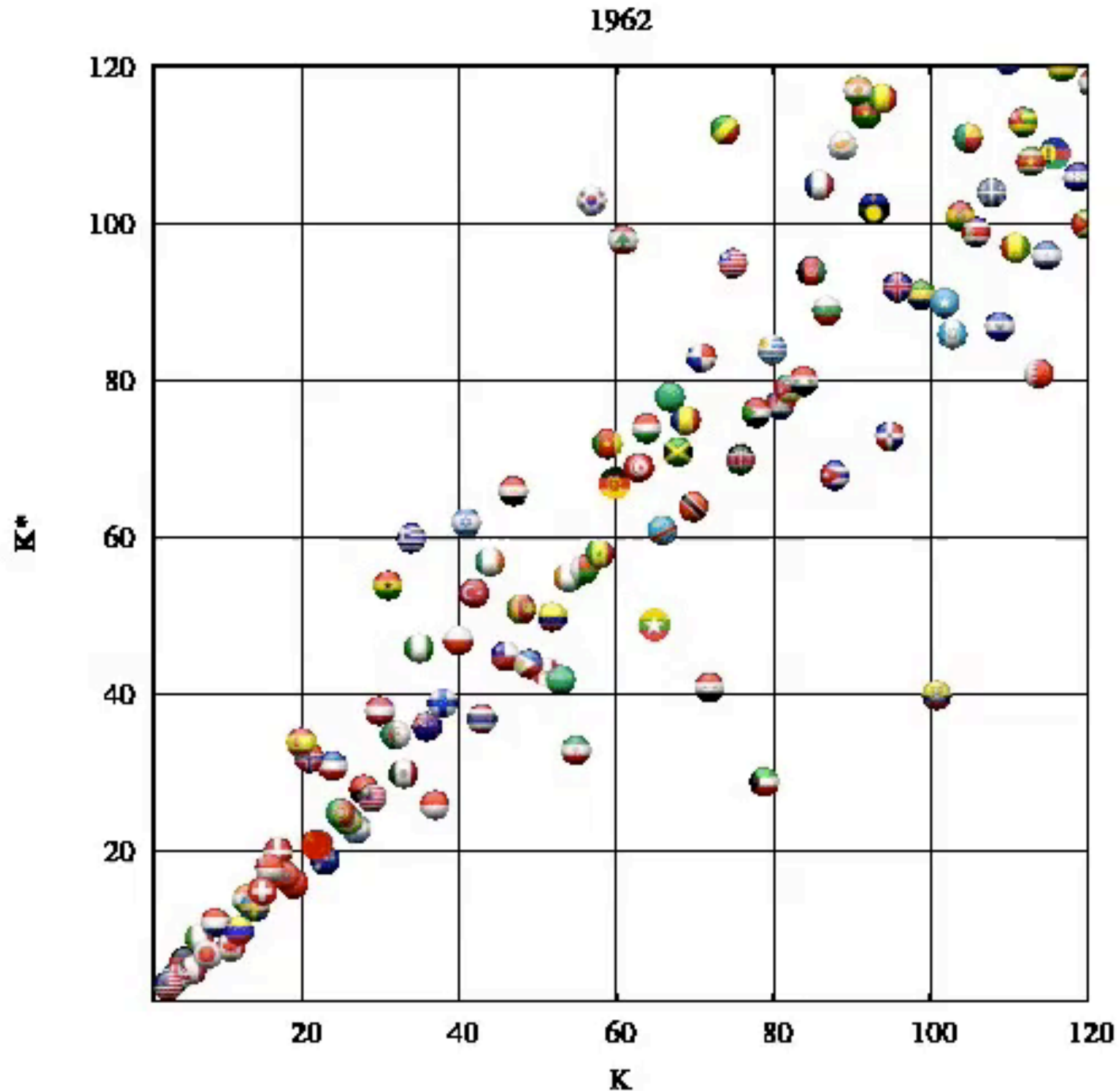$$M_{i,j} = \epsilon_i \epsilon_j / ij \qquad \epsilon_{i,j} \in [0,1)$$

(preserves Zipf law)

t:: all commodities (1962, 2008);
b: crude petroleum (2008), random model
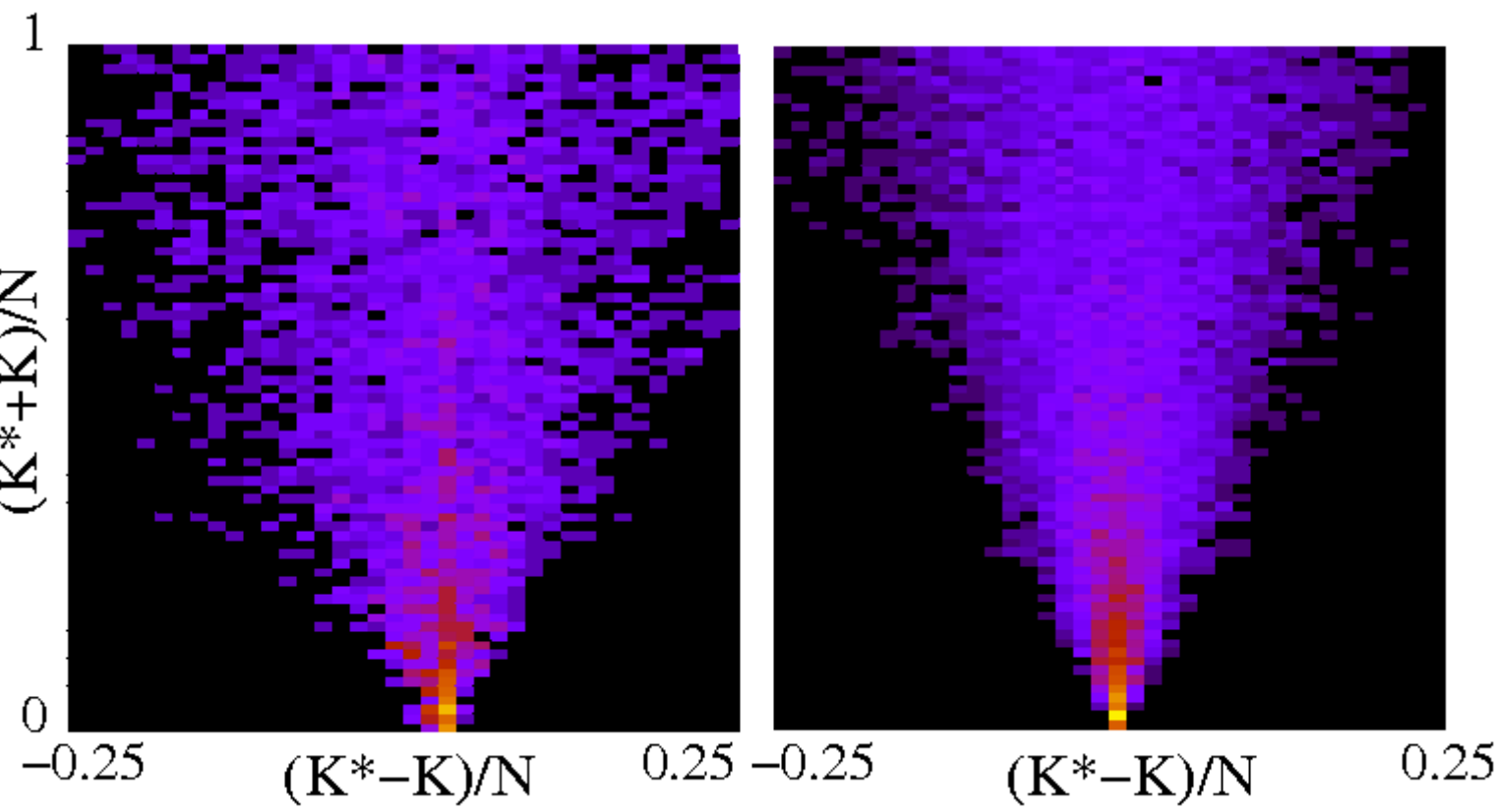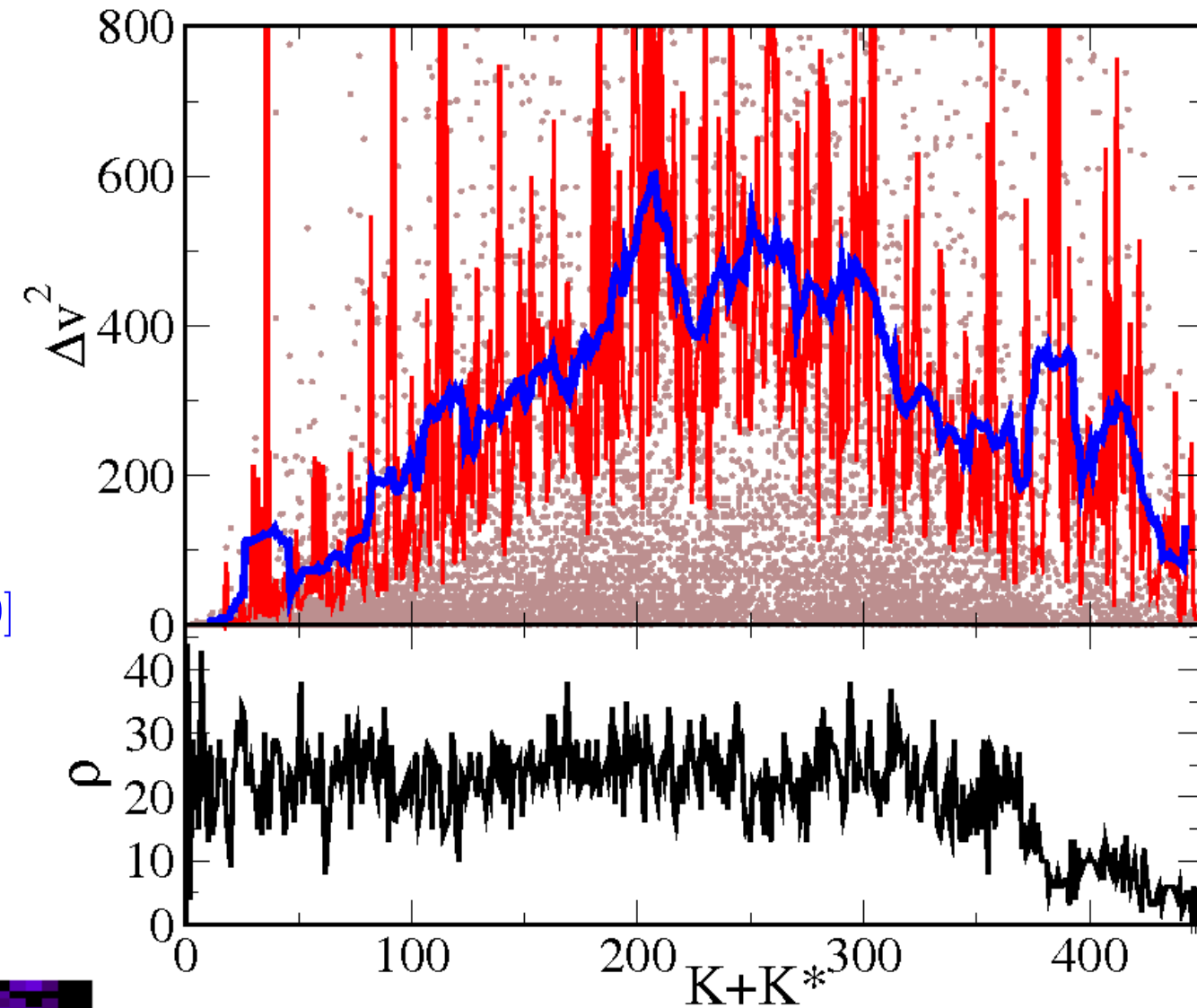
2d rank evolution

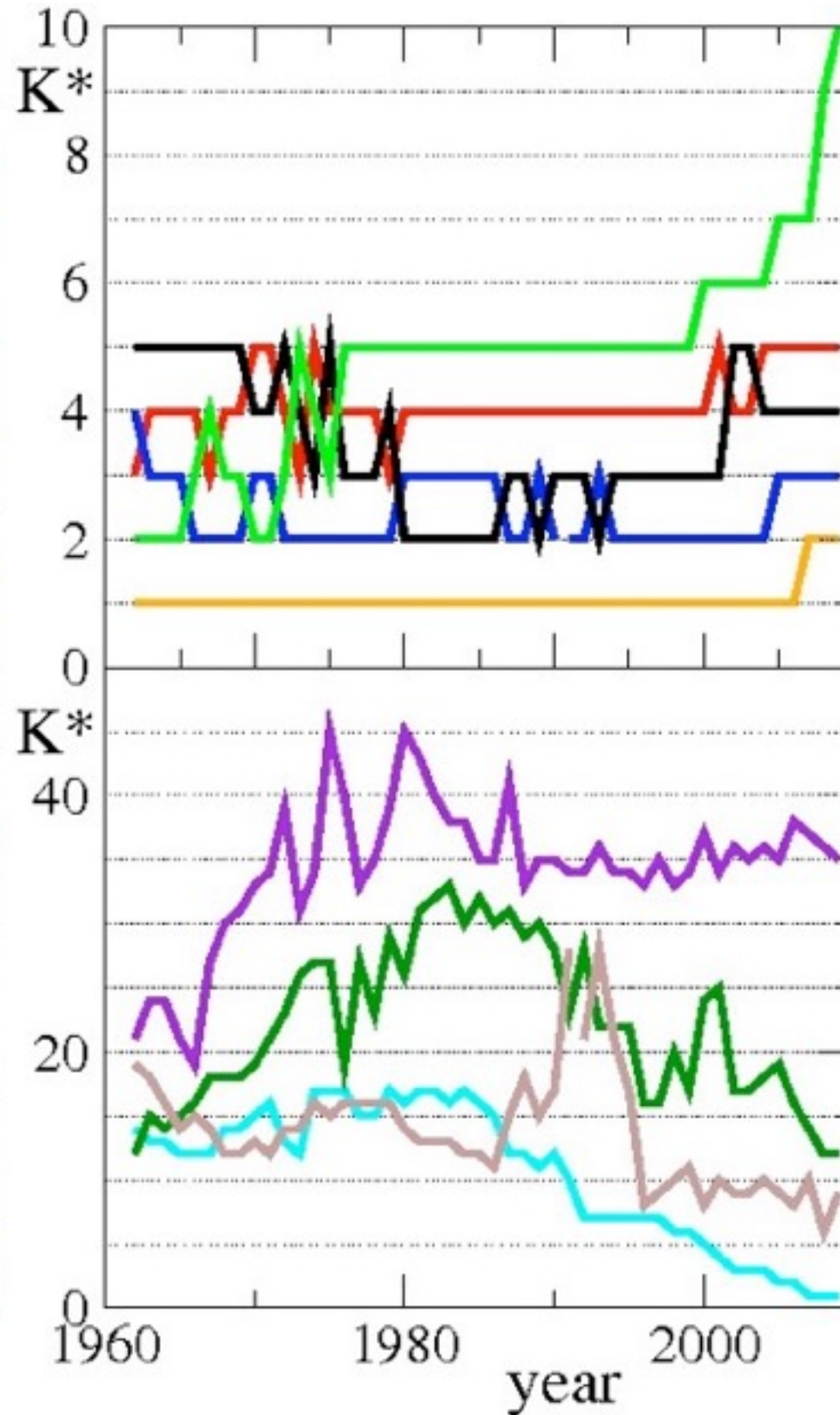
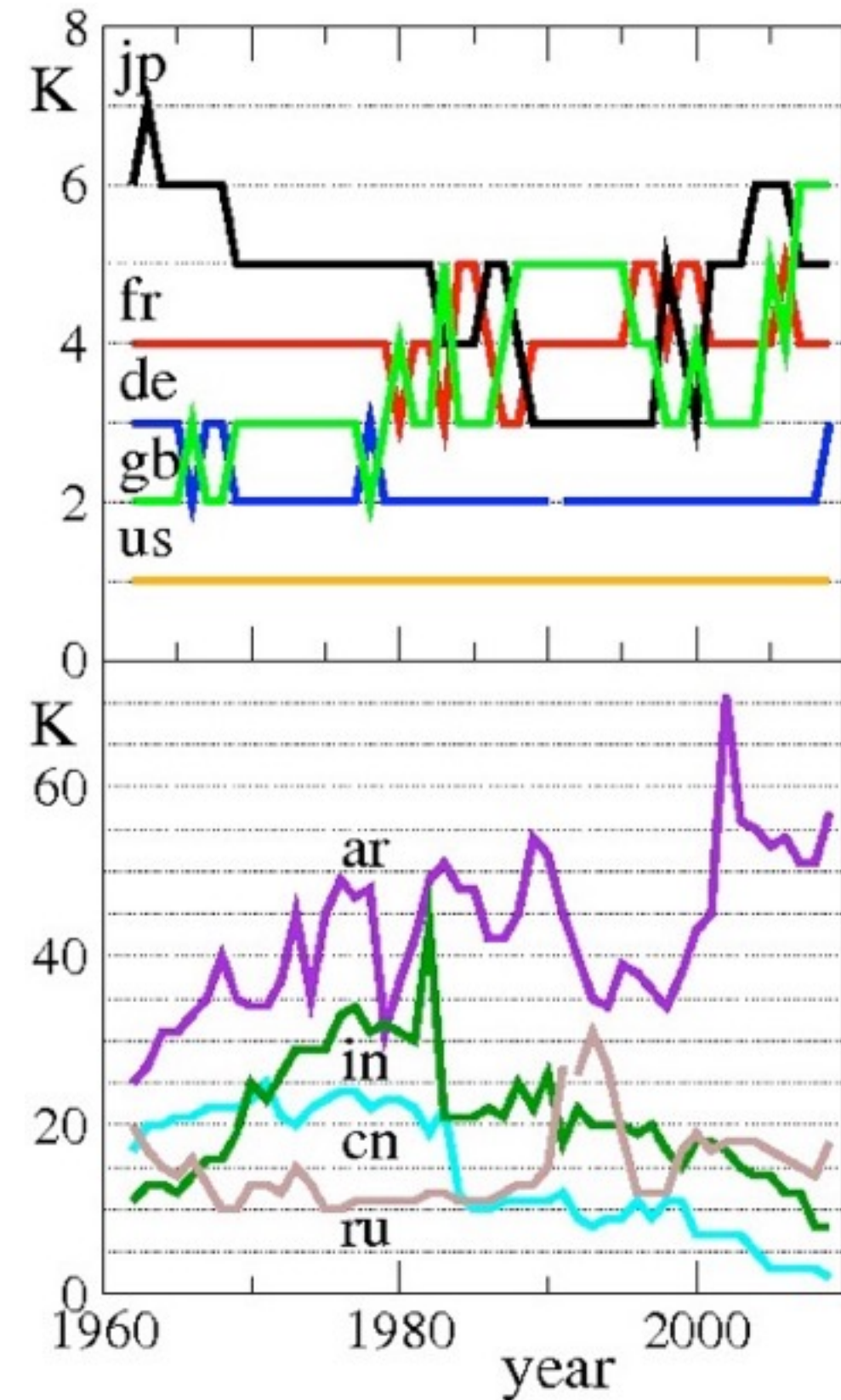1962

2d rank evolution

Velocity square vs. K+K*

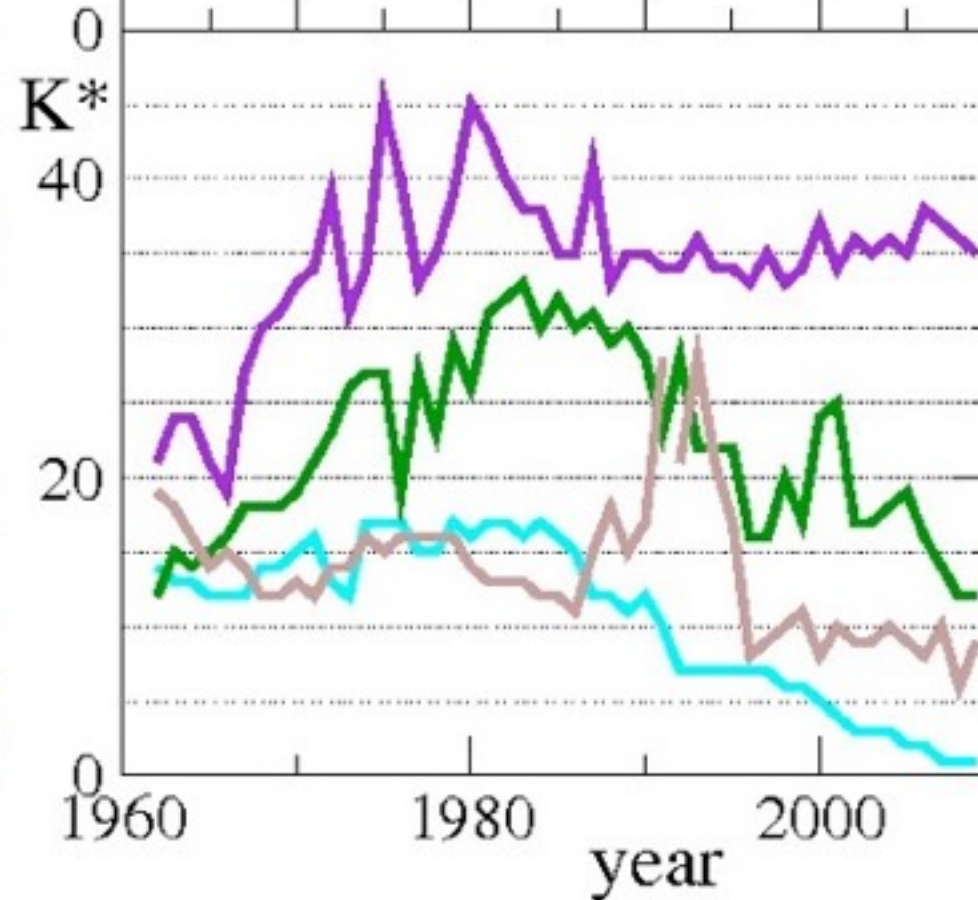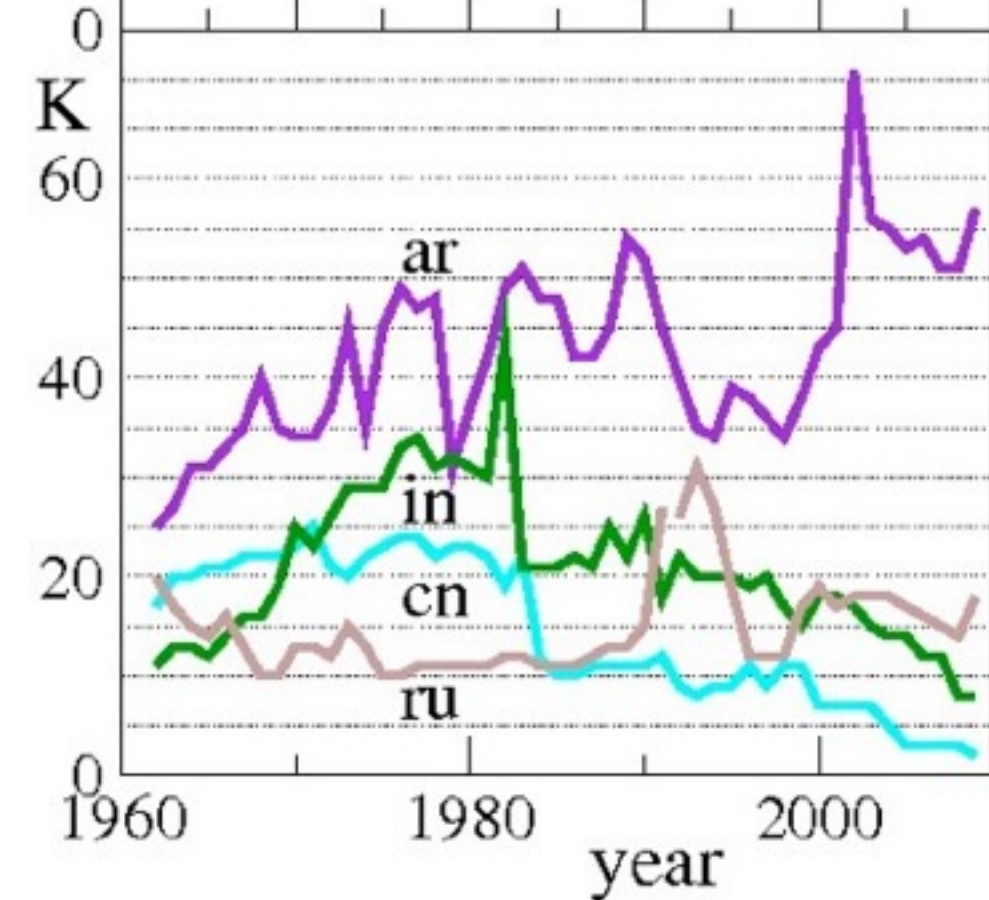$$\Delta v^2 = [K(t) - K(t-1)]^2 + [K^*(t) - K^*(t-1)]^2$$

average per $K + K^*$

average in $[K + K^* - 10, K + K^* + 10]$

2d rank evolution

Japan
France
Fed. Rep. of Germany
and Germany
Great Britain (sublimation?)
USA

Argentina
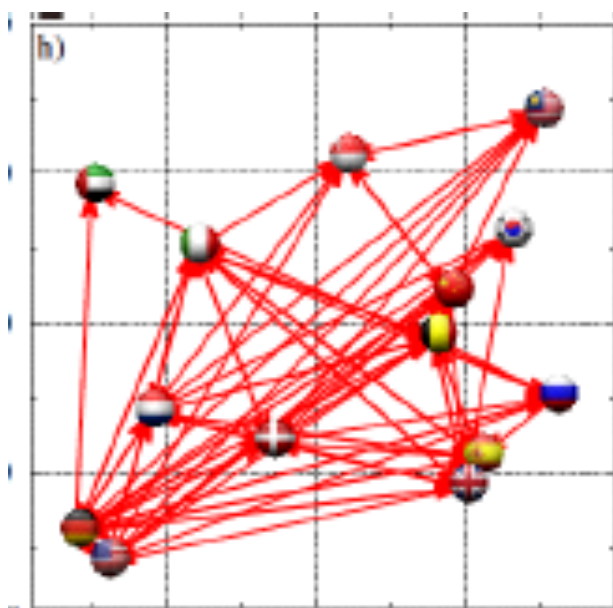India
China (deposition)
USSR and Russian Fed.

Google of multi-product WTN

1) WTN (all com. or 1 prod)
N=227

non-interacting products

2) WTN (multiprod)
N=13847

personalized vector $\longrightarrow$ prop to Vp of each c

2nd iteration $\longrightarrow$ reduced Pp for all c

$$v_i = \frac{V_c^p}{N_c \sum_{p'} V_c^{p'}}, \quad v_i^* = \frac{V_c^{*p}}{N_c \sum_{p'} V_c^{*p'}},$$

$$v'(i) = \frac{P_p}{N_c}, \quad v'^*(i) = \frac{P_p^*}{N_c}.$$
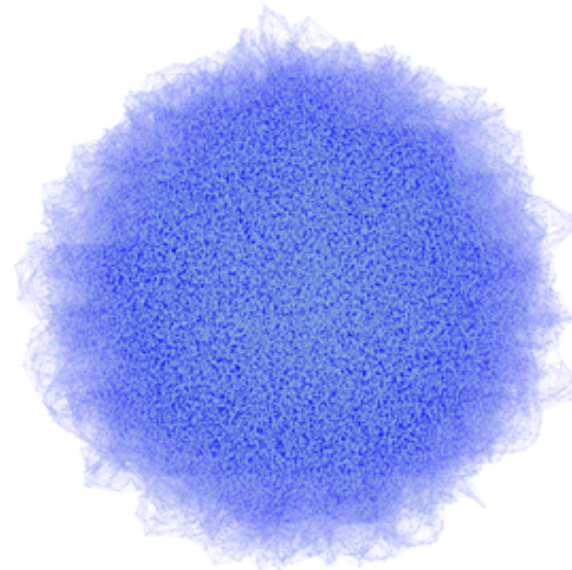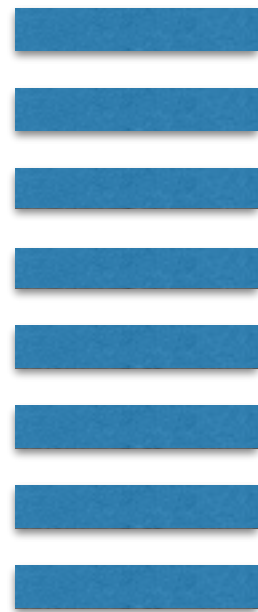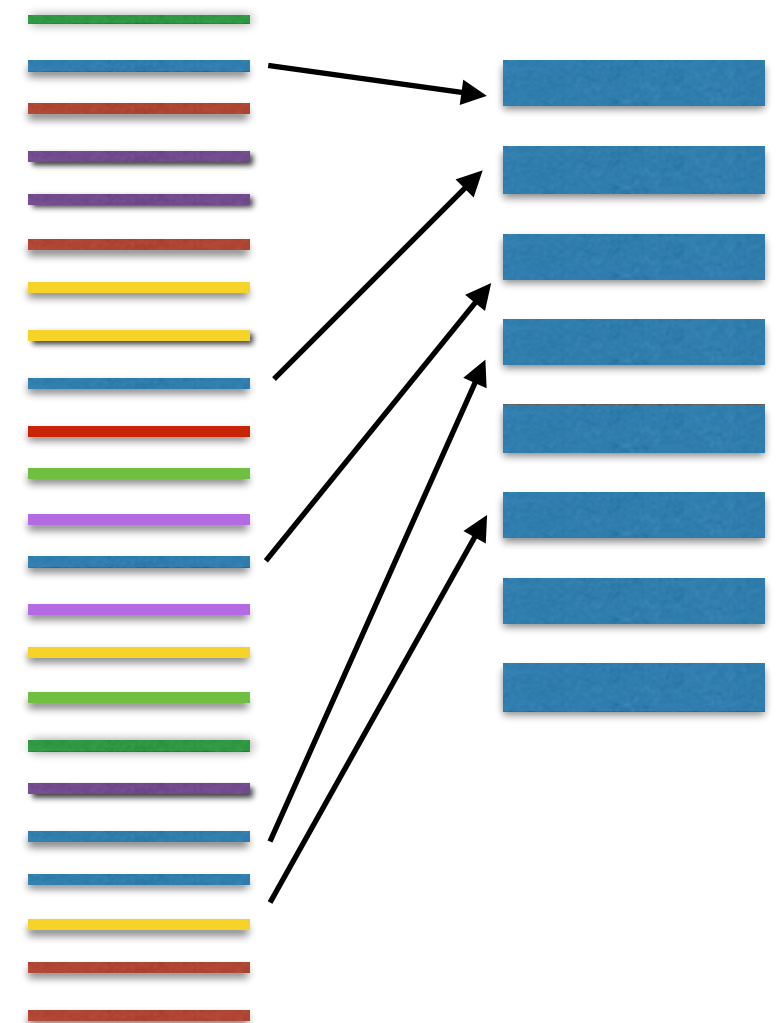
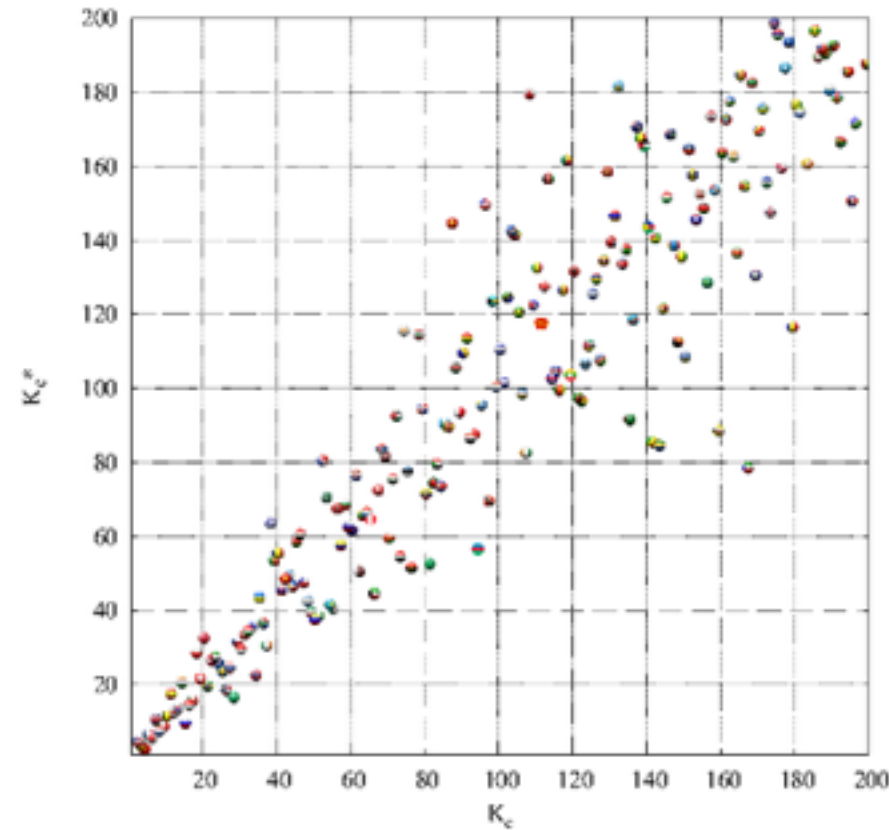L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011),
http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank
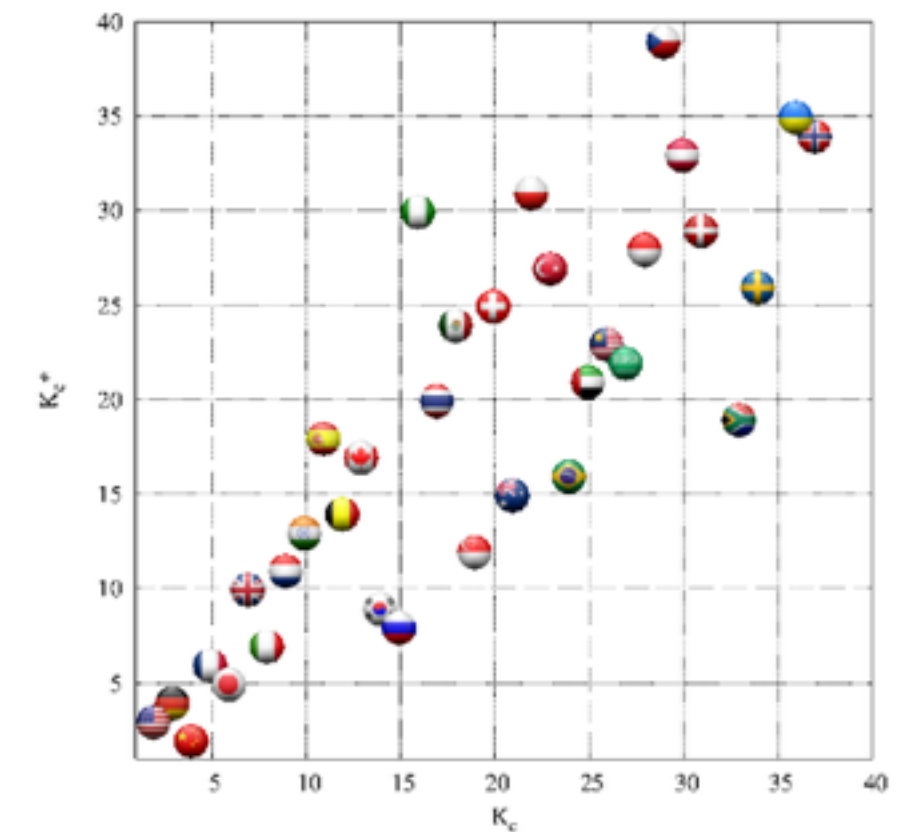
L. Ermann and D.L. Shepelyansky, EPJB (2015).

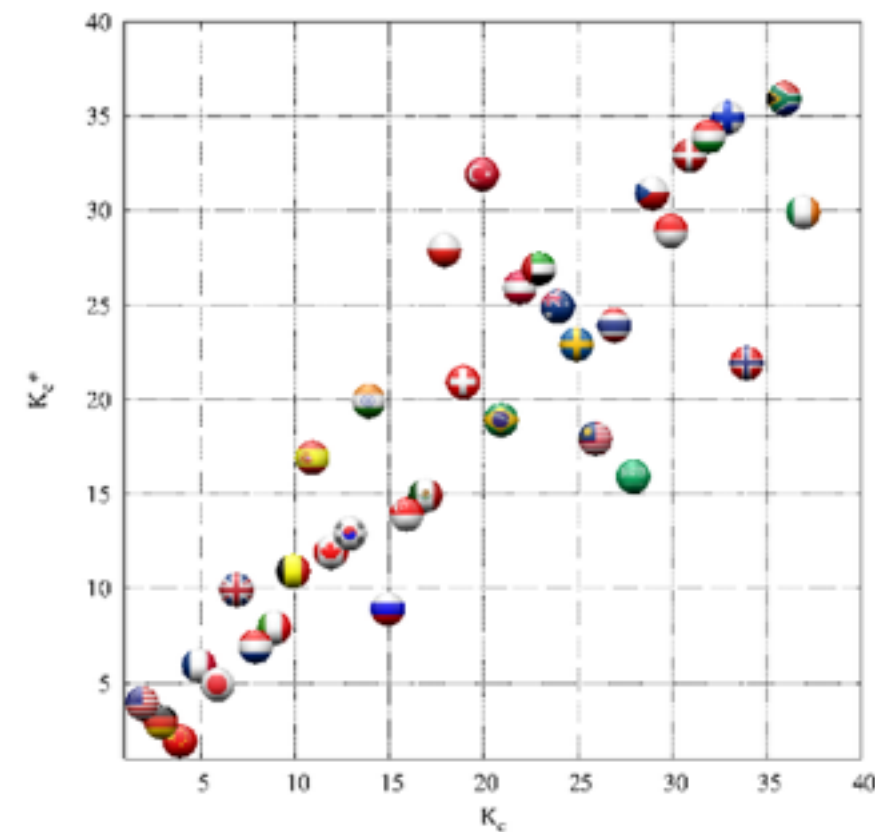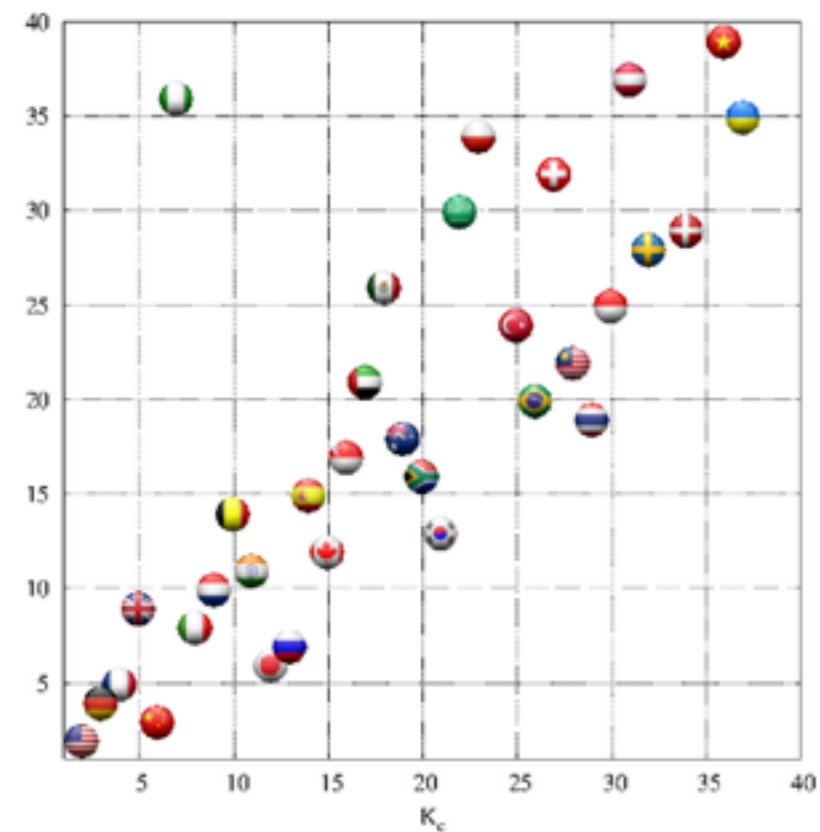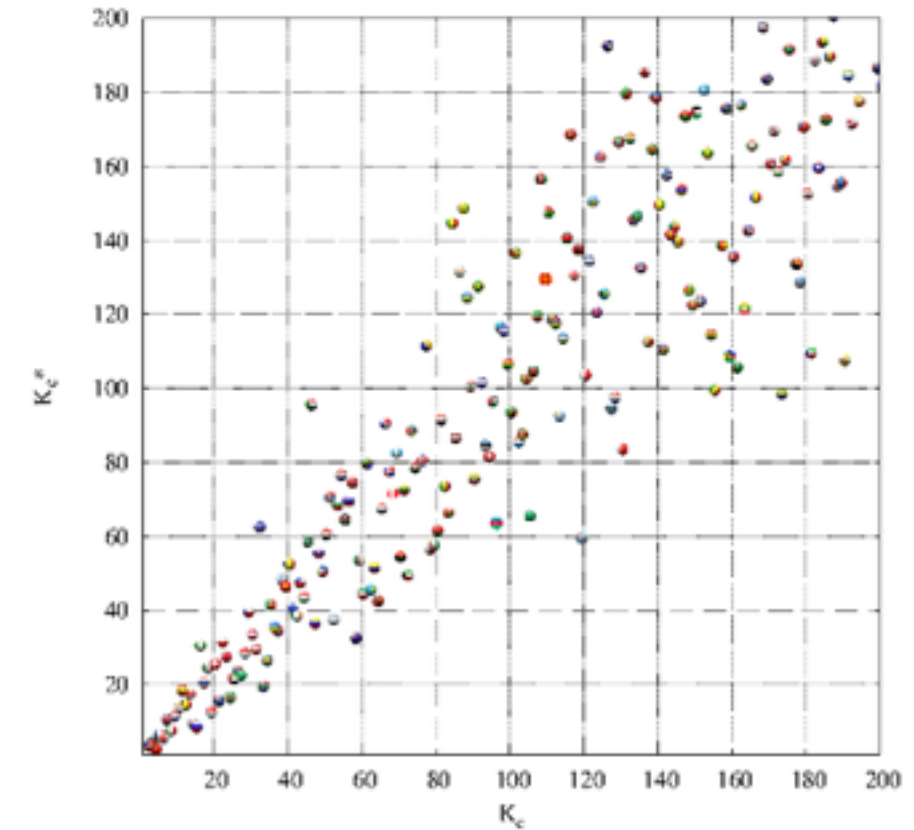# 2d ranking of countries (multiproducts)
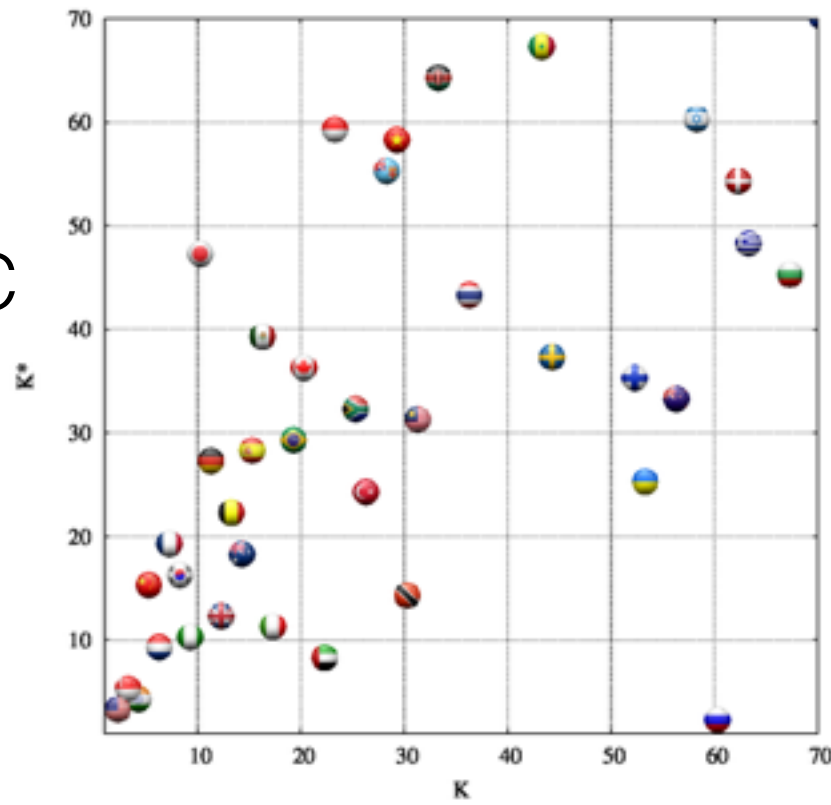
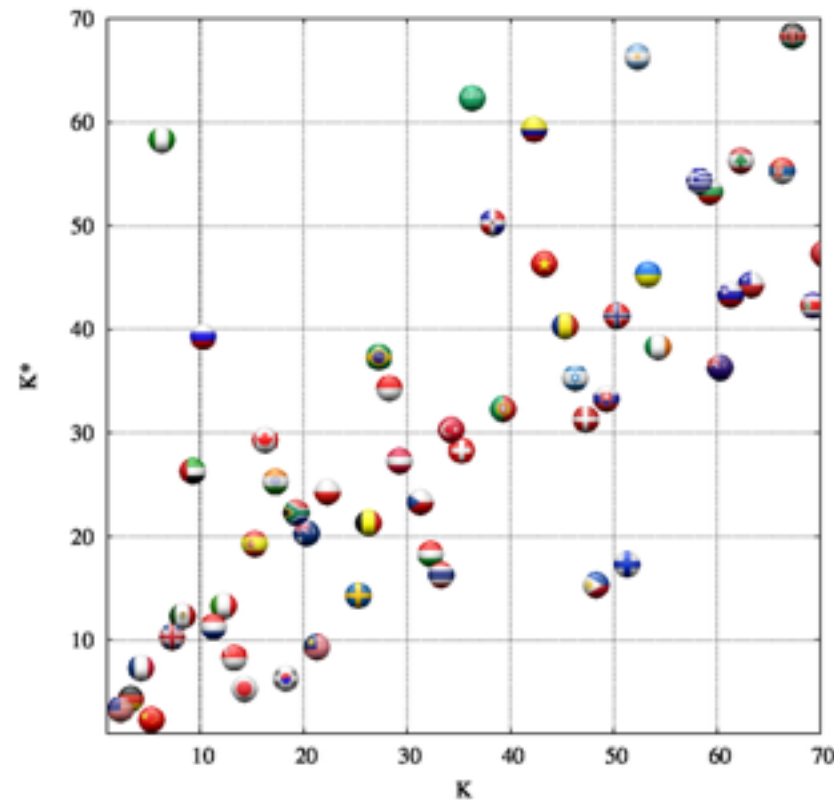

PageRank–CheiRank
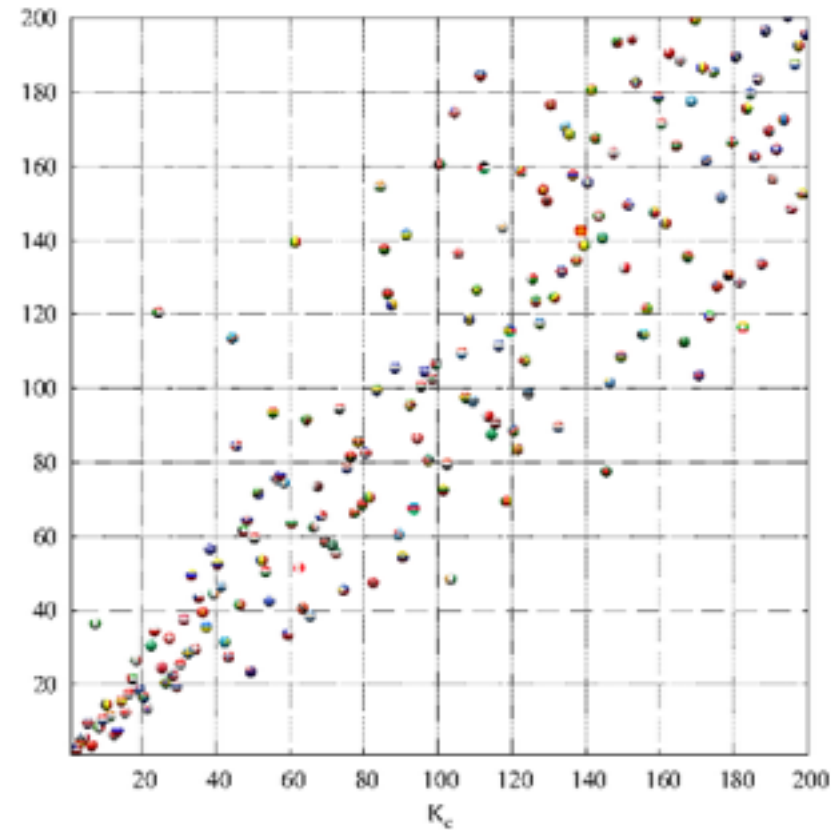
ImportRank–ExportRank

PageRank–CheiRank: all commodities

2d reduced ranks

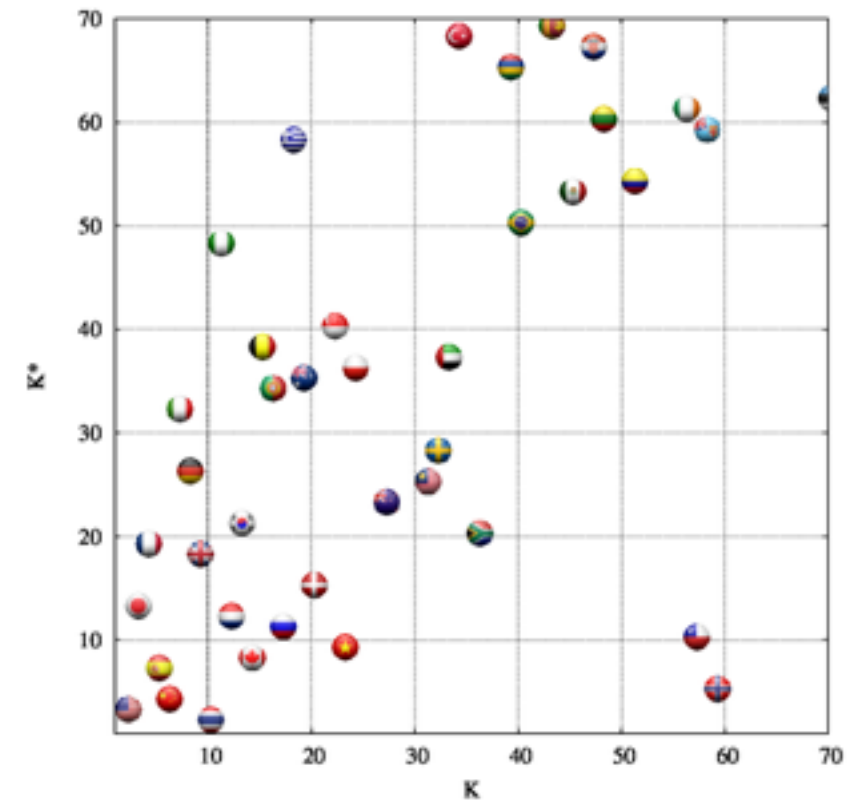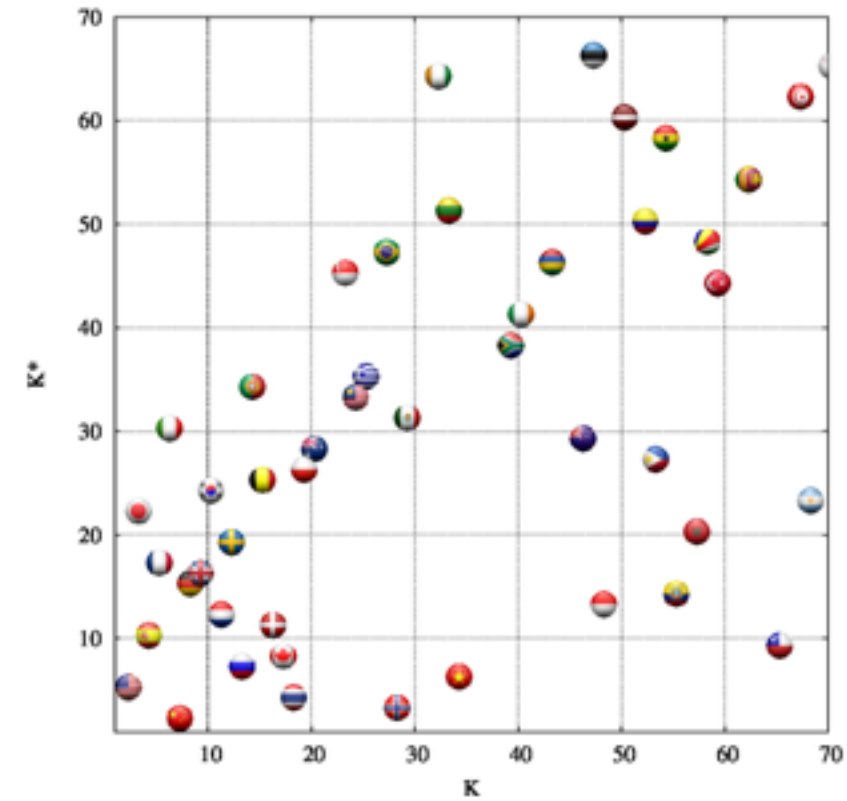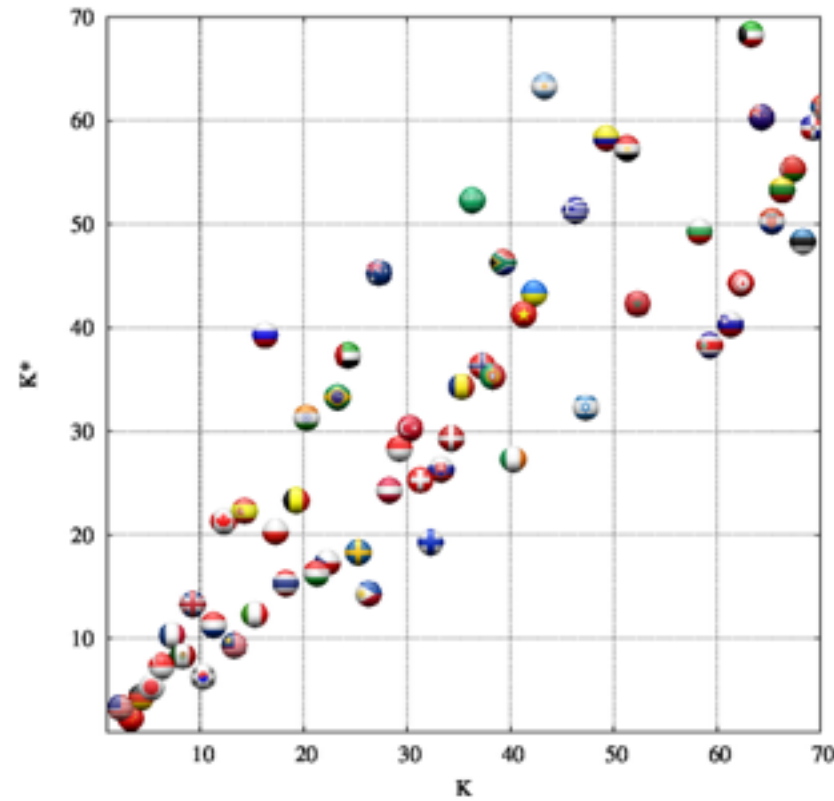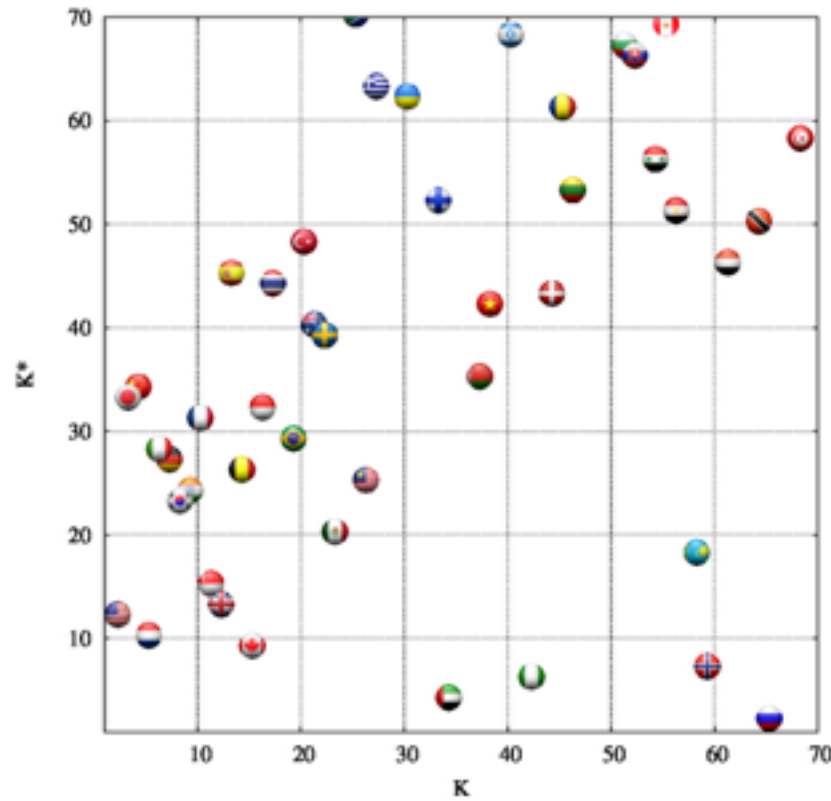
33 – Petroleum and petroleum products
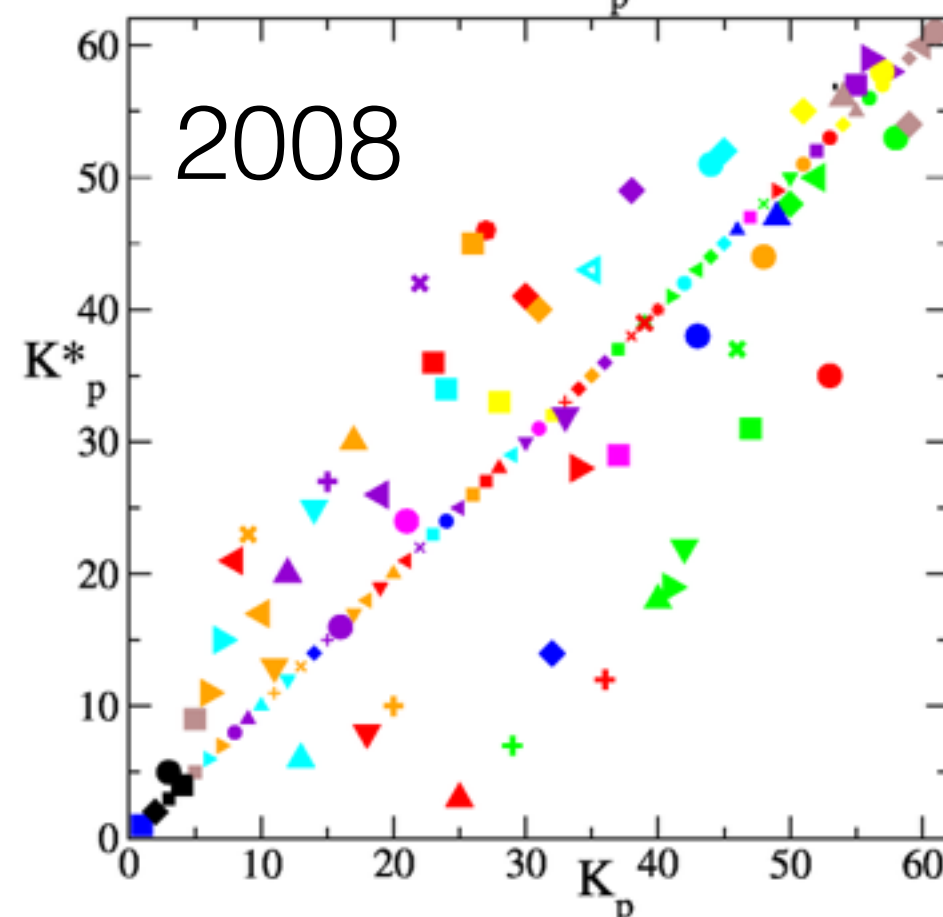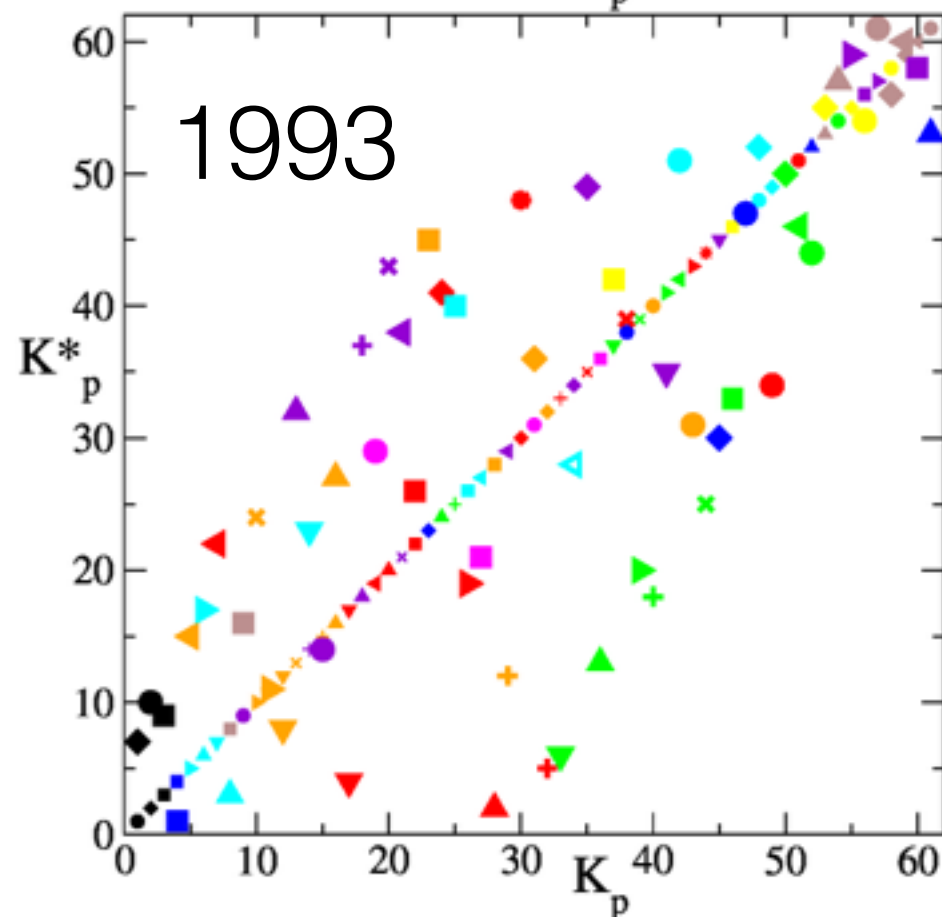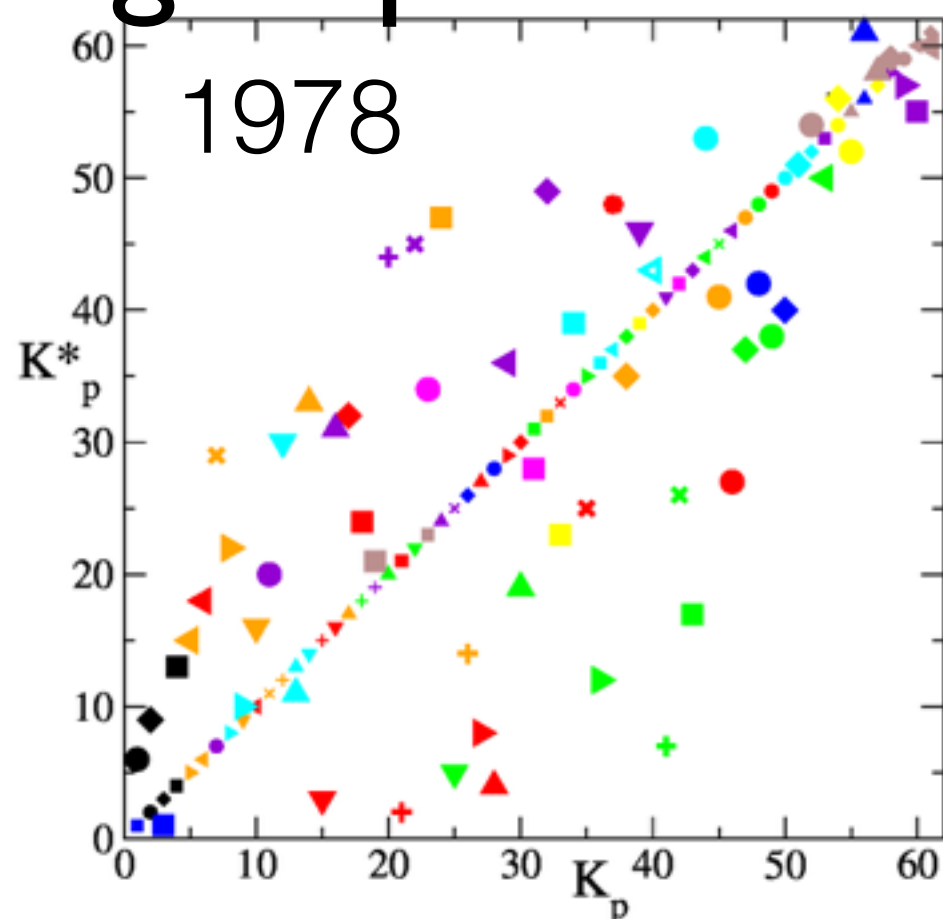
72 – Electrical machinery, apparatus, and appliances

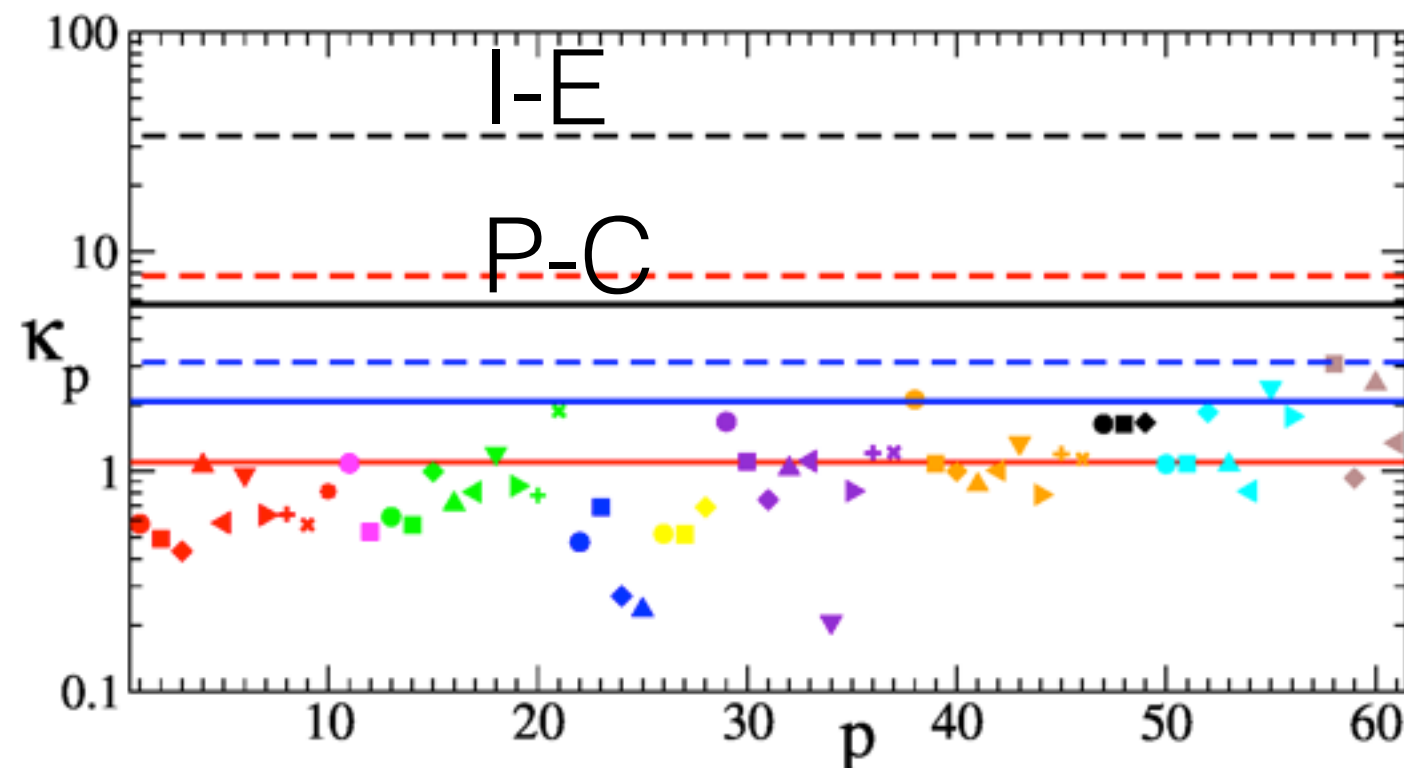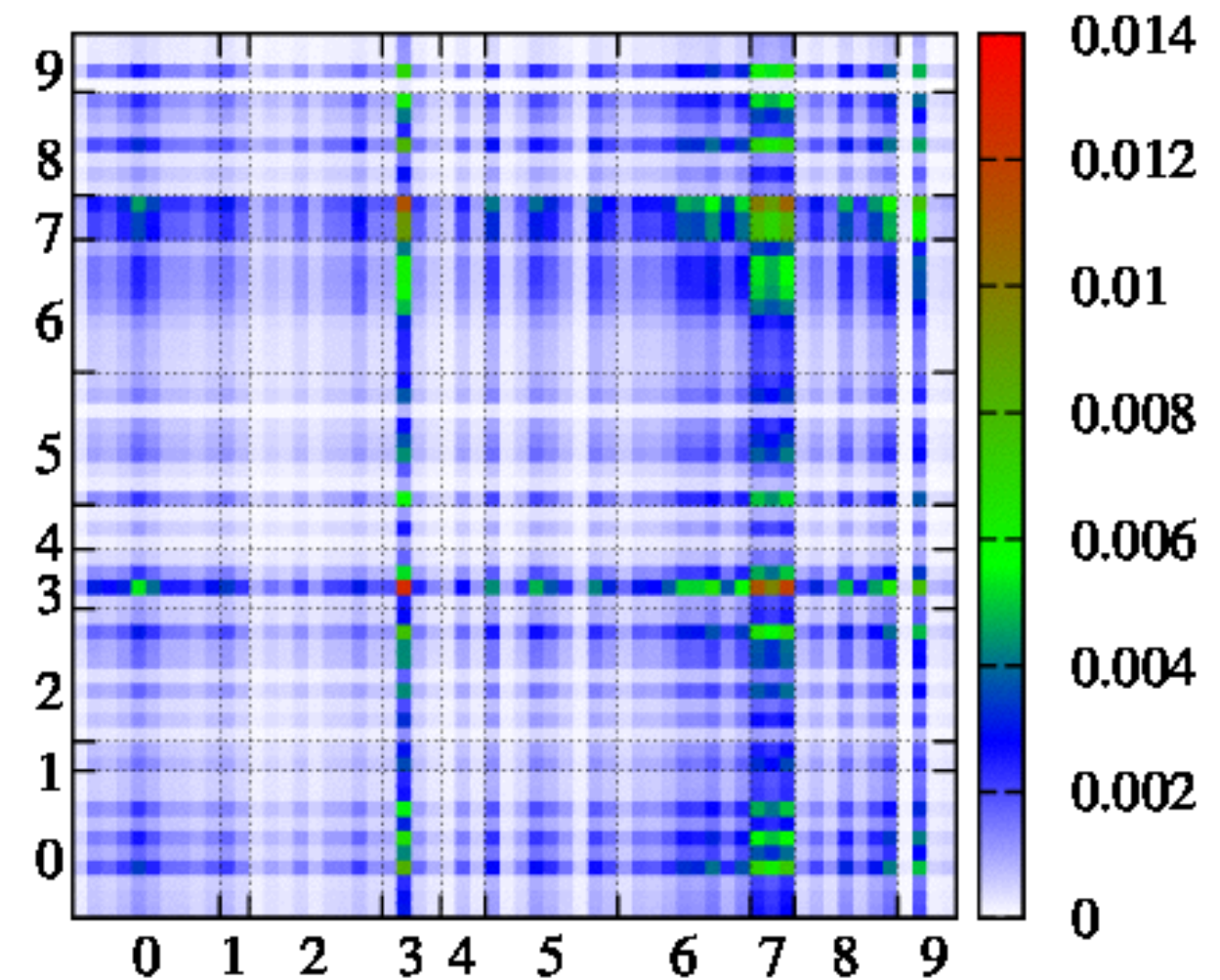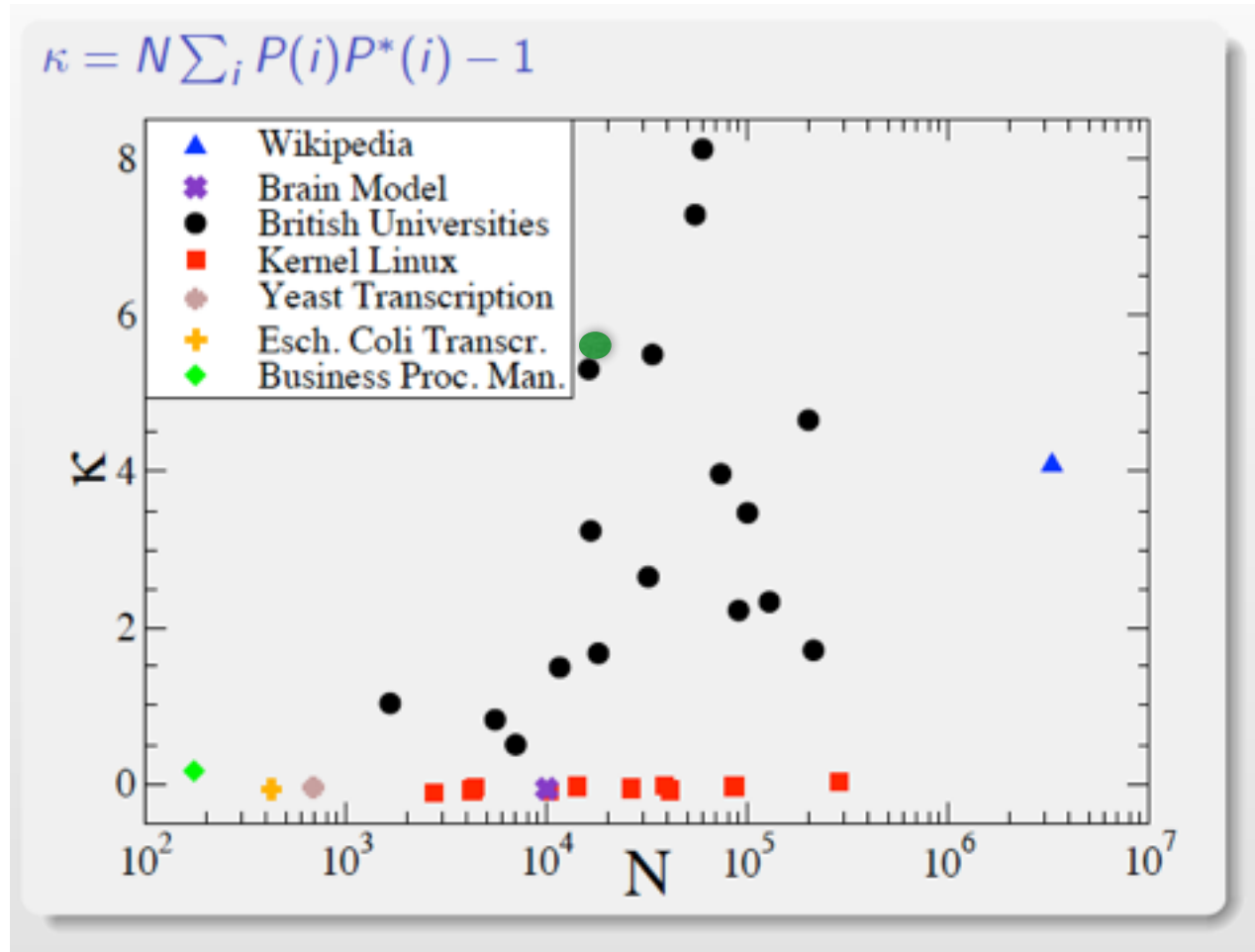03 – Fish and fish preparations

P-C

I-E

2d ranking of products

1963

1978

1993

2008
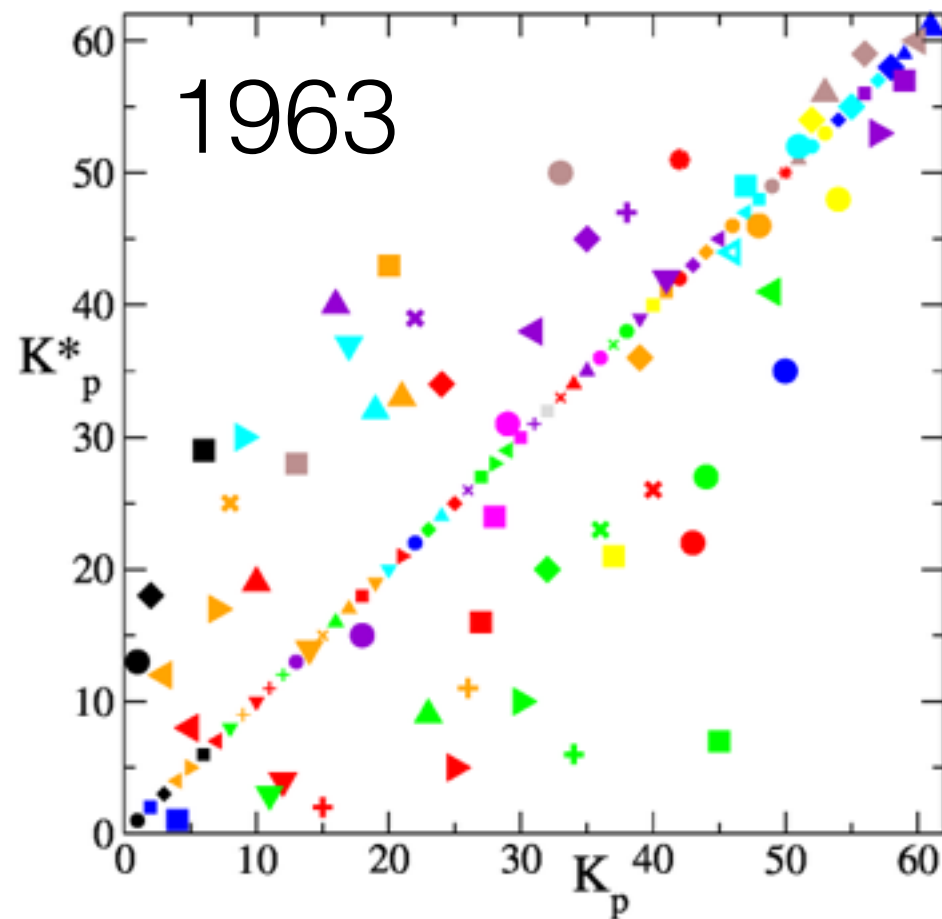
Live animals
Meat and meat preparations
Dairy products and eggs
Fish and fish preparations
Cereals and cereal preparations
Fruit and vegetables
Sugar, sugar prep. and honey
Coffee, tea, cocoa, spices & man
Feed. Stuff for animals
Miscellaneous food preparations
Beverages
Tobacco and tobacco manuf.
Hides, skins and fur skins, undress.
Oil seeds, oil nuts and oil kernels
Crude rubber incl. synth & recl.
Wood, lumber and cork
Pulp and paper
Textile fibres, not manuf., & waste
Crude fertilizers & crude minerals
Metalliferous ores and metal scrap
Crude animal & vegetable mat.
Coal, coke and briquettes
Petroleum and petroleum products
Gas, natural and manufactured
Electric energy
Animal oils and fats
Fixed vegetable oils and fats
Animal & veg. oils & fats (proc.)
Chemical elements & compounds
Crude chem. from coal, petr. & gas
Dyeing, tanning & colouring mat.

Medicinal & pharmaceutical prod.
Perfume mat., toilet & clean. prep.
Fertilizers, manufactured
Explosives and pyrotechnic prod.
Plastic materials, etc.
Chemical materials and products
Leather. Manuf. & dressed fur skin
Rubber manufactures, nes
Wood & cork manuf. exc. furniture
Paper, paperboard and manuf.
Textile yarn, fabrics, etc.
Non metallic mineral manuf., nes
Iron and steel
Non ferrous metals
Manufacture of metal, nes
Machinery, other than electric
Electrical machinery, apparatus and appliances
Transport equipment
Sanitary, plumbing, heating and lighting fixt.
Furniture
Travel goods, handbags and similar articles
Clothing
Footwear
Scientif & control instrum, photogr pds, clocks
Miscellaneous manufactured articles, nes
Postal packages not class. According to kind
Special transact. Not class. According to kind
Animals, nes, incl. Zoo animals, dogs and cats
Firearms of war and ammunition therefor
Coin, other than gold coin, not legal tender

PageRank CheiRank correlator

$$\kappa = N \sum_i P(i) P^*(i) - 1$$

Legend:
- Wikipedia
- Brain Model
- British Universities
- Kernel Linux
- Yeast Transcription
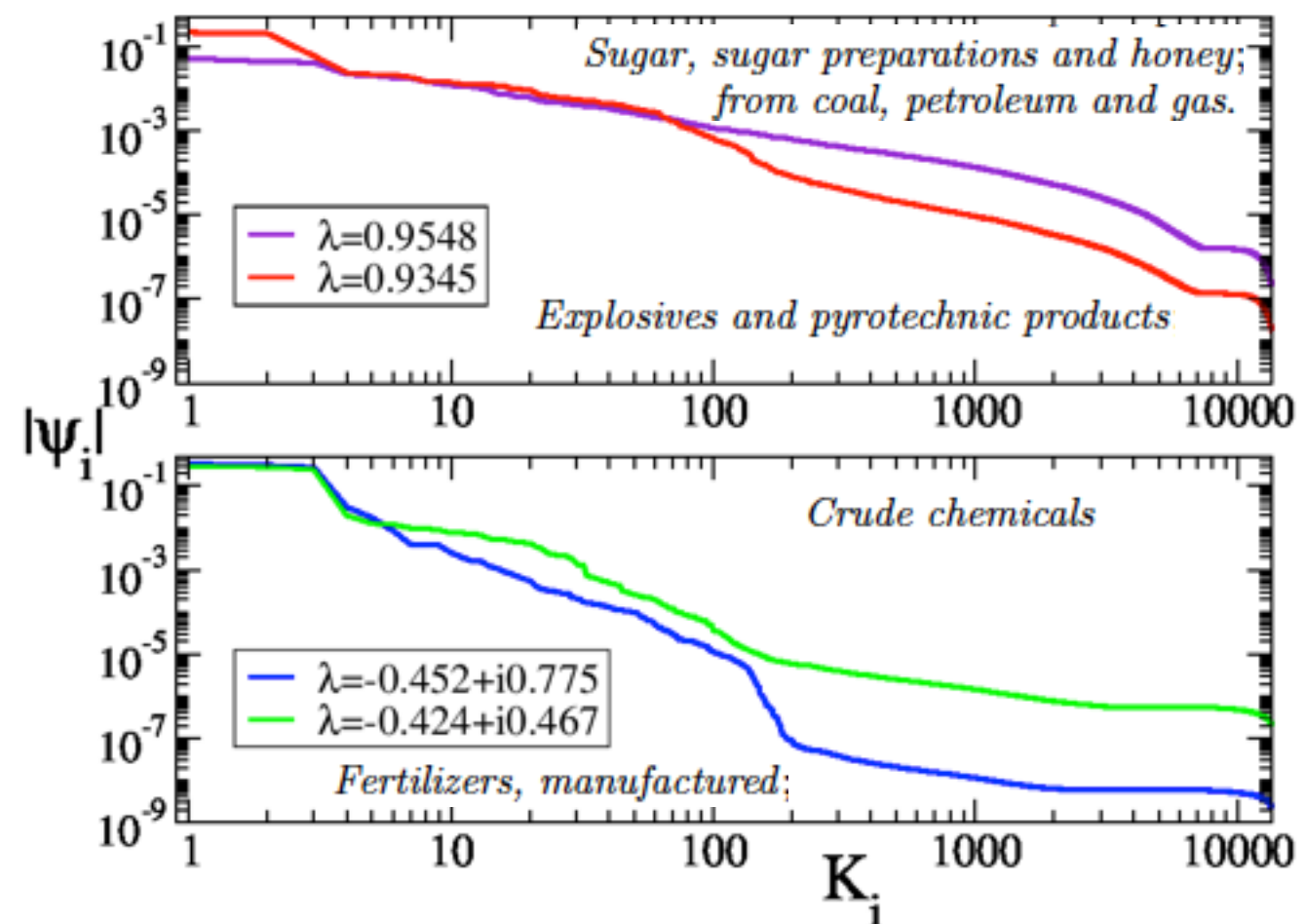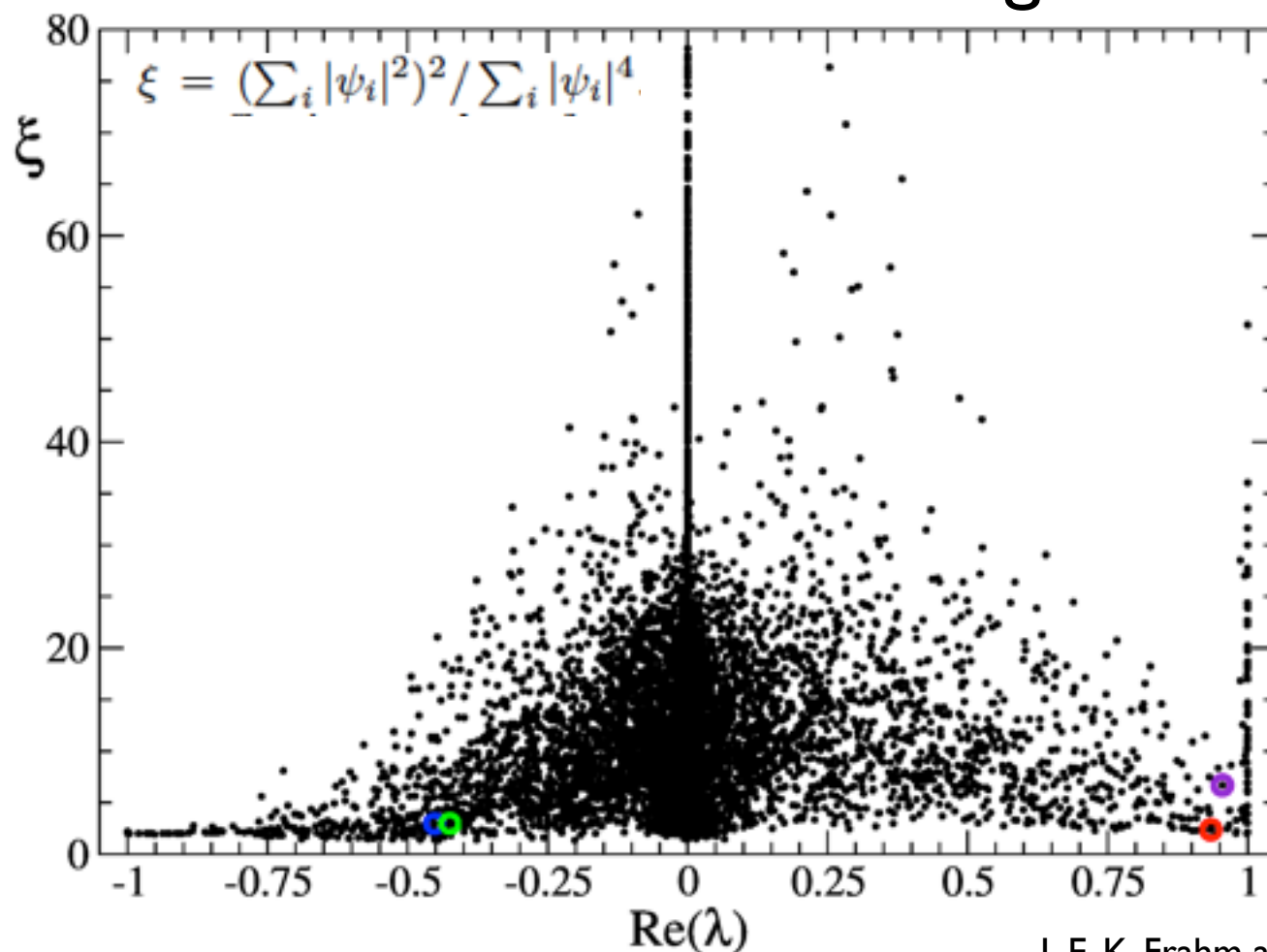- Esch. Coli Transcr.
- Business Proc. Man.

I-E

P-C

$$\kappa_{pp'} = N_c \sum_{c=1}^{N_c} \left[\frac{P(p + (c-1)N_p) P^*(p' + (c-1)N_p)}{\sum_{c'} P(p + (c'-1)N_p) \sum_{c''} P^*(p' + (c''-1)N_p)} \right] - 1$$

$$\kappa_p = \kappa_{pp'} \delta_{p,p'}$$

multi-prod WTN spectrum

K_i	$\|\psi_i\|$	country prod: 57	$\|\psi_i\|$	country prod:06	$\|\psi_i\|$	country prod:56	$\|\psi_i\|$	country prod:52
1	0.052	USA	0.216	Mali	0.332	Brazil	0.288	Japan
2	0.044	Tajikistan	0.201	Guinea	0.304	Bolivia	0.279	Rep. of Korea
3	0.042	Kyrgyzstan	0.059	USA	0.274	Paraguay	0.245	China
4	0.022	France	0.023	Germany	0.031	Argentina	0.020	Australia
5	0.021	Mexico	0.021	Mexico	0.017	Uruguay	0.013	USA
6	0.018	Italy	0.021	Canada	0.009	Chile	0.012	U Arab Em
7	0.018	Canada	0.018	UK	0.004	Portugal	0.010	Canada
8	0.015	Germany	0.015	Israel	0.004	Angola	0.010	Singapore
9	0.013	U Arab Em	0.015	C d'Ivoire	0.004	Spain	0.009	Germany
10	0.012	Qatar	0.014	Japan	0.003	France	0.008	New Zealand

eigenstate communities

$$\xi = \frac{(\sum_i |\psi_i|^2)^2}{\sum_i |\psi_i|^4}$$

Sugar, sugar preparations and honey; from coal, petroleum and gas.

$\lambda=0.9548$
$\lambda=0.9345$

Explosives and pyrotechnic products

Crude chemicals

$\lambda=-0.452+i0.775$
$\lambda=-0.424+i0.467$

Fertilizers, manufactured;

L.E, K. Frahm and D.L. Shepelyansky, EPJB 86, 193
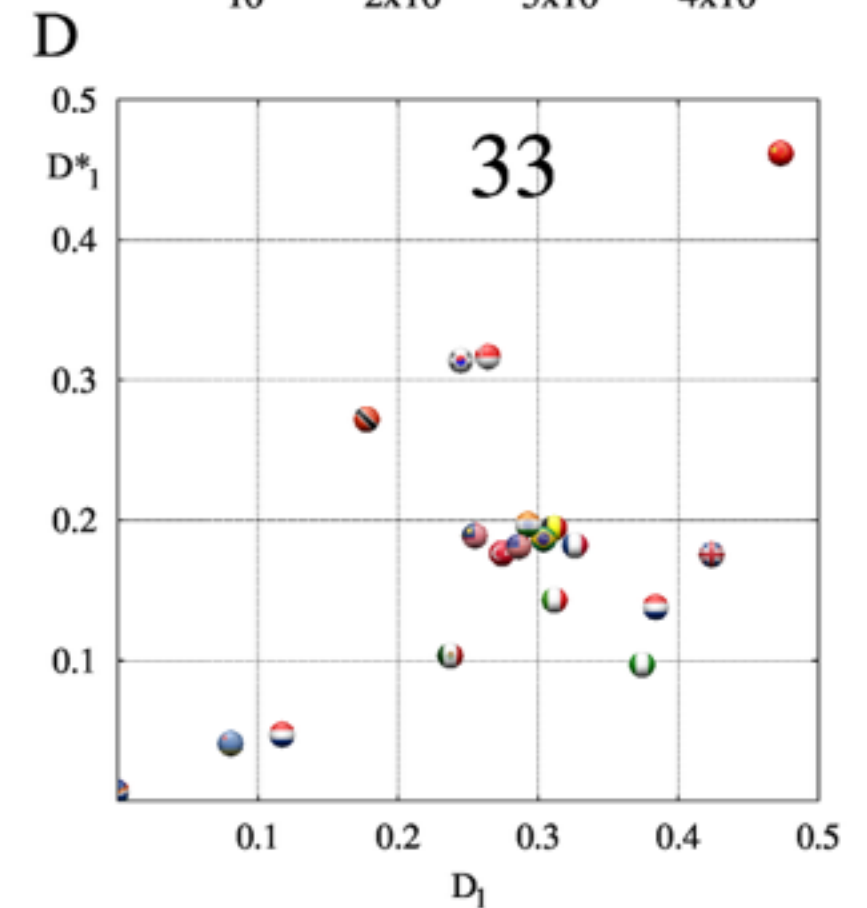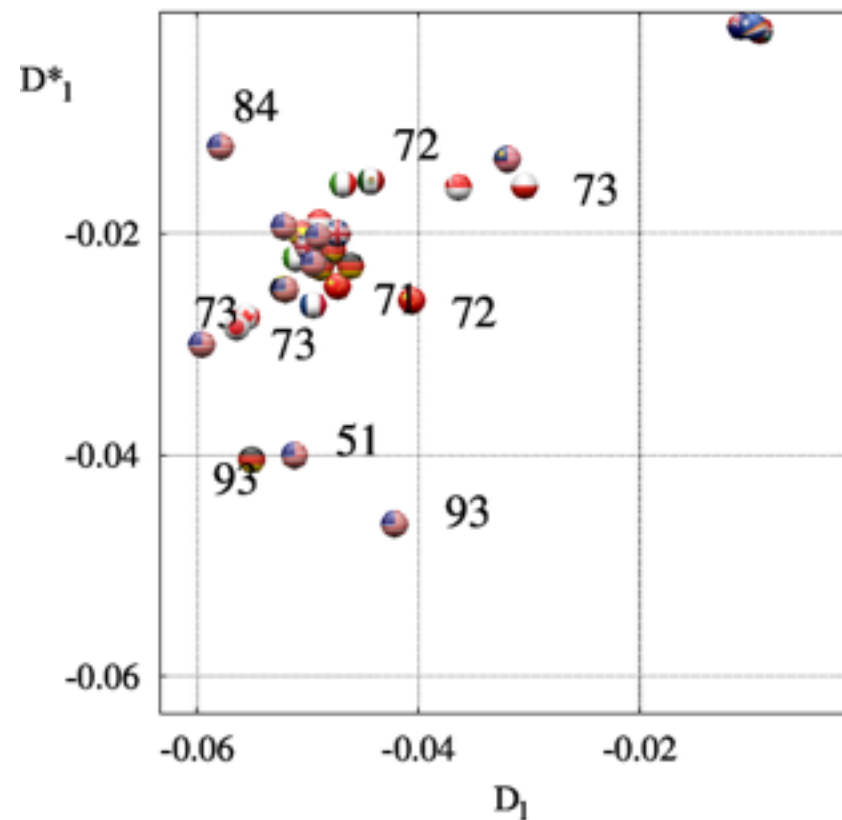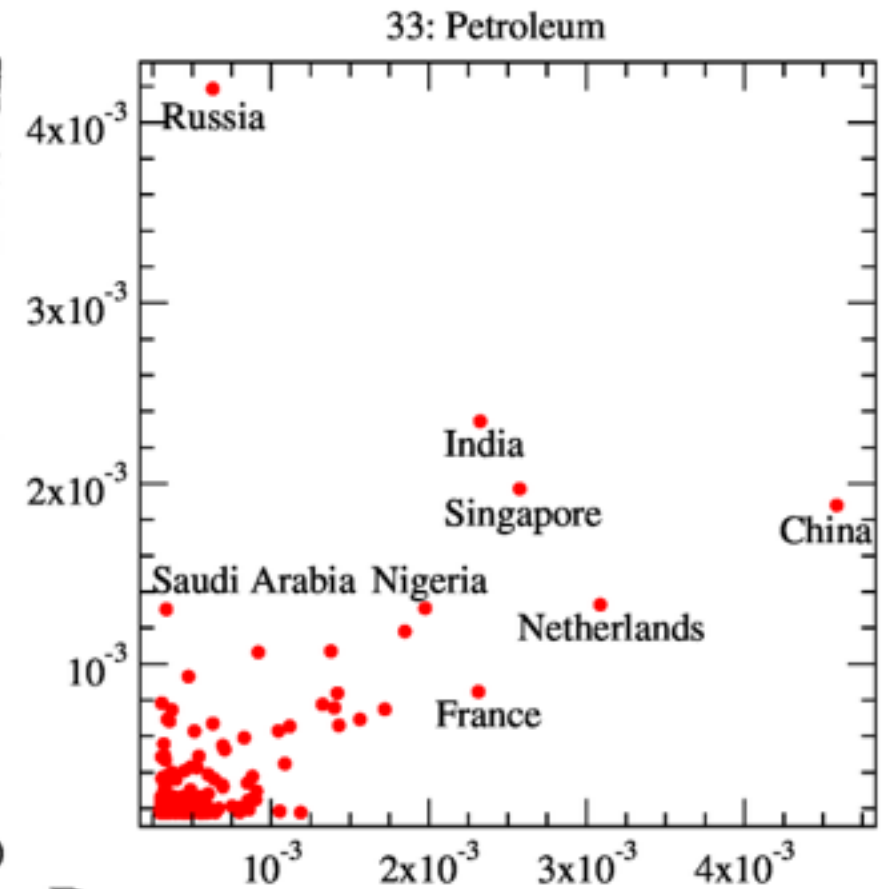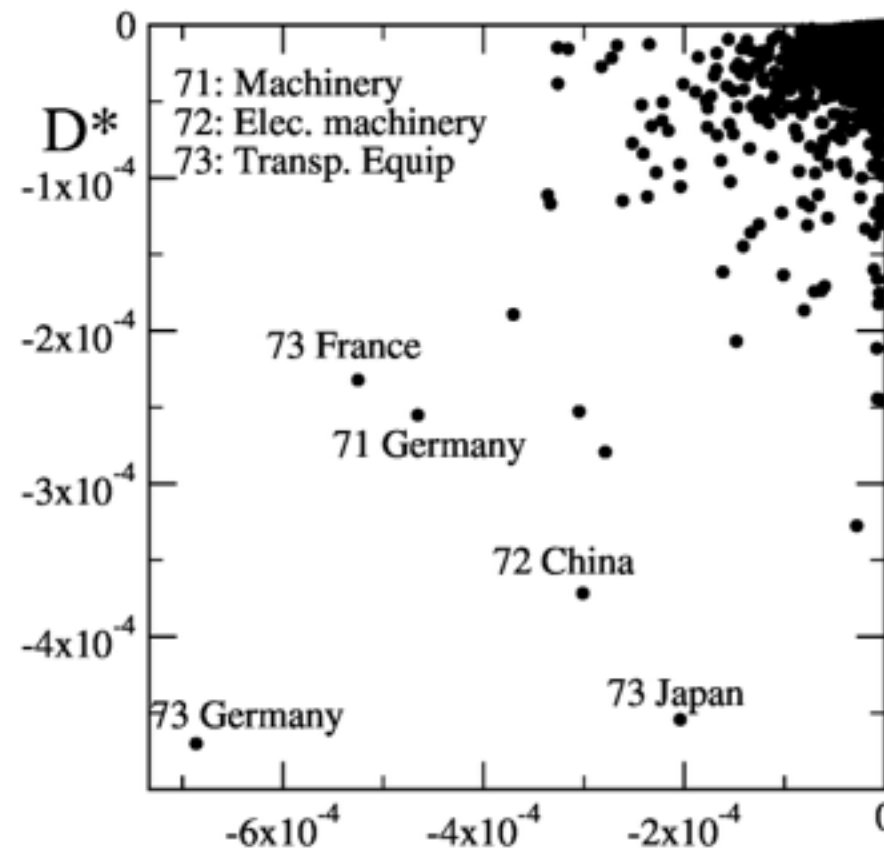
Sensitivity to price variation

$$D = dP/d\delta = \Delta P/\delta$$

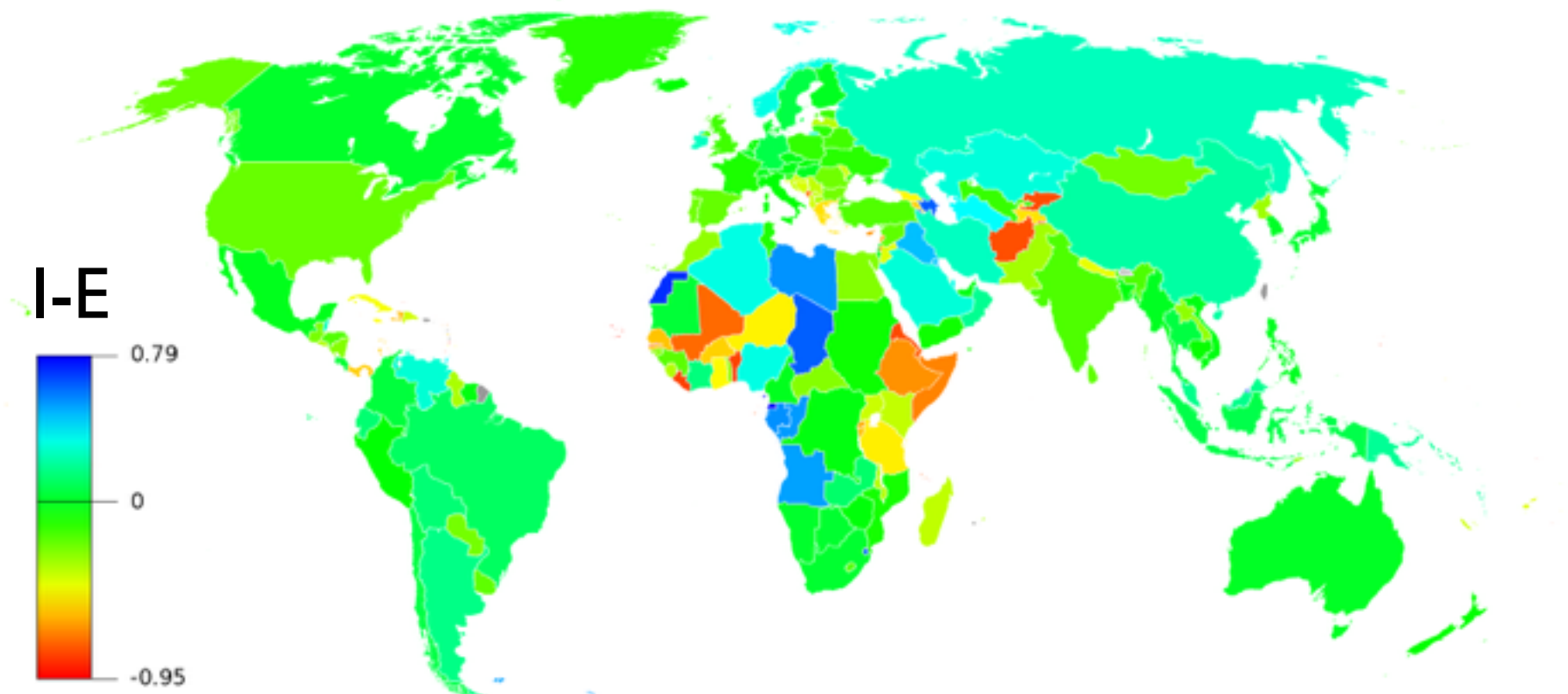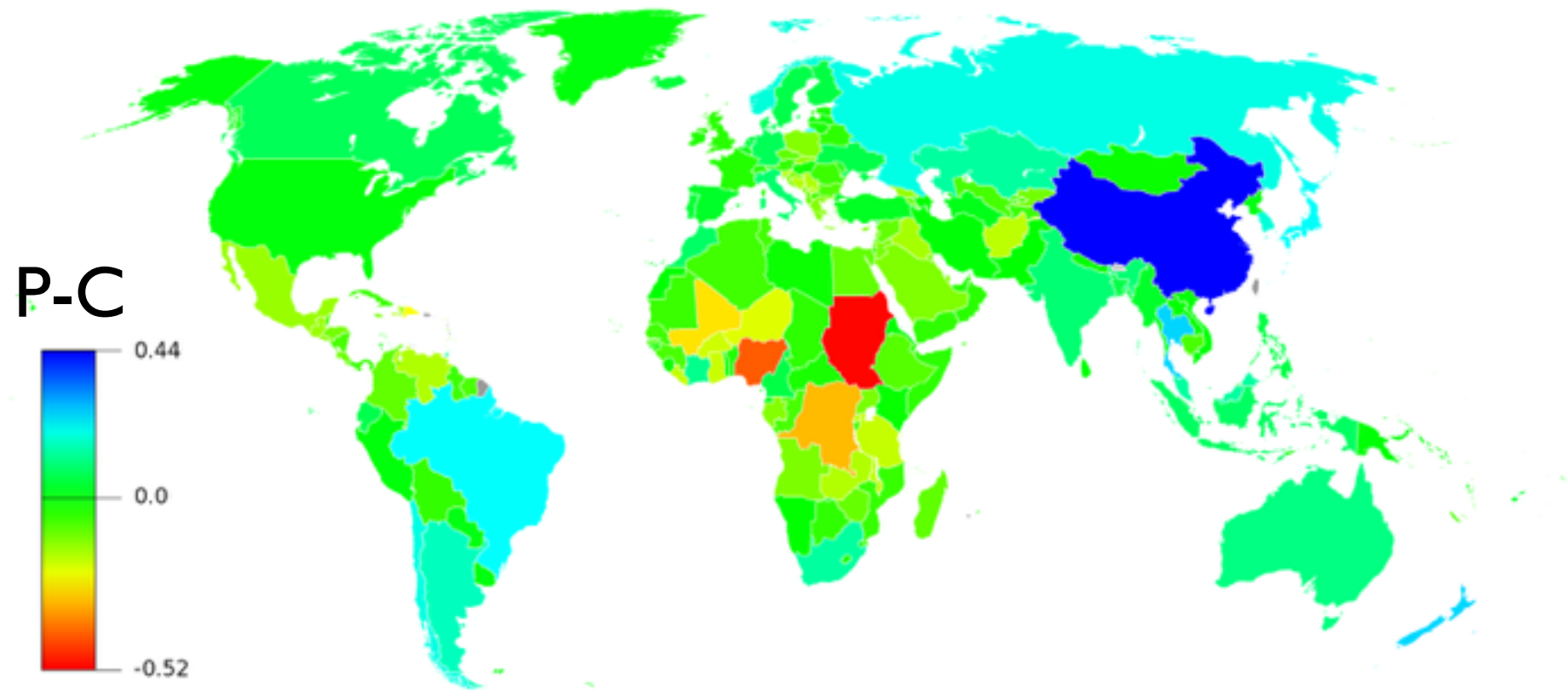$$D^* = dP^*/d\delta = \Delta P^*/\delta.$$

$$\delta' = 0.01, 0.03, 0.05$$

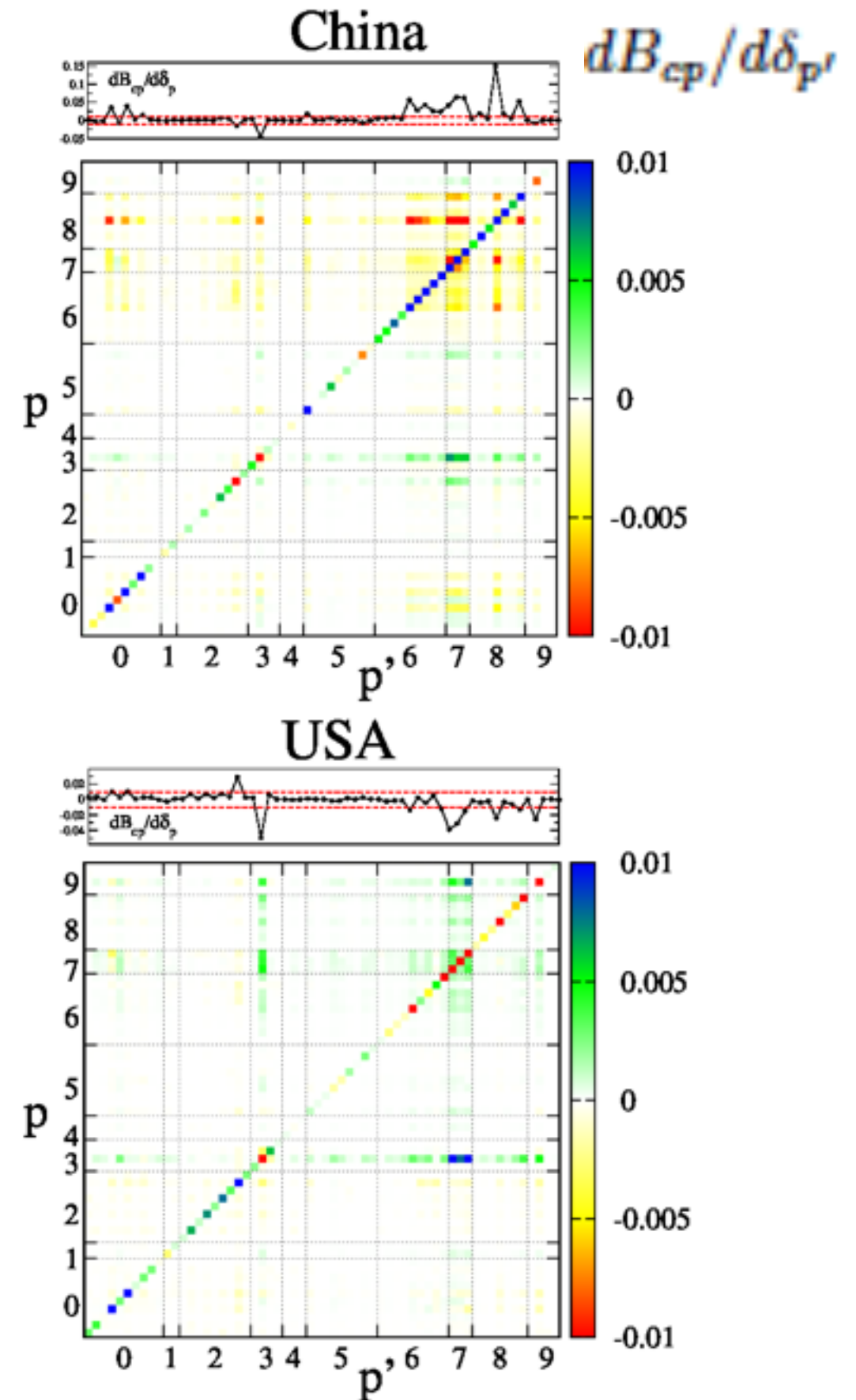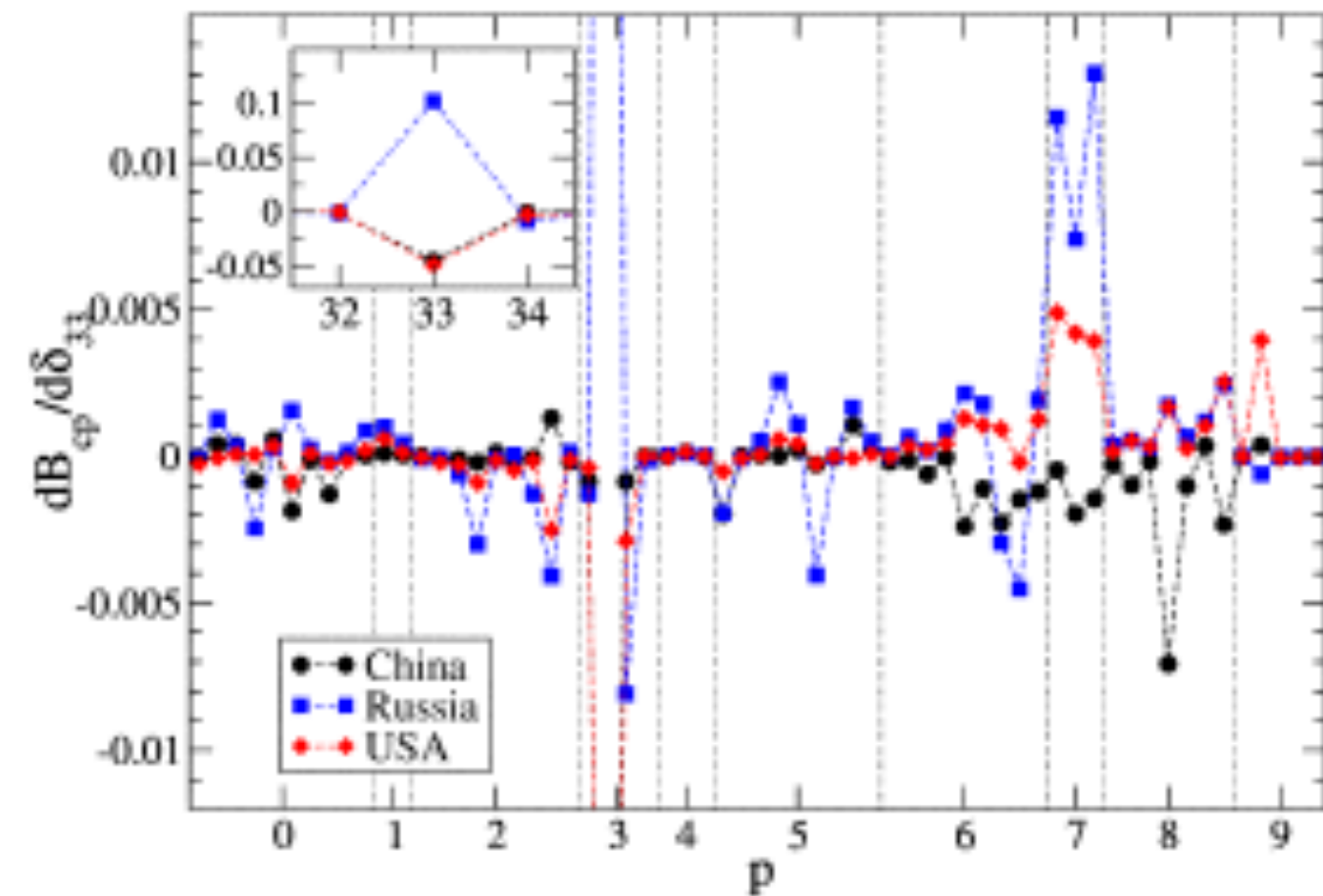$$D_l = D/P \quad \text{and} \quad D_l^* = D^*/P^*$$

$$K_2 \leq 50,$$

Country balance

$$B_c = \sum_p (P_{cp}^* - P_{cp}) / \sum_p (P_{cp}^* + P_{cp}) = (P_c^* - P_c)/(P_c^* + P_c).$$

P-C

0.44
0.0
-0.52

I-E

0.79
0
-0.95

Sensitivity to price variation II

Nestedness

biogeography

bipartite networks: species - sites (islands, plants, etc)

1937 Hulten
1957 Darlington
1975 Daubenmire

Causes: rates of extinction and colonialization
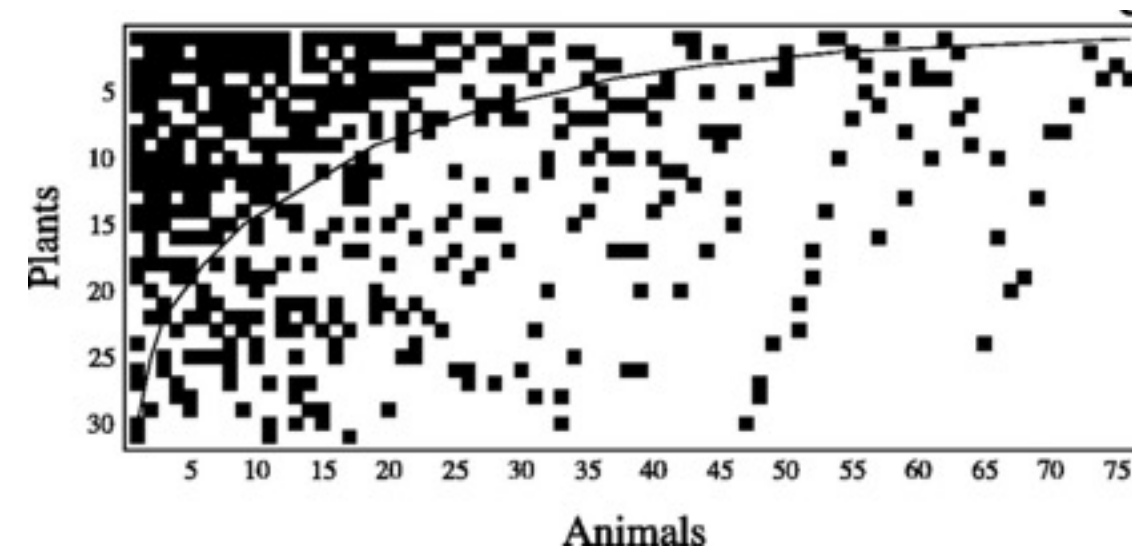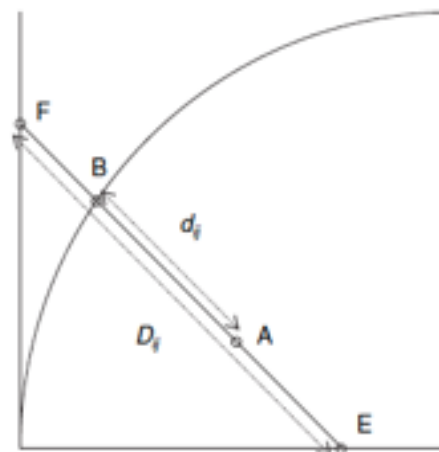(ay least 7 mechanisms)

Bacteria
Susceptible ←————————————→ Hard to Infect

Generalists ←————————————→ Specialists
Phages
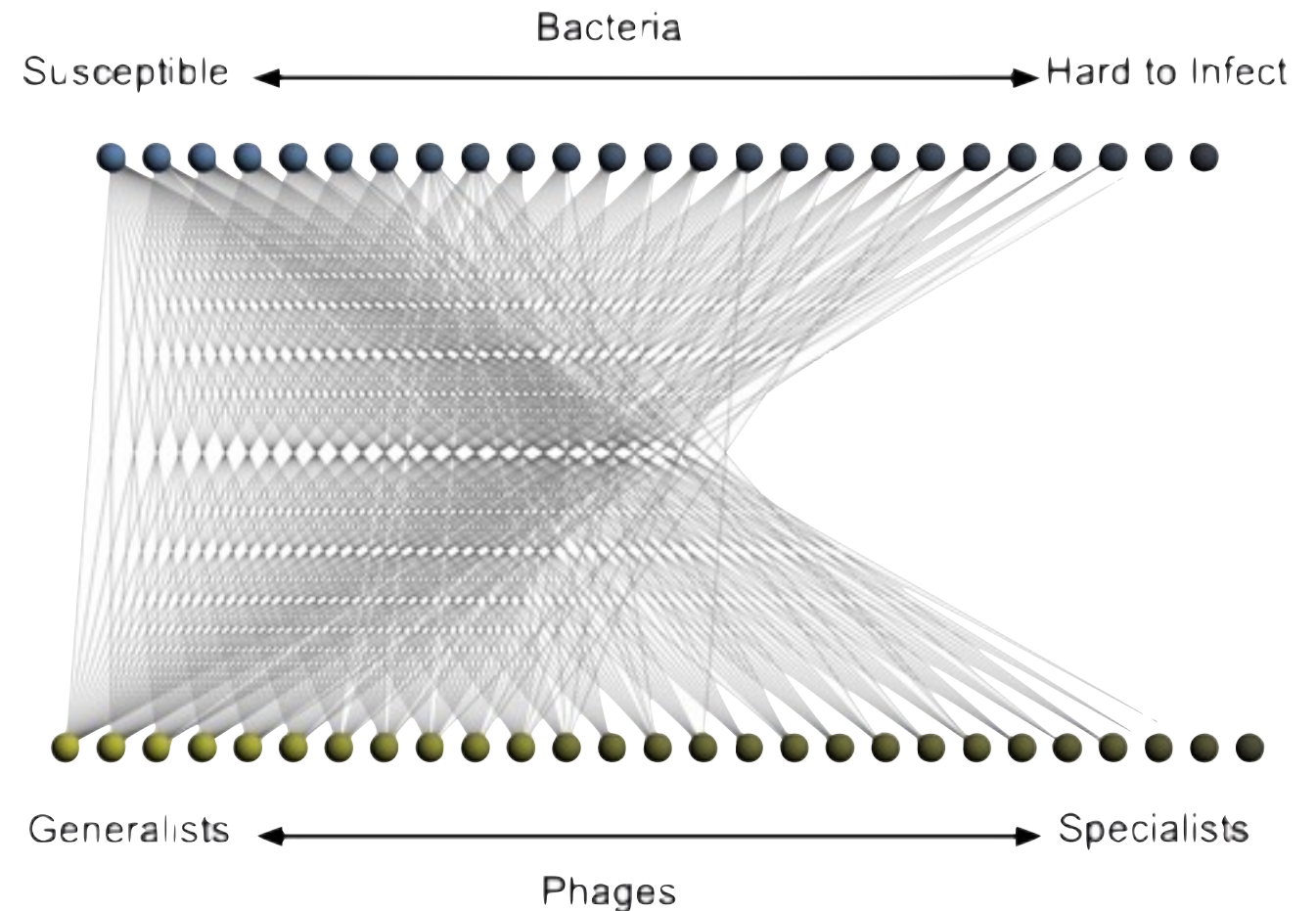
quantifying nestedness

BINMATNEST

M.A. Rodriguez-Girones and L. Santamaria,
Journal of Biogeography 33, 924 (2006)

isocline

$$f(x;p) = \frac{0.5}{n} + \frac{n-1}{n} \cdot \left(1 - \left(1 - \frac{m \cdot x - 0.5}{m-1}\right)^P\right)^{1/P}.$$
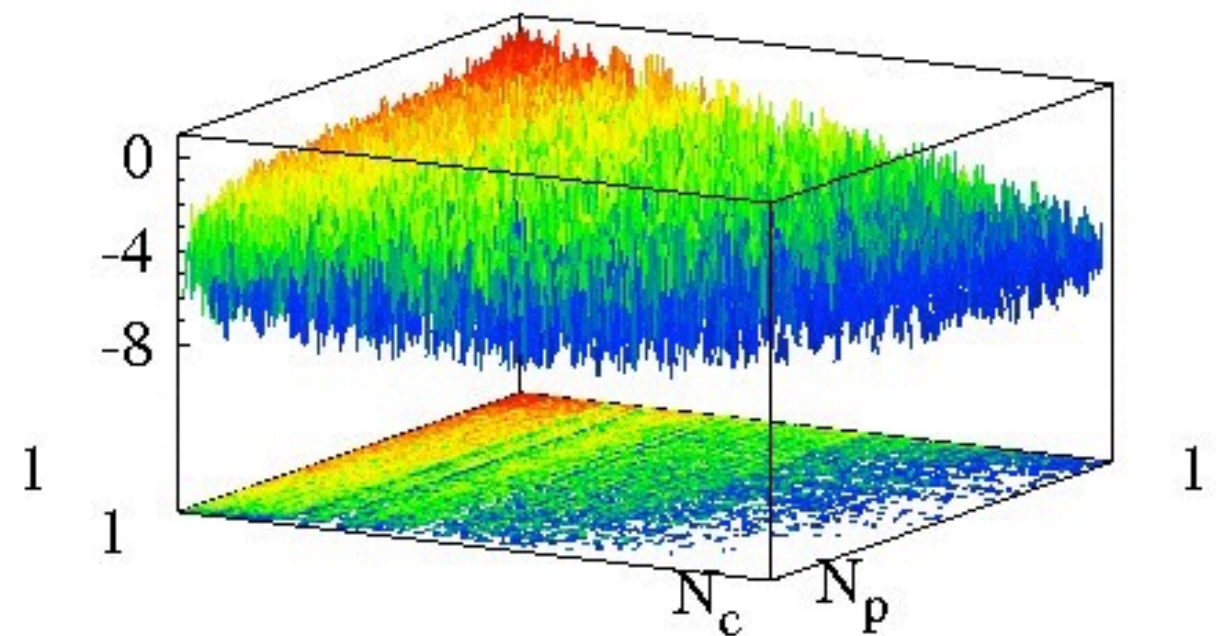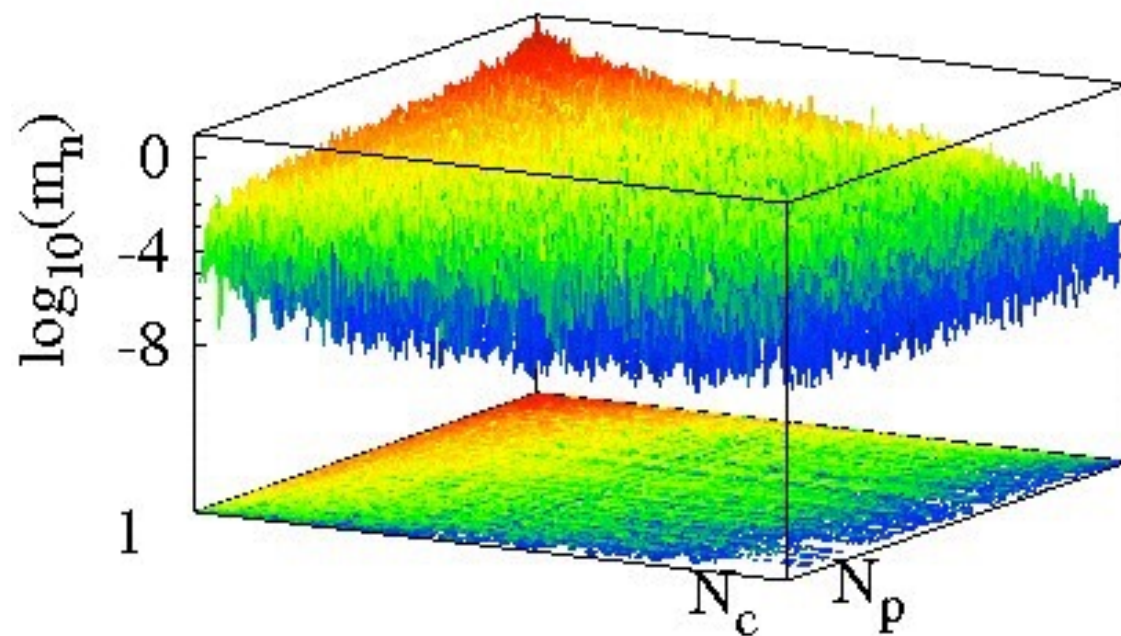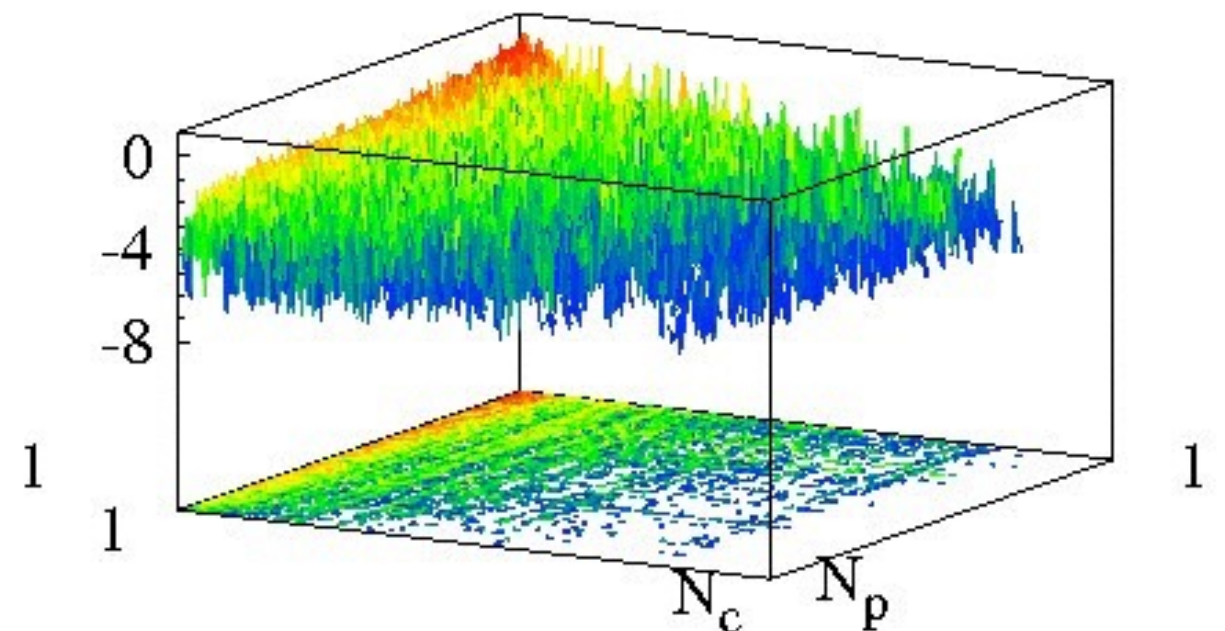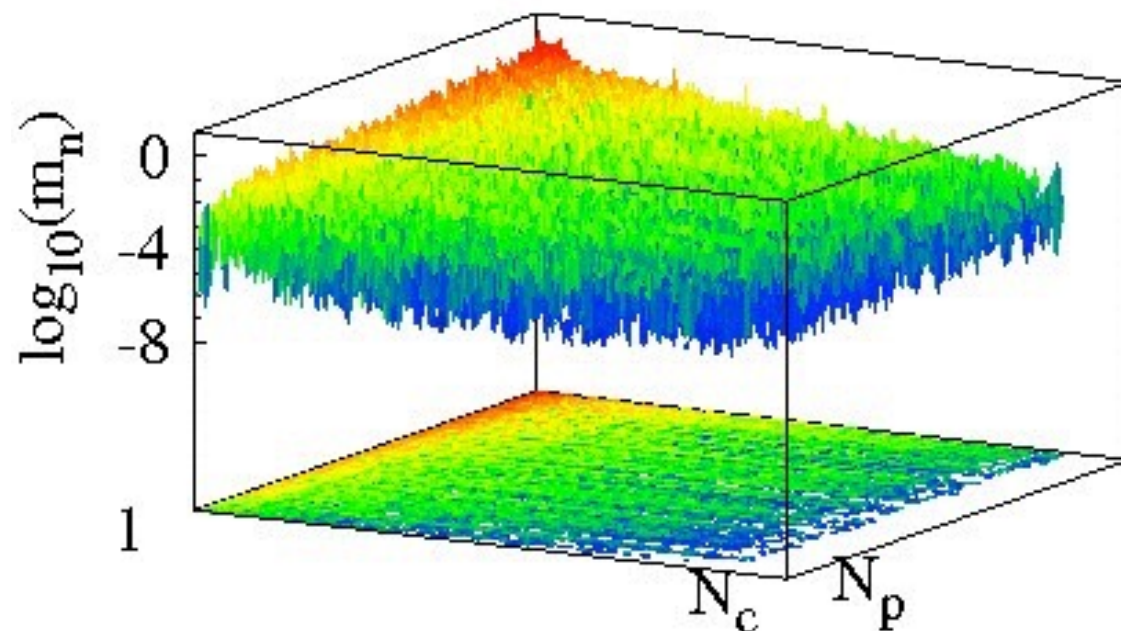
Nestedness

$$m^{(i,e)} = M^{(i,e)}/M_{\max}$$

$$M_{p,c}^{(i)} = \sum_{c'=1}^{N_c} M_{c,c'}^p \qquad\qquad M_{p,c}^{(e)} = \sum_{c'=1}^{N_c} M_{c',c}^p$$
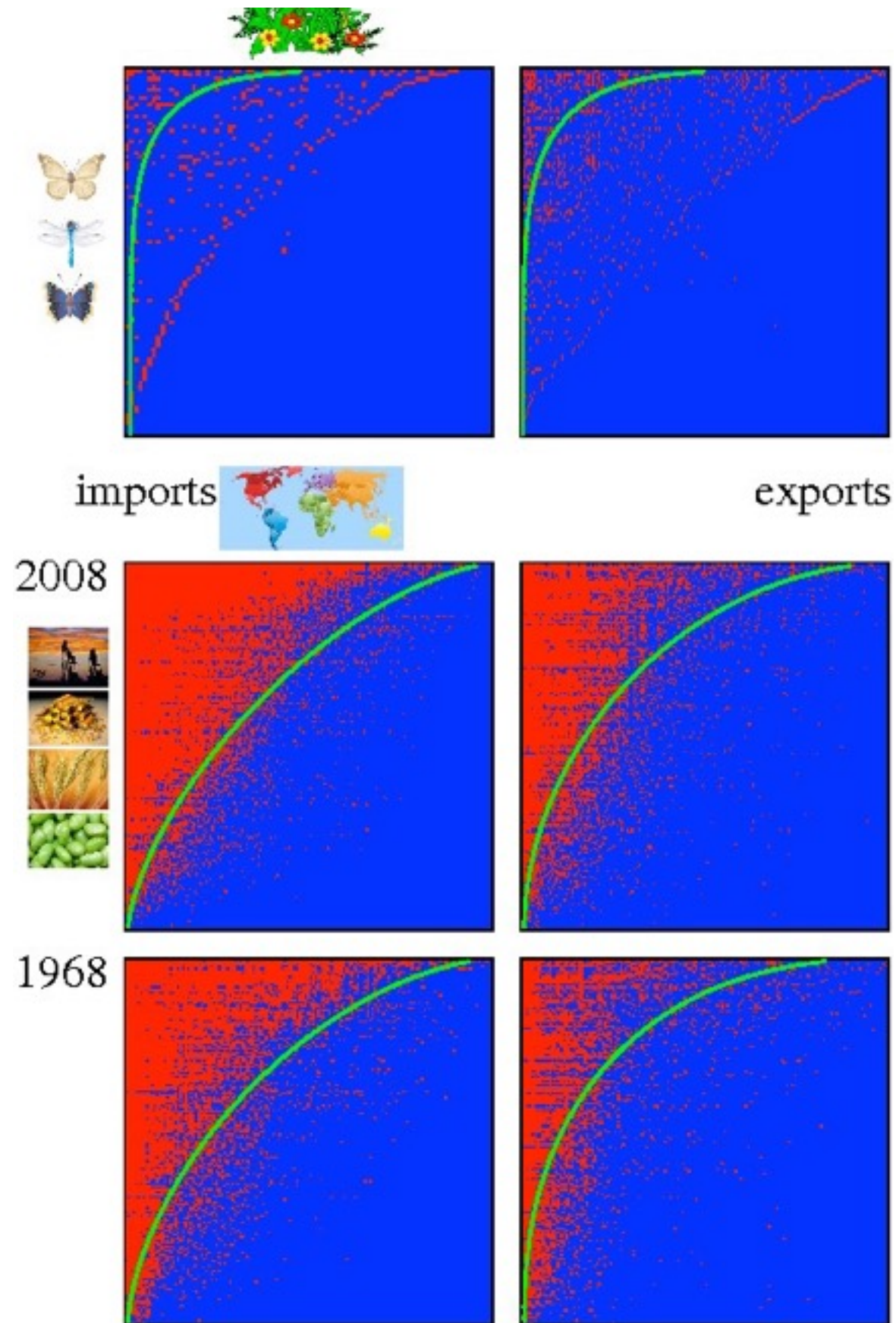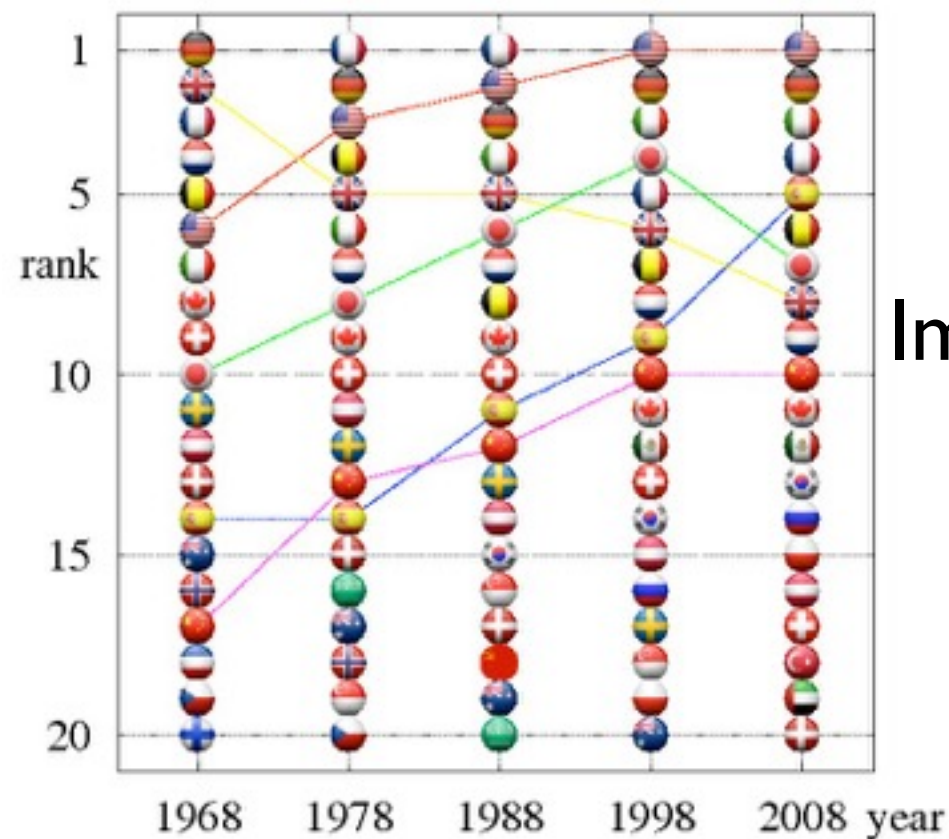
2008

1968

Binary mutualistic networks

$$Q_{c,p}^{(i,e)} = \begin{cases} 1 & \text{if } m_{c,p}^{(i,e)} \geq \mu \\ 0 & \text{if } m_{c,p}^{(i,e)} < \mu \end{cases}$$
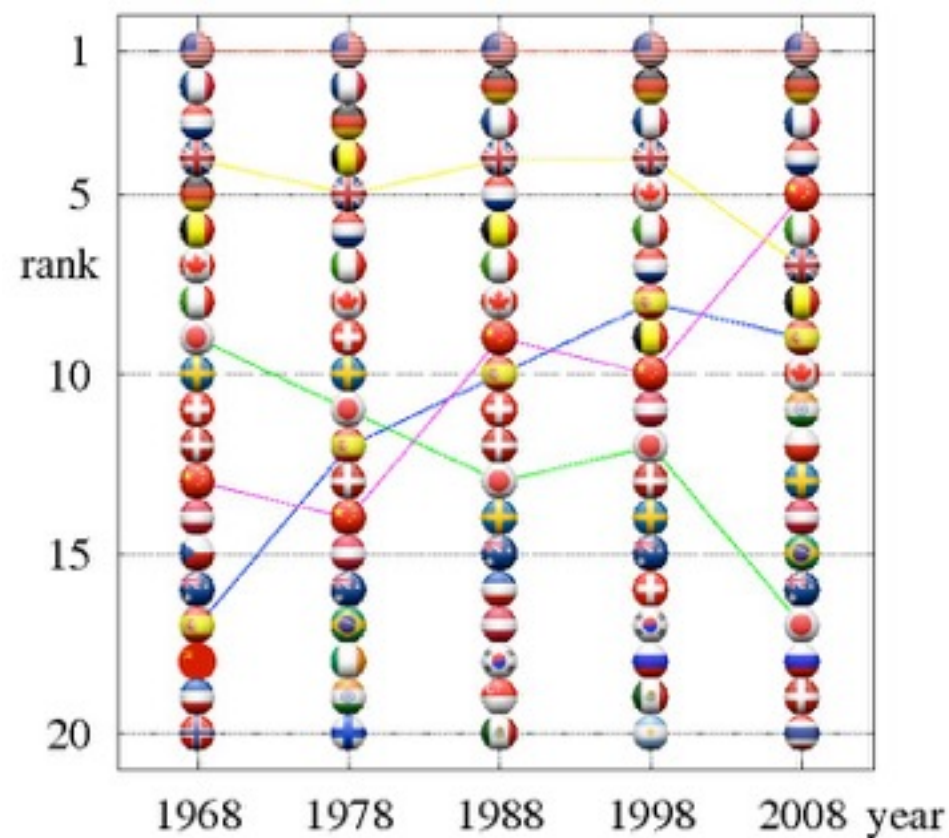

imports   exports

2008

1968

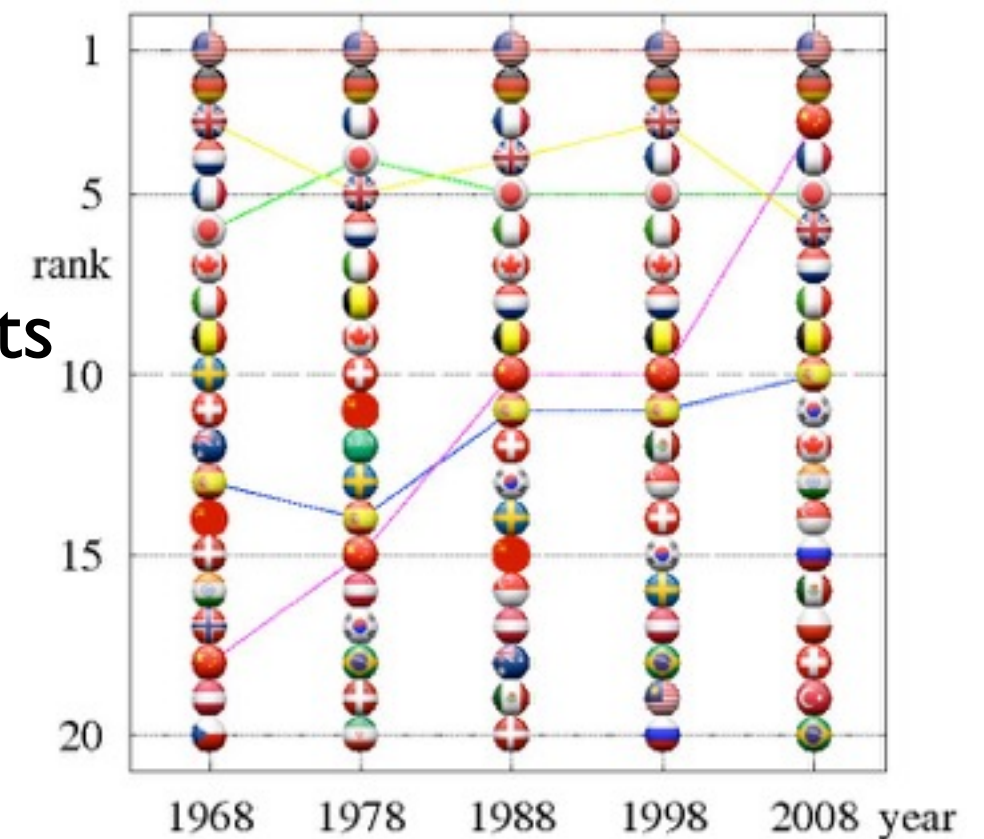# Ecolorank of countries
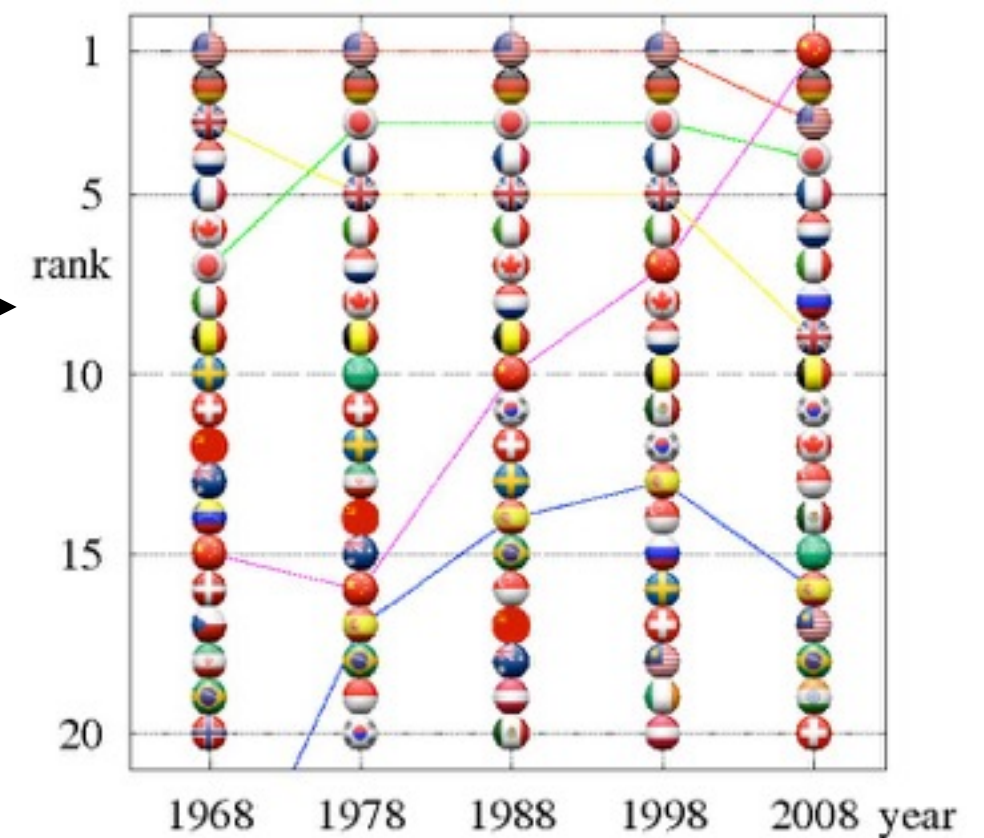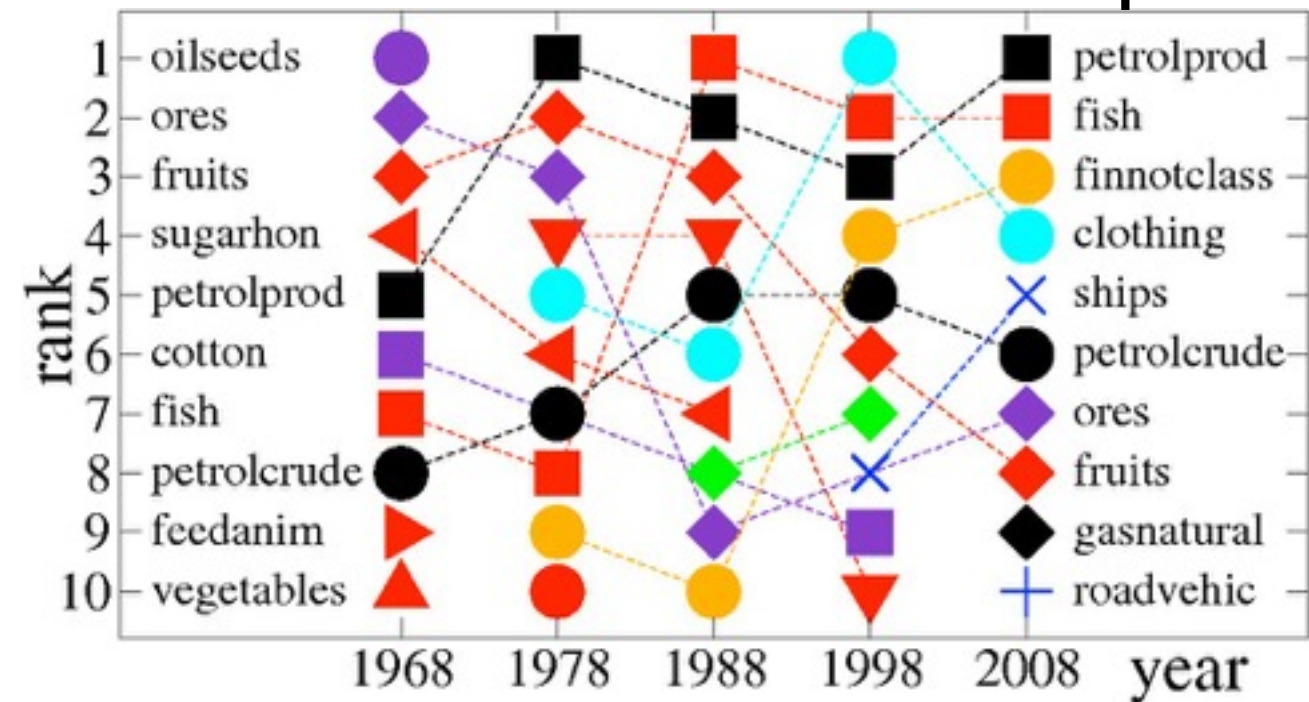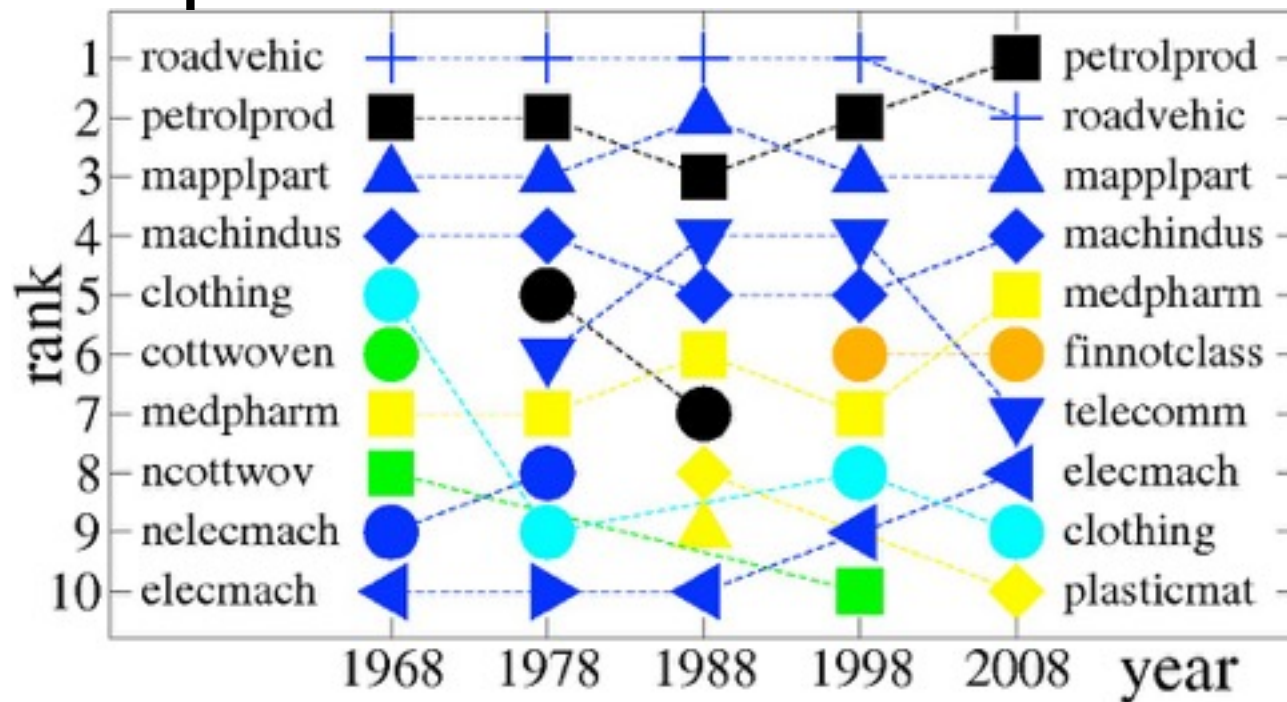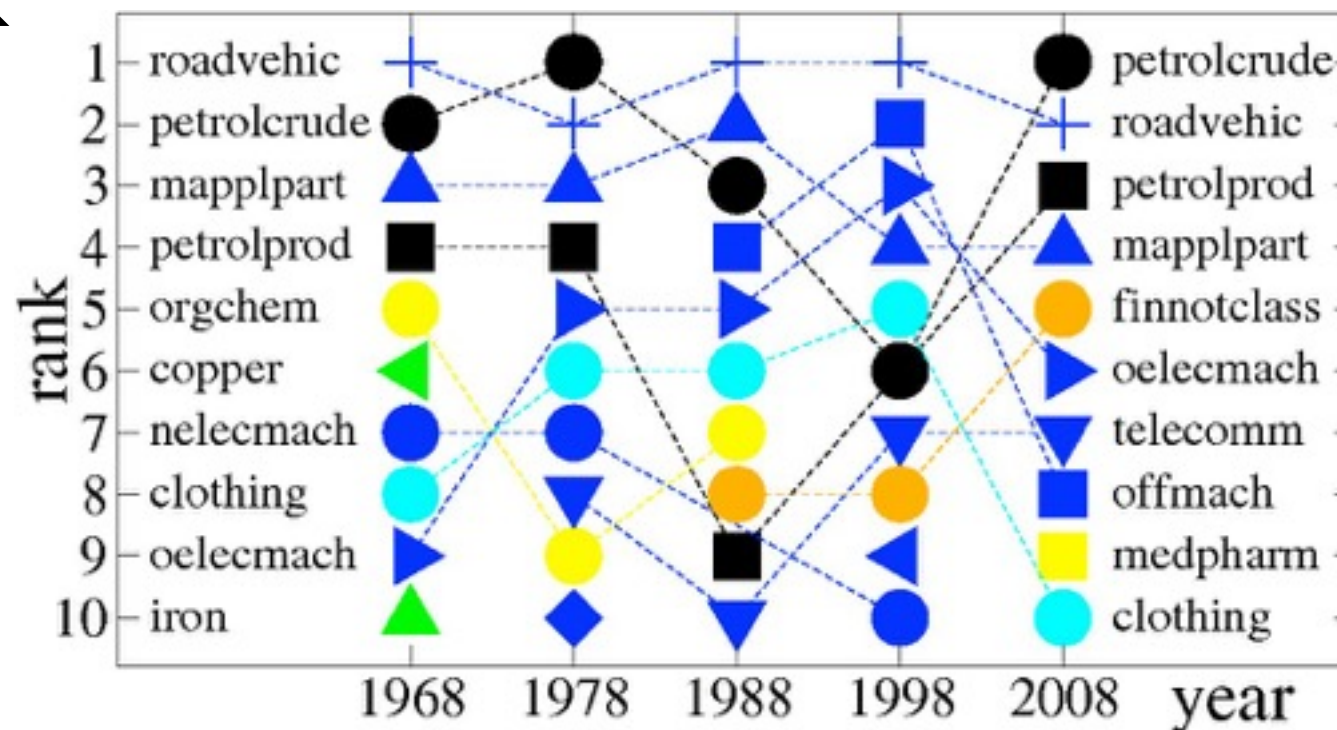

Imports   Exports

money
ranking

Ecolorank of products

imports

exports

money rank

L. ERMANN Multiprod WTN

July 1st 2015, Luchon

Conclusions

- Google matrix of the WTN (democratic in countries, global network properties):
  1) one product of all comm. (Nc)
  2) multiprod (Nc x Np)
  2d-ranking, spectrum, communities in eigenstates, correlation between P-C, comparison with I-E, new tool for trade analysis
- Asymmetry in products
- Time evolution analysis
- Sensitivity to price variation (weak coupling between products) would lead to prediction of crisis and time evolution
- Nestedness and EcoloRank

Messi beaucoup!

2d global ranks
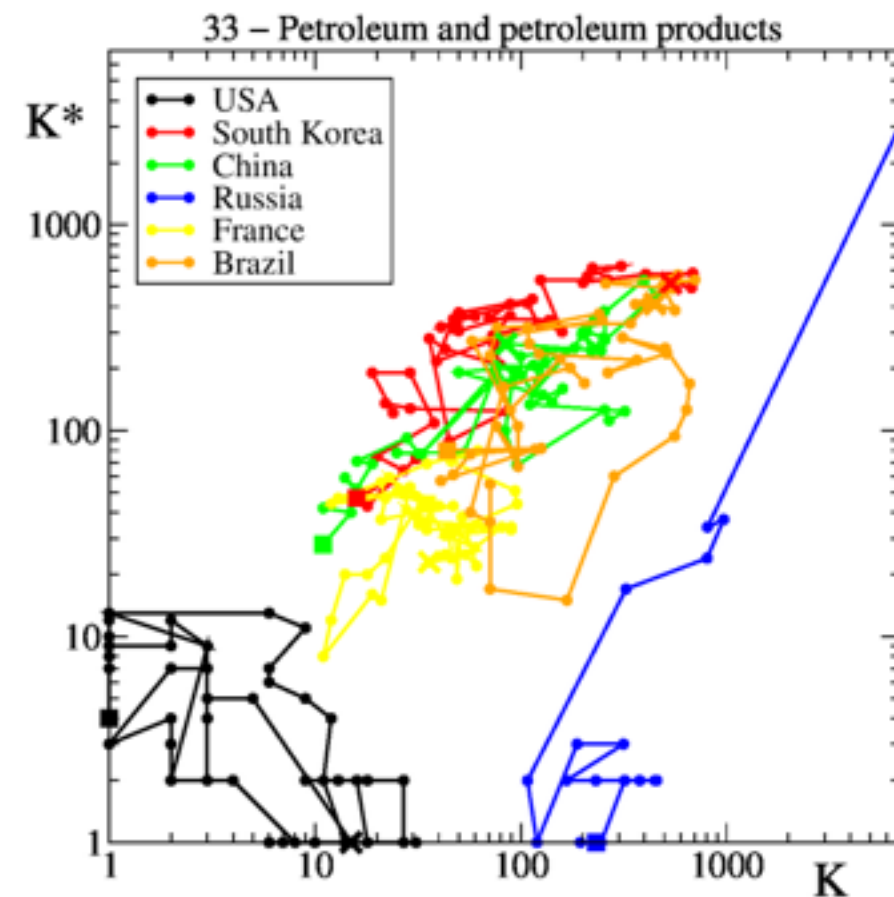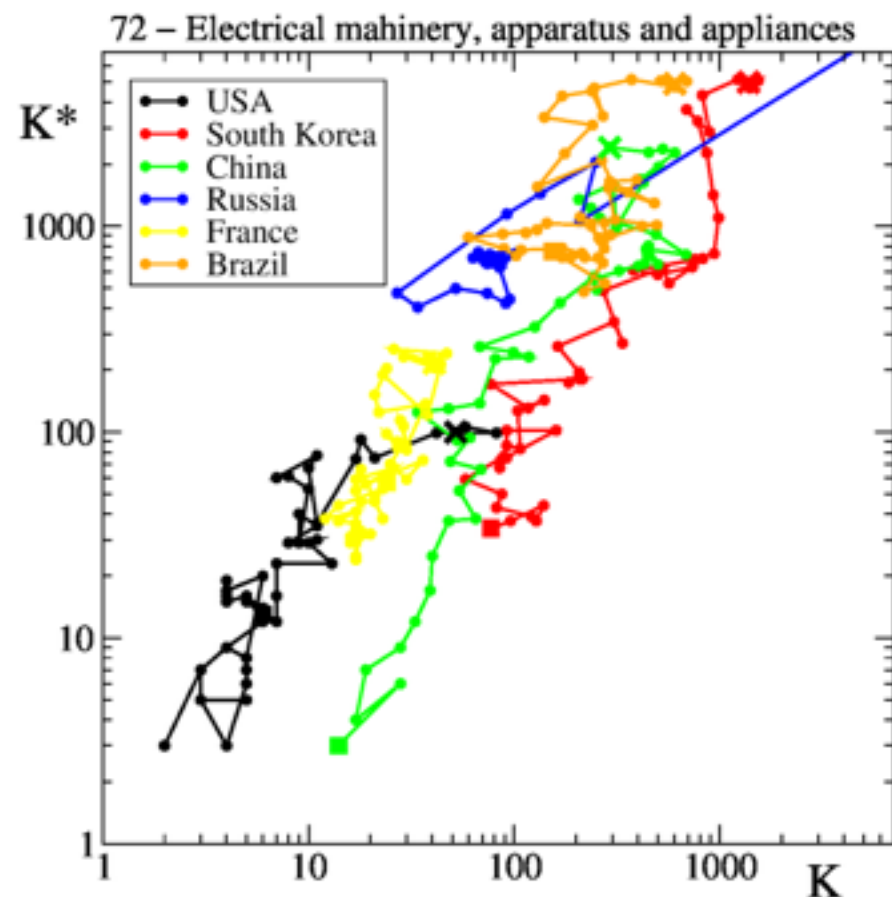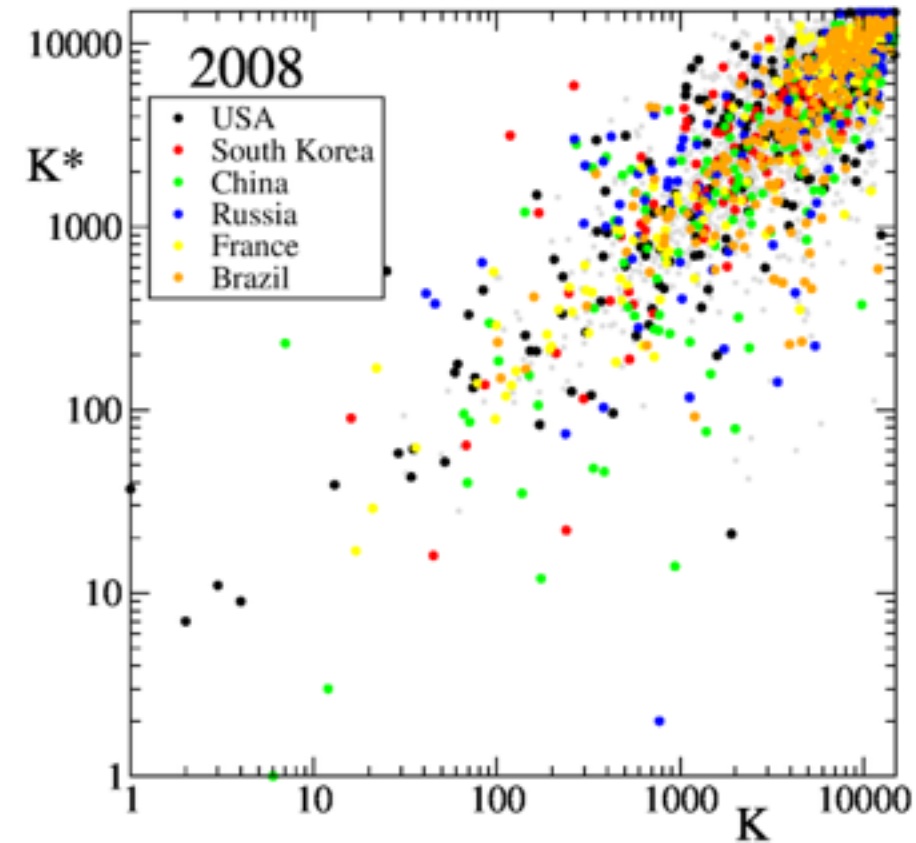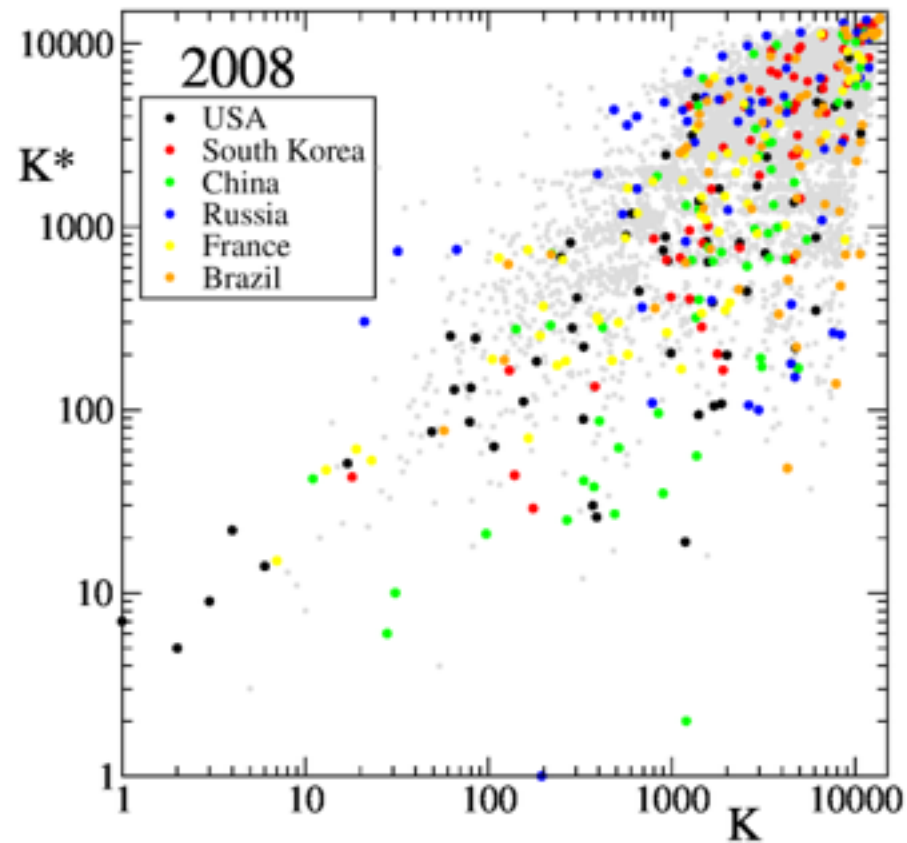
2d global ranks
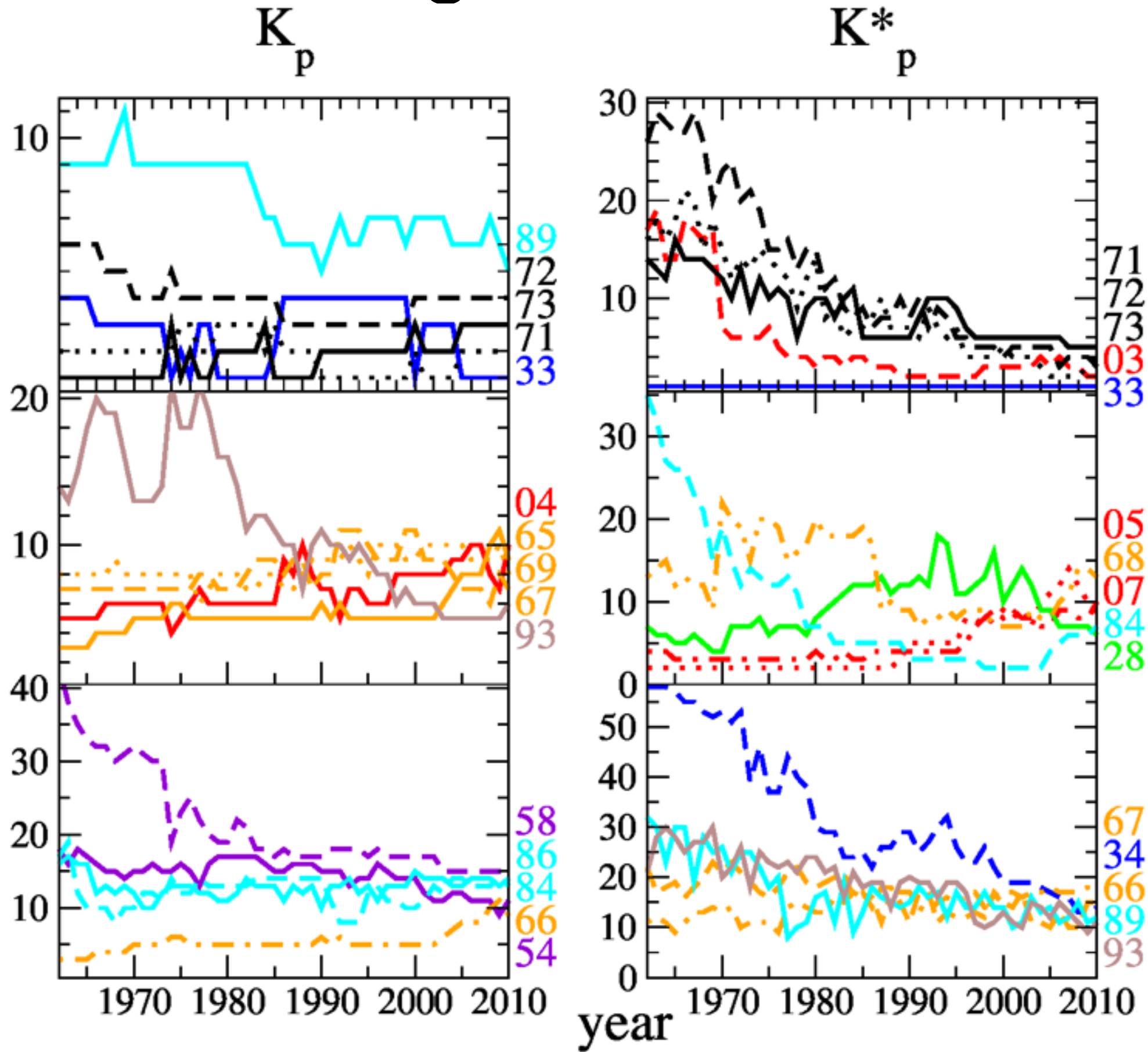
2d global ranks

product names

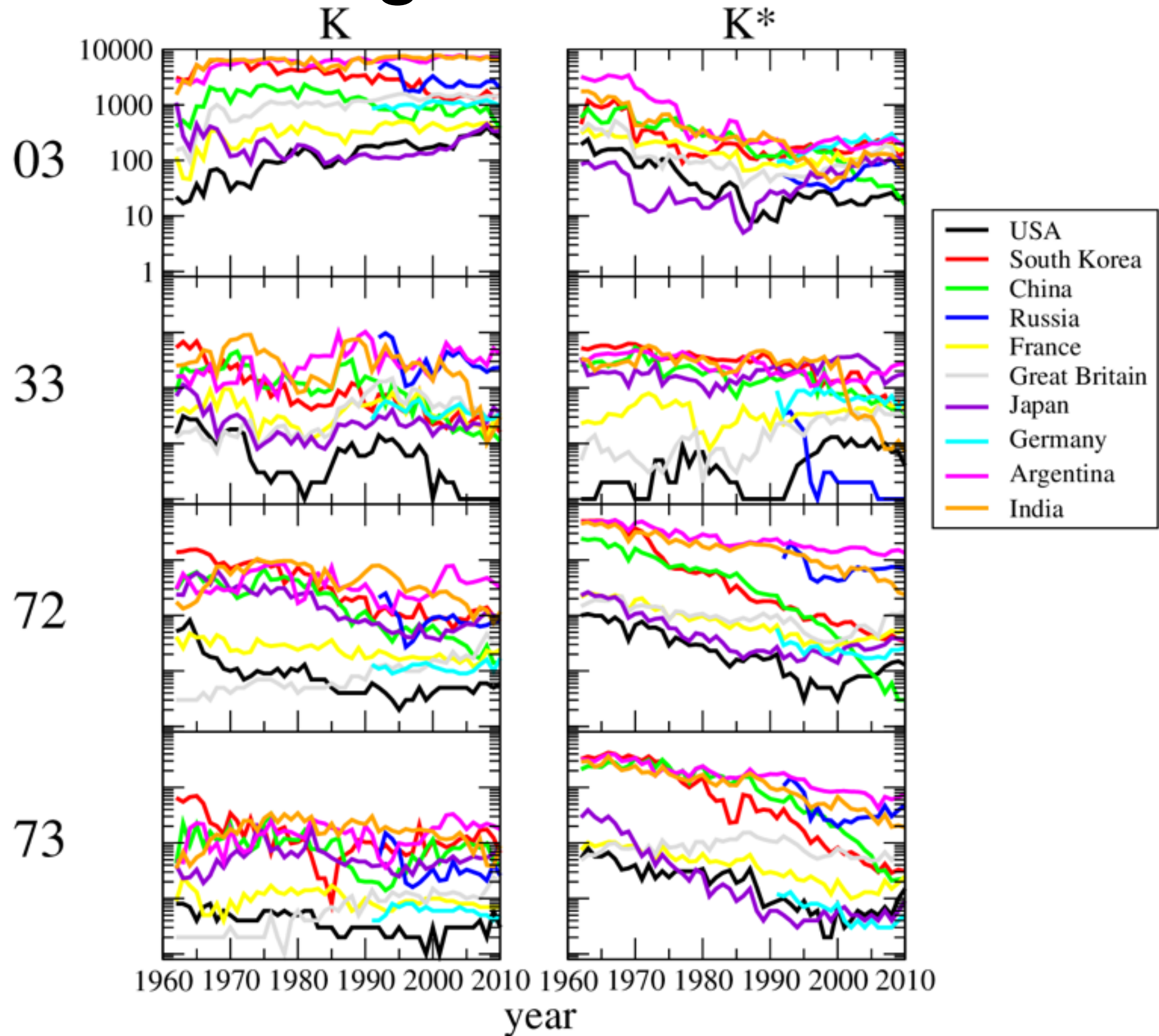| code	name	code	name
00	Live animals	54	Medicinal and pharmaceutical products
01	Meat and meat preparations	55	Perfume materials, toilet & cleansing preptions
02	Dairy products and eggs	56	Fertilizers, manufactured
03	Fish and fish preparations	57	Explosives and pyrotechnic products
04	Cereals and cereal preparations	58	Plastic materials, etc.
05	Fruit and vegetables	59	Chemical materials and products, nes
06	Sugar, sugar preparations and honey	61	Leather, lthr. Manufs., nes & dressed fur skins
07	Coffee, tea, cocoa, spices & manufacs. Thereof	62	Rubber manufactures, nes
08	Feed. Stuff for animals excl. Unmilled cereals	63	Wood and cork manufactures excluding furniture
09	Miscellaneous food preparations	64	Paper, paperboard and manufactures thereof
11	Beverages	65	Textile yarn, fabrics, made up articles, etc.
12	Tobacco and tobacco manufactures	66	Non metallic mineral manufactures, nes
21	Hides, skins and fur skins, undressed	67	Iron and steel
22	Oil seeds, oil nuts and oil kernels	68	Non ferrous metals
23	Crude rubber including synthetic and reclaimed	69	Manufactures of metal, nes
24	Wood, lumber and cork	71	Machinery, other than electric
25	Pulp and paper	72	Electrical machinery, apparatus and appliances
26	Textile fibres, not manufactured, and waste	73	Transport equipment
27	Crude fertilizers and crude minerals, nes	81	Sanitary, plumbing, heating and lighting fixt.
28	Metalliferous ores and metal scrap	82	Furniture
29	Crude animal and vegetable materials, nes	83	Travel goods, handbags and similar articles
32	Coal, coke and briquettes	84	Clothing
33	Petroleum and petroleum products	85	Footwear
34	Gas, natural and manufactured	86	Scientif & control instrum, photogr gds, clocks
35	Electric energy	89	Miscellaneous manufactured articles, nes
41	Animal oils and fats	91	Postal packages not class. According to kind
42	Fixed vegetable oils and fats	93	Special transact. Not class. According to kind
43	Animal and vegetable oils and fats, processed	94	Animals, nes, incl. Zoo animals, dogs and cats
51	Chemical elements and compounds	95	Firearms of war and ammunition therefor
52	Crude chemicals from coal, petroleum and gas	96	Coin, other than gold coin, not legal tender
53	Dyeing, tanning and colouring materials		