Graph Theory Applications

Vadim Lozin

DIMAP – Center for Discrete Mathematics and its Applications

Mathematics Institute

University of Warwick

Theory vs applications

Theory vs applications

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Theory Application

Theory vs applications

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Theory Application

Practice $\longrightarrow$ Theory $\longrightarrow$ Practice

Seven Bridges of Königsberg

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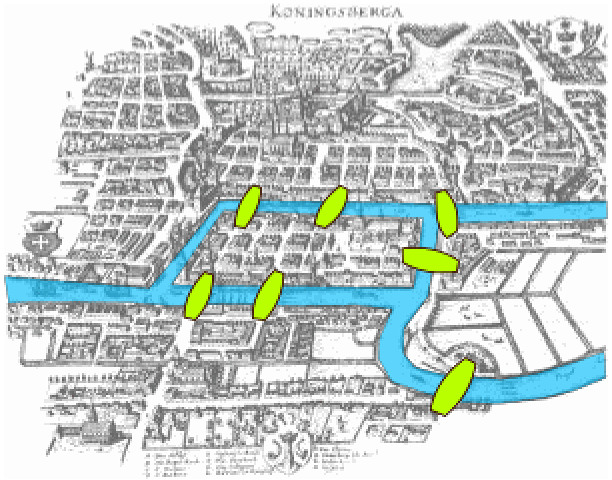
The problem was to find a walk through the city that would cross each bridge once and only once.



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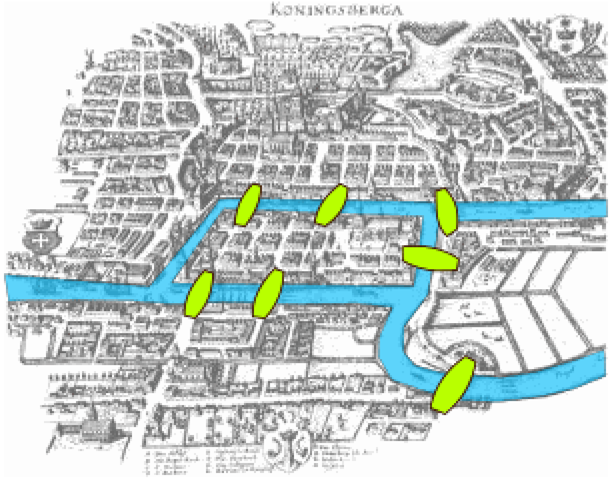

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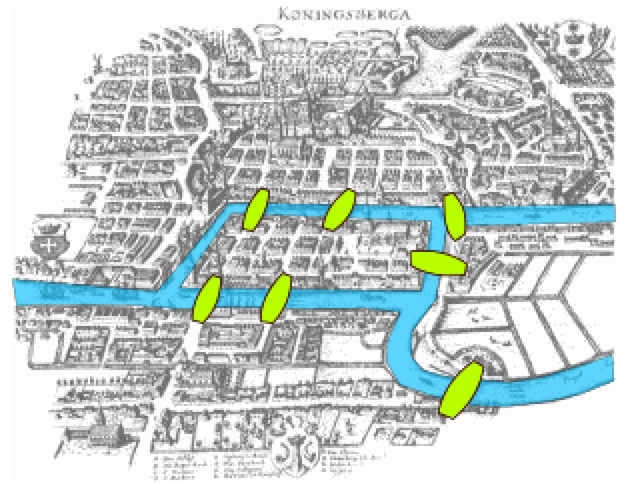

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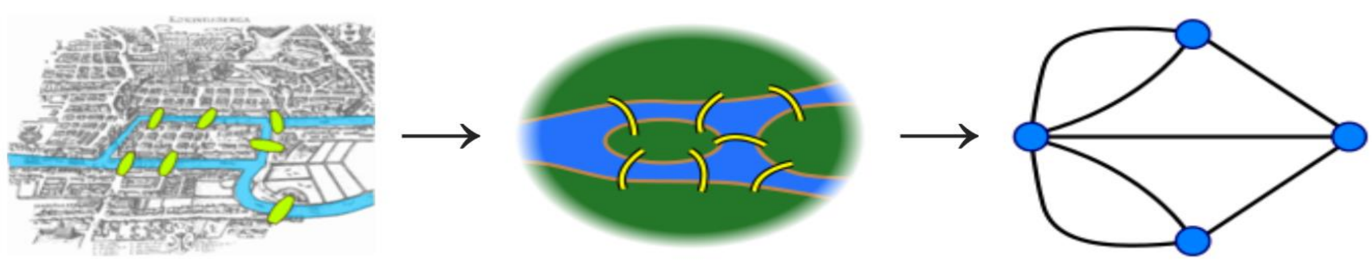
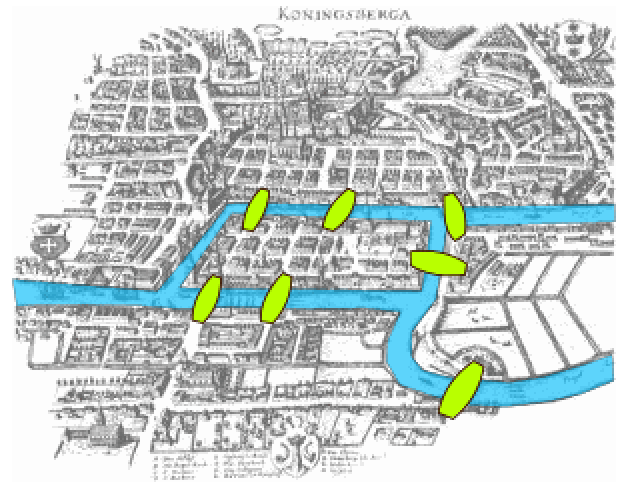


Seven Bridges of Königsberg

A graph is Eulerian if it has a cycle containing every edge of the graph exactly once.



Theorem. A graph is Eulerian if and only if it is connected and every vertex of the graph has even degree.

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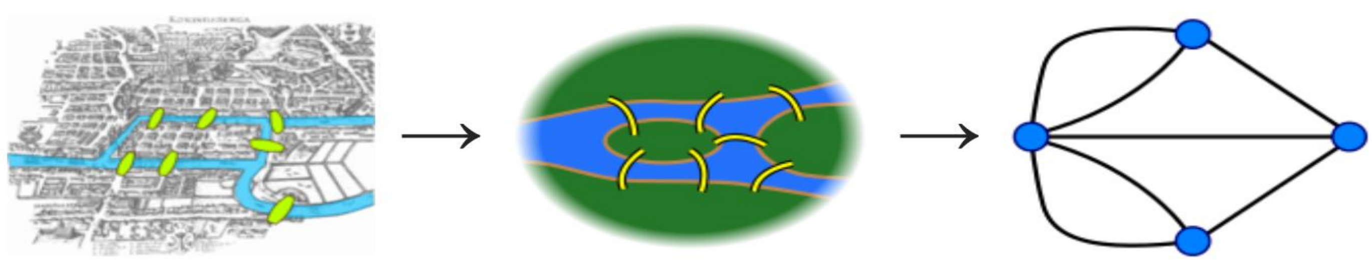

The Internet and Social Networks

The Internet and Social Networks

Internet Mathematics publishes conceptual, algorithmic, and empirical papers focused on large, real-world complex networks such as the web graph, the Internet, online social networks, and biological networks.

Small World Networks

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Theorem. The diameter of almost all graphs is 2.

$$\frac{\text{The number of n-vertex graphs}}{\text{The number of n-vertex graphs of diameter 2}} \xrightarrow[n\to\infty]{} 1$$

My Small World

My Small World

Professor Vadim Lozin

Vadim Lozin

Professor of Mathematics

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Email: V.Lozin@warwick.ac.uk

Personal Home Page

Teaching Responsibilities 2014/15:

Term 1: MA4J3 Graph Theory

Term 2: MA252 Combinatorial Optimisation

Research

Interests: Graph theory, combinatorics, discrete mathematics

Grants: Clique-width of graphs , Stability in graphs

Team: Viktor Zamaraev (postdoc), Aistis Atminas (Ph.D. student), Andrew Collins (Ph.D. student), Igor Razgon (external member)

Supervision: Former posdocs, Ph.D. students, M.Sc. students, etc.

Visits: KTH, EPFL, KAUST, UFRJ, etc.

Talks: Plenary and others

Centre for Discrete Mathematics and its Applications

The Centre for Discrete Mathematics and its Applications (DIMAP) has been established in March 2007 by the University of Warwick, partially funded by an EPSRC Science and Innovation Award EP/D063191/1 of £3.8 million. The Centre builds on a collaboration among

- the Department of Computer Science,
- the Warwick Mathematics Institute, and
- the Operational Research and Management Sciences group in the Warwick Business School.

The DIMAP is co-located in the adjacent new Computer Science and Mathematics buildings and it is directed by a Management Board led by Prof. Artur Czumaj, with the advice on scientific direction from the DIMAP Advisory Board.

Vision for the Centre

DIMAP is a multidisciplinary research centre supporting an internationally competitive programme of research in discrete modelling, algorithmic analysis, and combinatorial (discrete) optimisation. It aims to support a thriving Industrial Affiliates Programme, and develop collaborative research rooted in discrete mathematics, involving researchers at other UK universities. The Centre also contributes to the development of undergraduate modules and taught postgraduate modules within degrees offered by the participating departments. With a number of internationally renowned researchers, an extensive programme of scientific seminars (including Combinatorics Seminar), international workshops and visiting researchers, and a multidisciplinary angle, DIMAP is one of the leading international research centres in discrete mathematics and its applications in computer science and operational research.

Participating Groups:

Department of Computer Science

Foundations of Computer Science (FoCS) Research Group

Warwick Mathematics Institute

Operational Research and Management Sciences Group

Events:

DIMAP Seminar

Combinatorics Seminar

NEW! **DIMAP Logic Day**, June 1, 2015

WBS research translates into practical success

Archived Article • 28 January 2010 • Feature

Research by Vladimir Deineko, WBS Associate Professor of Operational Research, has recently provided two examples of academic research translating successfully into practice.

He has been leading research into designing optimal routes for commercial waste collection services in Coventry City Council over recent months. This research by WBS faculty together with members from the Computer Science department has been supported by the EPSRC and Warwick's Centre for Discrete Mathematics and its Applications (DIMAP). Now completed, initial results show that applying new algorithms developed as a result of the research project can bring about up to 20 percent savings in transportation costs such as fuel consumption, servicing costs and wear and tear on vehicles.

In addition, Vladimir has been building on and developing a tool using combinatorial type algorithms, together with Doctoral student Thomas Ridd, to allocate cohorts of students into equitable teams, taking into account the need for an even spread across the cohort of different backgrounds, skills, and cultural origins. Their work has been picked up by Warwick Ventures, where experts are currently looking into the commercialisation of this tool.

Vladimir comments, "As scholars, we are always happy when our papers are published in top research journals and are highly referenced by our colleagues. Over the past two years I have been working with practitioners, implementing my theoretical results into practical tools, for example, software prototypes. I have found this highly exciting and enjoyable, and it is rewarding to see how the tools you have created make a real change in everyday work, and how people who use these tools are so impressed with the results they can get by...just clicking a button."

Dr Vladimir Deineko

Associate Professor (Reader) of Operational Research

Operational Research & Management Sciences Group

Email: Vladimir.Deineko@wbs.ac.uk / Tel: 024 765 24501

Biography

Years of teaching experience in a variety of cultural environments; formerly associate professor at Dnepropetrovsk State University, Ukraine, and invited researcher at University of Technology, Graz, Austria. Participation in consultancy projects related to problem solving in industry, commerce, and the public sector.

Research Interests

Algorithmic aspects of the problem solving process with the main focus on the analysis of efficiently solvable cases of hard optimisation problems such as travelling salesman problem and quadratic assignment problem; design and implementation of exact and approximate algorithms for combinatorial optimisation problems: vehicle routing problem, bin packing problem, network optimisation problems etc.

Recent Publications [all...]

Journal Articles

- Eranda Cela Vladimir Deineko Gerhard Woeginger. "Well-solvable cases of the QAP with block-structured matrices"
  Discrete Applied Mathematics published online (2015)
- Eranda Cela Vladimir Deineko Gerhard Woeginger. "Linearizable special cases of the QAP"
  Journal of Combinatorial Optimization to appear (2015)
- Vladimir Deineko, Gerhard Woeginger. "Another Look at the Shoelace TSP: The Case of Very Old Shoes."
  Lecture Notes In Computer Science Fun with Algorithms (2014): 125-136.
- Dr Vladimir Deineko, Bettina Klinz,Alexander Tiskin,Gerahard Woeginger.
  "Four-point conditions for the TSP: The complete complexity classification"
  Discrete Optimization 14 (2014): 147-159.
- Vladimir Deineko, Gerhard Woeginger. "Two hardness results for core stability in hedonic coalition formation games"
  Discrete Applied Mathematics 161 (2013): 1837-1842.

Travelling Salesman Problem

Travelling salesman problem (**TSP**): Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

Applications of the TSP

Much of the work on the TSP is motivated by its use as a platform for the study of general methods that can be applied to a wide range of discrete optimization problems. This is not to say, however, that the TSP does not find applications in many fields. Indeed, the numerous direct applications of the TSP bring life to the research area and help to direct future work.

The TSP naturally arises as a subproblem in many transportation and logistics applications, for example the problem of arranging school bus routes to pick up the children in a school district. This bus application is of important historical significance to the TSP, since it provided motivation for Merrill Flood, one of the pioneers of TSP research in the 1940s. A second TSP application from the 1940s involved the transportation of farming equipment from one location to another to test soil, leading to mathematical studies in Bengal by P. C. Mahalanobis and in Iowa by R. J. Jessen. More recent applications involve the scheduling of service calls at cable firms, the delivery of meals to homebound persons, the scheduling of stacker cranes in warehouses, the routing of trucks for parcel post pickup, and a host of others.

Home

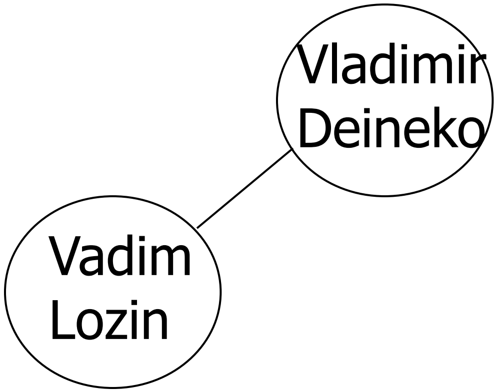
Applications
Genome
Starlight
Scan Chains
DNA
Whizzkids
Baseball
> Coin Collection
Airport Tours
USA Trip
Sonet Rings
Power Cables

Genome Sequencing

An old application of the TSP is to schedule the collection of coins from payphones throughout a given region. A modified version of Concorde's Chained Lin-Kernighan heuristic was used to solve a variety of coin collection problems. The modifications were needed to handle 1-way streets and other features of city-travel that make the assumption that the cost of travel from x to y is the same as from y to x unrealistic in this scenario.

Next Application

My Small World



Four Colour Problem

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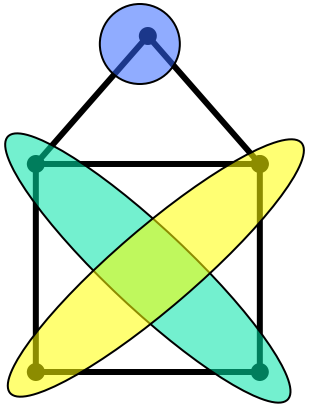

Definition. Vertex colouring is an assignment of colours to the vertices of the graph in which any two adjacent vertices receive different colours.

The minimum number of colours needed to colour the vertices of a graph G is the *chromatic number* of G.

Four Colour Theorem

Kenneth Appel and Wolfgang Haken at the University of Illinois announced, on June 21, 1976 that they had proven the theorem. Appel and Haken found an unavoidable set of 1,936 reducible configurations which had to be checked one by one by computer. This reducibility part of the work was independently double checked with different programs and computers.

In 2005, Benjamin Werner and Georges Gonthier formalized a proof of the theorem inside the Coq proof assistant (an interactive theorem prover). This removed the need to trust the various computer programs used to verify particular cases; it is only necessary to trust the Coq kernel.

Scheduling via Colouring

Assume that we have to schedule a set of interfering jobs, i.e. jobs that cannot be executed at the same time (for example, they use a shared resource).

We need to determine the minimum *makespan* , i.e. the minimum time required to finish the jobs.

Let G be the *conflict graph* of the jobs:

the vertices of the graph corresponds to the jobs,
the edges correspond to jobs that are in conflict.

The chromatic number of the graph equals the minimum *makespan*.

ALAIN HERTZ

Professor
Department of Mathematical and Industrial Engineering

ALAIN HERTZ
D.Sc. (EPF Lausanne)

- **Research**
- Publications at Polytechnique
- Laureates

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alain.hertz@polymtl.ca

Web link(s)
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RESEARCH

Research interests

- Combinatorial Optimization
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- Heuristics and metaheuristics
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- Scheduling problems
- Vehicle routing problems

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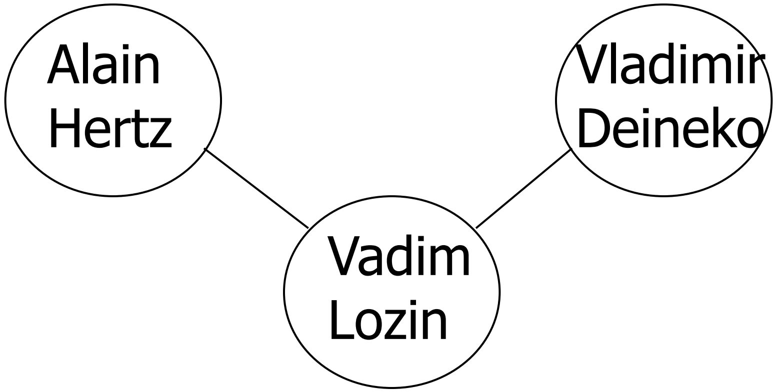
Gamache, M., Hertz, A., Ouellet, J. O. (2007). A Graph Coloring Model for a Feasibility Problem in Monthly Crew Scheduling With Preferential Bidding. *Computers & Operations Research, 34(8),* p. 2384-2395.

My Small World



Ramsey Game

This two player game requires a sheet of paper and pencils of two colors, say red and blue. Six points on the paper are chosen, with no three in line. Now the players take a pencil each, and take turns drawing a line connecting two of the chosen points. The first player to complete a triangle of her own color loses. Can the game ever result in a draw?

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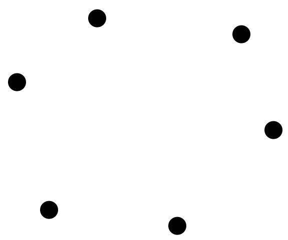

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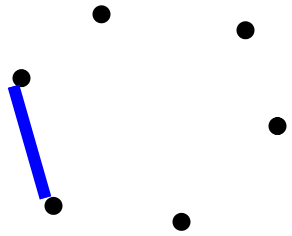

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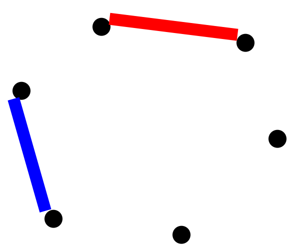

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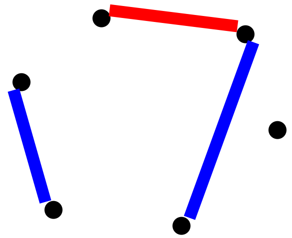

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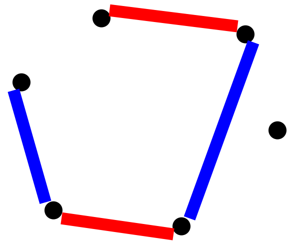

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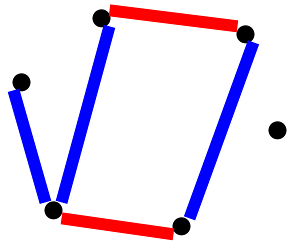

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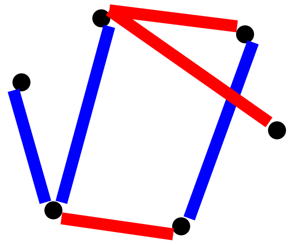

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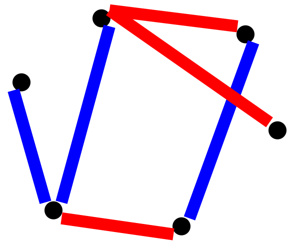
Claim. *Any coloring of the edges of the complete graph on 6 vertices with 2 colors contains a monochromatic triangle.*



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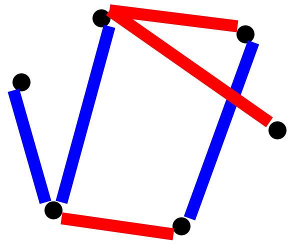

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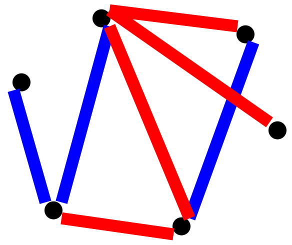

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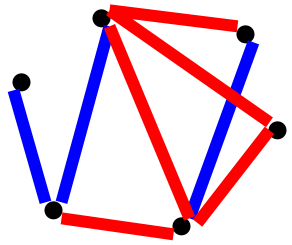

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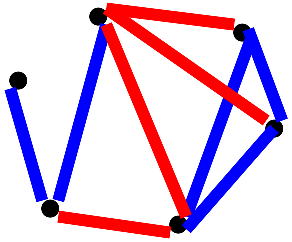
Otherwise, these three neighbours create a blue tringle.



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Born	22 February 1903 Cambridge
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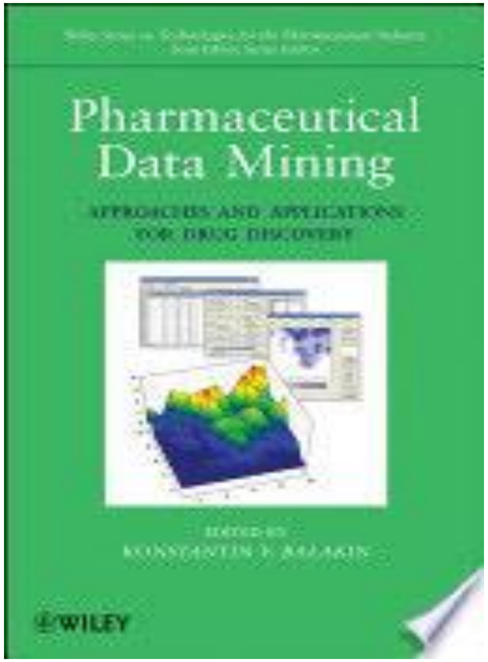
Ramsey Theory and Data Mining



"Ramsey theory predicts that more elaborate patterns will emerge as the number of data points increases".

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$42<R(5,5)<50$

Erdős asks us to imagine an alien force, vastly more powerful than us, landing on Earth and demanding the value of $R(5, 5)$ or they will destroy our planet. In that case, he claims, we should marshal all our computers and all our mathematicians and attempt to find the value. But suppose, instead, that they ask for $R(6, 6)$. In that case, he believes, we should attempt to destroy the aliens.

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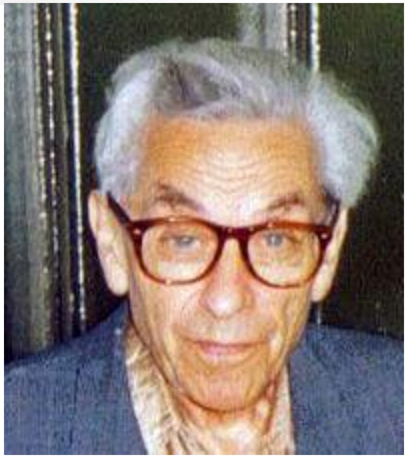
Paul Erdős



Paul Erdős was a Hungarian mathematician. He was one of the most prolific mathematicians of the 20th century. Erdős pursued problems in combinatorics, graph theory, number theory, classical analysis, approximation theory, set theory, and probability theory.

Born	26 March 1913
Died	20 Sept. 1996 (aged 83)

–1996)

Publications of Paul Erdös

Items of Interest Related to Erdös Numbers

The Erdös Number Project

This is the website for the Erdös Number Project, which studies research collaboration among mathematicians.

*The site is maintained by **Jerry Grossman** at **Oakland University**. **Patrick Ion**, a retired editor at **Mathematical Reviews**, and **Rodrigo De Castro** at the **Universidad Nacional de Colombia, Bogota** provided assistance in the past. Please address all comments, additions, and corrections to Jerry at **grossman@oakland.edu**.*

Erdös numbers have been a part of the **folklore of mathematicians** throughout the world for many years. For an introduction to our project, a description of what Erdös numbers are, what they can be used for, who cares, and so on, choose the "What's It All About?" link below. To find out who **Paul Erdös** is, look at this **biography** at the MacTutor History of Mathematics Archive, or choose the "Information about Paul Erdös" link below. Some useful information can also be found in **this Wikipedia article**, which may or may not be totally accurate.

WHAT'S INSIDE:

- **What's It All About?**: General overview, including our (admittedly arbitrary) rules for what counts as a research collaboration.
- **The Data**: Lists of all of Paul Erdös's coauthors and their respective coauthors, organized in various ways. There are also links to websites of or about Erdös's coauthors.
- **Facts about Erdös Numbers and Collaborations**: Statistical descriptions of Erdös number data, a file of the subgraph induced by Erdös coauthors, Erdös number record holders, facts about collaboration in mathematical research and the collaboration graph, including some information about publishing habits of mathematicians (for example, the median number of papers is 2, and the mean is about 7). This subpage has loads of information about the collaboration graph and Erdös numbers, including the distribution of Erdös numbers (they range up to 13, but the average is less than 5, and almost everyone with a finite Erdös number has a number less than 8) and "Erdös numbers of the second kind".
- **Famous Paths to Paul Erdös**: Fields Medalists and Nobel Prize winners have small Erdös numbers.
- **Compute Your Own Erdös Number**: It may be smaller than you think.
- **Research on Collaboration**: Papers on collaboration in scientific research, collaboration graphs and other small world graphs, and Erdös numbers. A lot of research is currently being done by various scientists on collaboration graphs and related topics.

Collaboration graph

- vertices are people
- edges connect people who collaborate
  (e.g. have a joint publication)

Collaboration distance is the length of a shortest path between two people in the collaboration graph

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The **Erdős number** is the distance to Paul Erdős.

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ISSN 2167-5163

University of Warwick

MOBILE ACCESS

Choose an Author Name

Please enter another Author name Lozin, Vadim V.

Enter another Author Name

Search | Erdös, Paul | Use Erdös

Free Tool

AMS AMERICAN MATHEMATICAL SOCIETY

Mirror Sites Providence, RI USA

AMERICAN MATHEMATICAL SOCIETY
MathSciNet Mathematical Reviews 75 years 1940-2015
ISSN 2167-5163

University of Warwick

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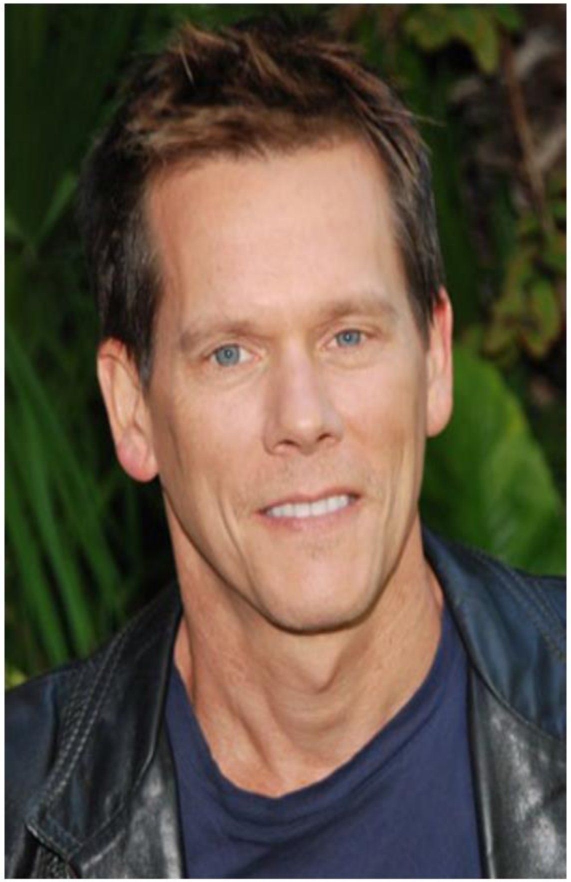
MR Erdos Number = 3

Vadim V. Lozin	coauthored with	Kathie Cameron	MR2069064 (2005a:05157)
Kathie Cameron	coauthored with	Horst Sachs	MR1273197 (95b:68090)
Horst Sachs	coauthored with	Paul Erdős[1]	MR0165515 (29 #2797)

Change First Author | Change Second Author | New Search

Free Tool

Kevin Bacon Number



- Example of an interesting use of graph theory!
- If we have a graph of actors
- Links indicate when 2 actors have worked on the same film
- The number of links between any actor and Kevin Bacon is that actor's Kevin Bacon number
- http://oracleofbacon.org/
- Use imdb for reference
- Let's try a couple....

Shortest Path

Problem. Find a shortest path between
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Applications: navigation,
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Dijkstra's Algorithm

Edsger Wybe Dijkstra

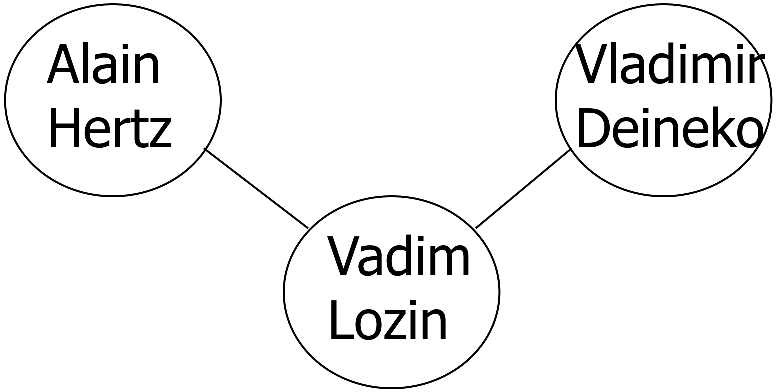
Born	11 May 1930 Rotterdam, Netherlands
Died	6 August 2002 (aged 72) Nuenen, Netherland

Did you know that

The difference in the speed of clocks at different heights above the earth is now of considerable practical importance, with the advent of very accurate navigation systems based on signals from satellites. If one ignored the predictions of general relativity theory, the position that one calculated would be wrong by several miles!

Stephen Hawking *A brief history of time*

My Small World



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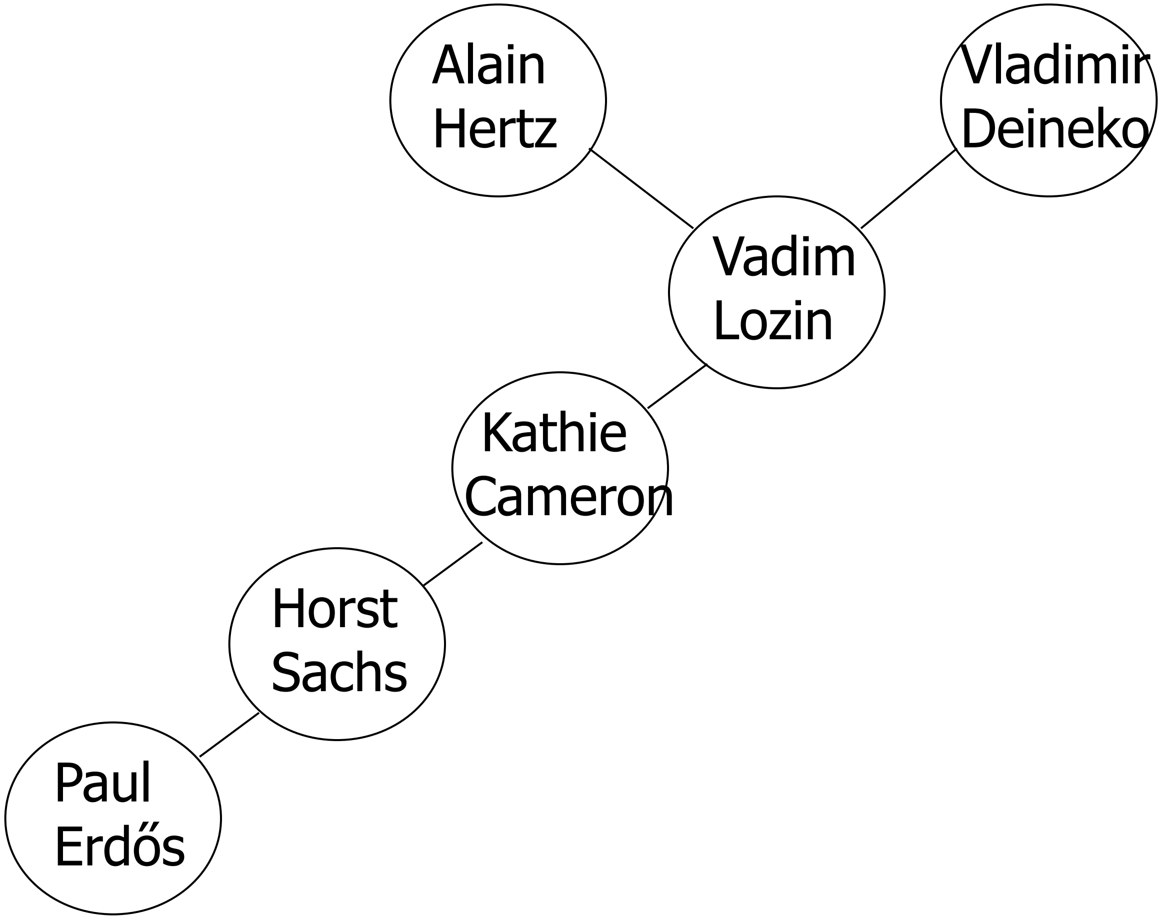

Marriage Problem:

- There are n boys and n girls.
- For each pair, it is either compatible or not.

Goal: find the maximum number of compatible pairs.

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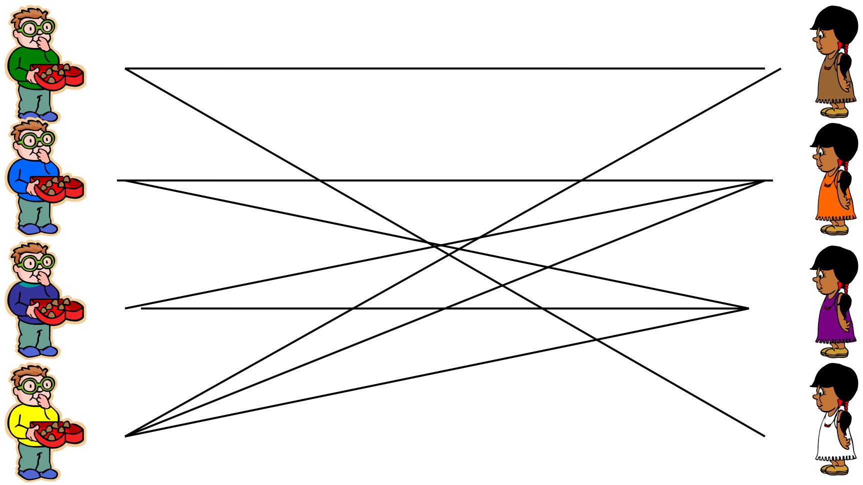
Bipartite graph



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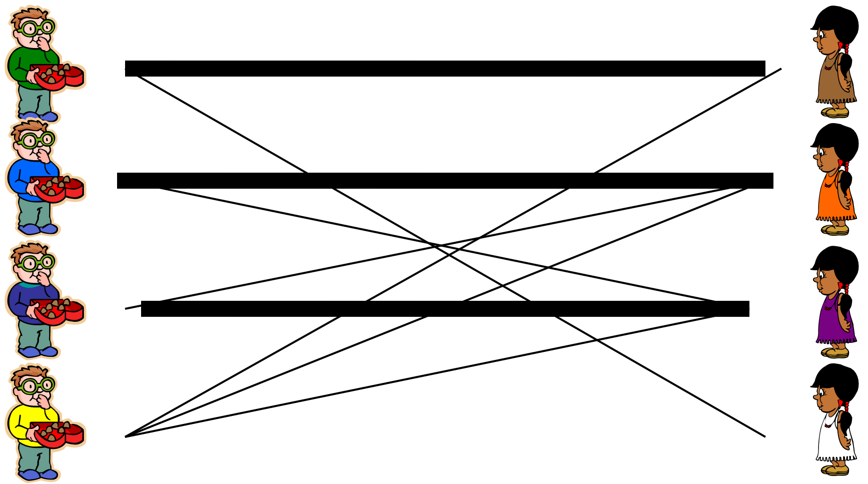

Definition. A <u>matching</u> in a graph is a subset of its edges no two of which share a vertex.

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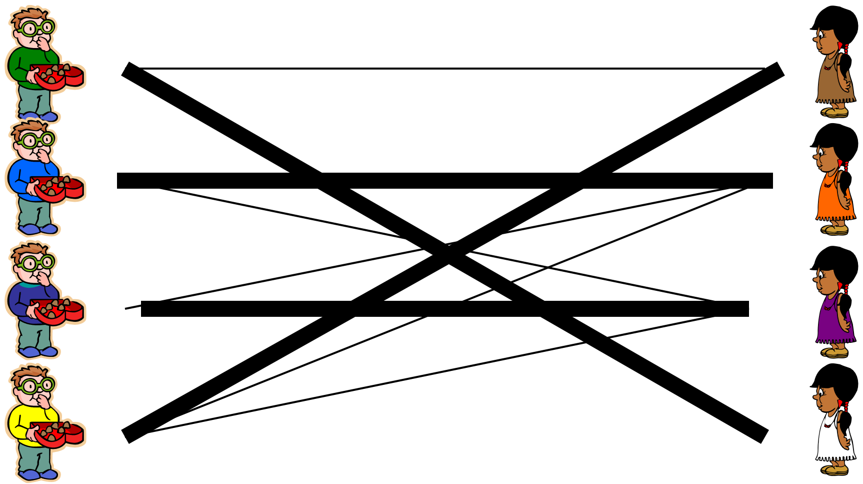

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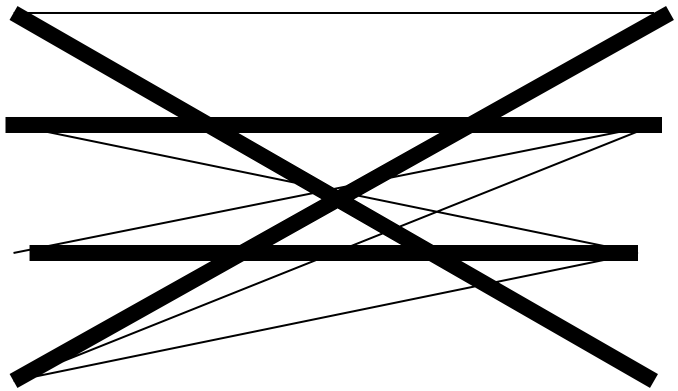
The maximum matching problem

Problem: Find a matching of maximum size

Bipartite graph



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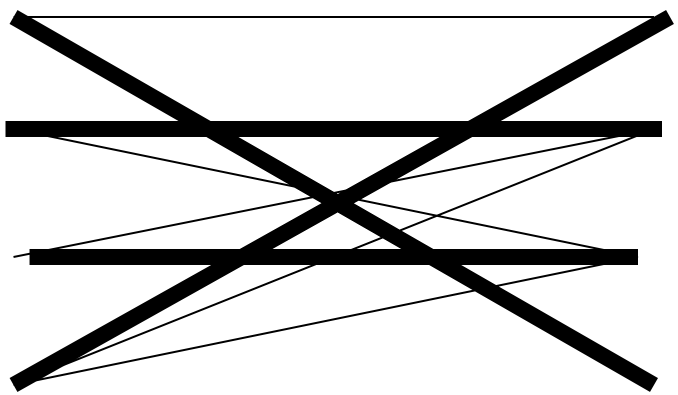
Maximum Flow



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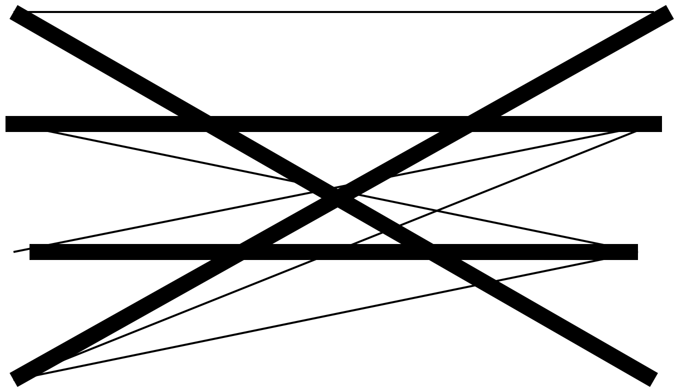

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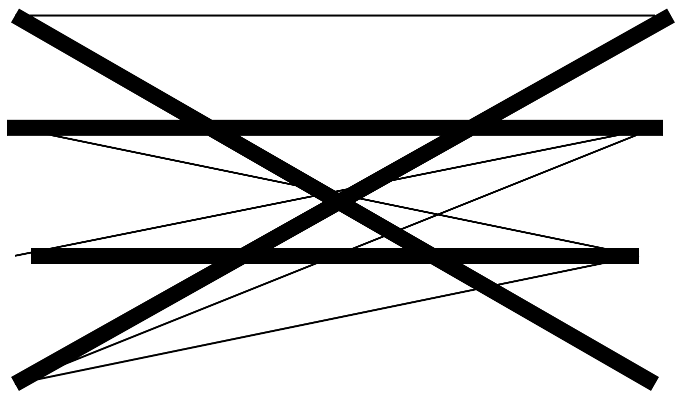

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The matching algorithm by Edmonds is one of the most involved of combinatorial algorithms.

Applications of Bipartite Matching to Problems in Object Recognition

Ali Shokoufandeh
Department of Mathematics and Computer Science
Drexel University
Philadelphia, PA

Sven Dickinson
Department of Computer Science and Center for Cognitive Science
Rutgers University
New Brunswick, NJ

Abstract

The matching of hierarchical (e.g., multiscale or multilevel) image features is a common problem in object recognition. Such structures are often represented as trees or directed acyclic graphs, where nodes represent image feature abstractions and arcs represent spatial relations, mappings across resolution levels, component parts, etc. Such matching problems can be formulated as *largest isomorphic subgraph* or *largest isomorphic subtree* problems, for which a wealth of literature exists in the graph algorithms community. However, the nature of the vision instantiation of this problem often precludes the direct application of these methods. Due to occlusion and noise, no significant isomorphisms may exists between two graphs or trees. In this paper, we review our application of a more general class of matching methods, called *bipartite matching*, to two problems in object recognition.

Stable Matching problem
Stable Marriage problem

Given *n* men and *n* women, where each person has ranked all members of the opposite sex in order of preference, marry the men and women together such that there are no two people of opposite sex who would both rather have each other than their current partners. When there are no such pairs of people, the set of marriages is deemed stable.

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Algorithms for finding solutions to the stable marriage problem have applications in a variety of real-world situations, perhaps the best known of these being in the assignment of graduating medical students to their first hospital appointments.

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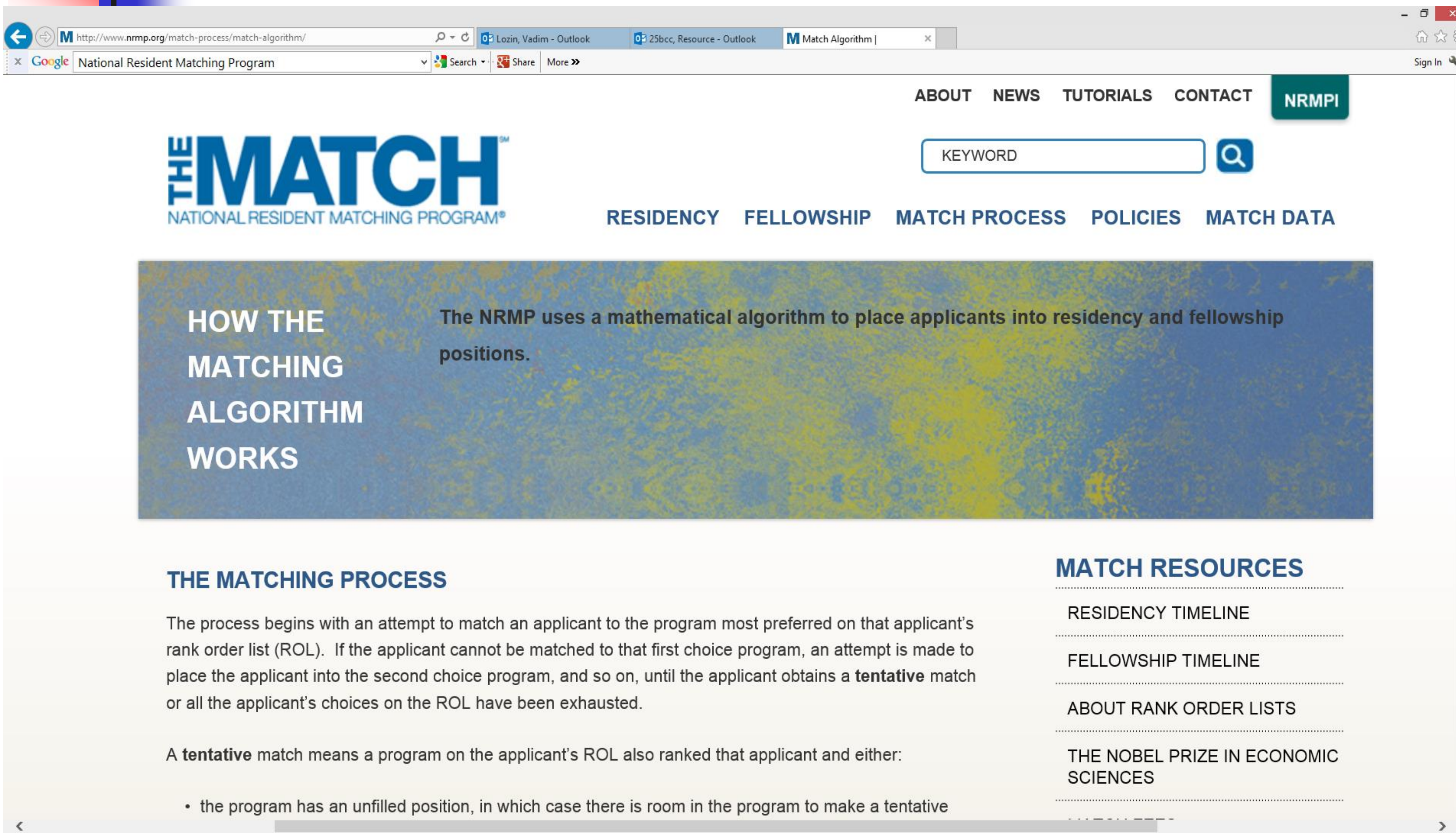


Economics

Nobel prize for economics won by Alvin Roth and Lloyd Shapley

Speed daters would be surprised to learn concept based on work by two academics awarded Nobel prize for economics

Larry Elliott and Josephine Moulds

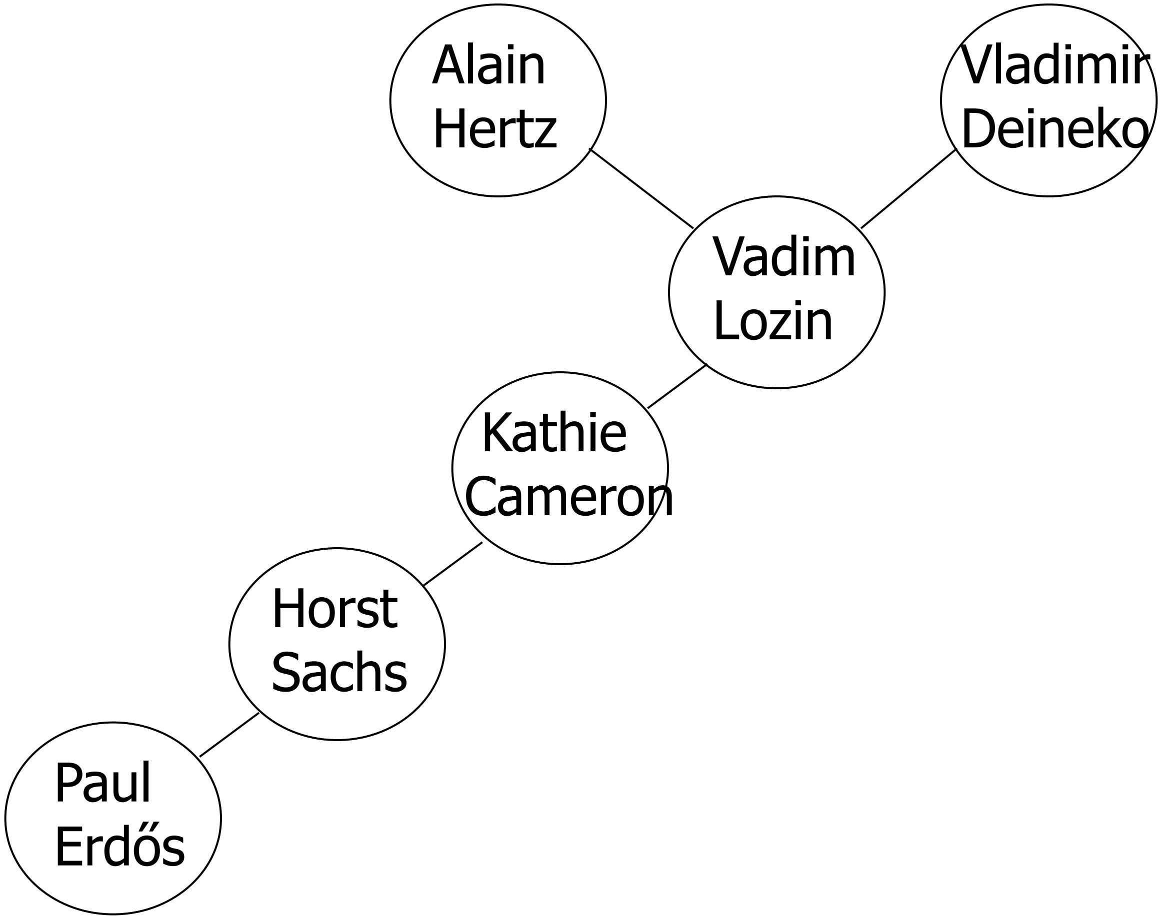
Monday 15 October 2012 18.54 BST

How to find a mate is one of the oldest problems in the world and one of many modern answers is to go on a speed dating evening. But most speed daters would be surprised to learn that the concept was based on academic work by an economist and a mathematician who have been honoured for their pioneering work with the Nobel prize for economics.

Fifty years ago, Lloyd Shapley became the godfather of modern matchmaking when he wrote a paper in which he sought to answer the question of how individuals in a group of people could be paired up when all had different views on who might be their best partner.

His work was later developed by Alvin Roth, who found other practical uses for the approach, including matching kidney donors with patients, and to make sure that students were allocated one of the schools of their choice.

My Small World



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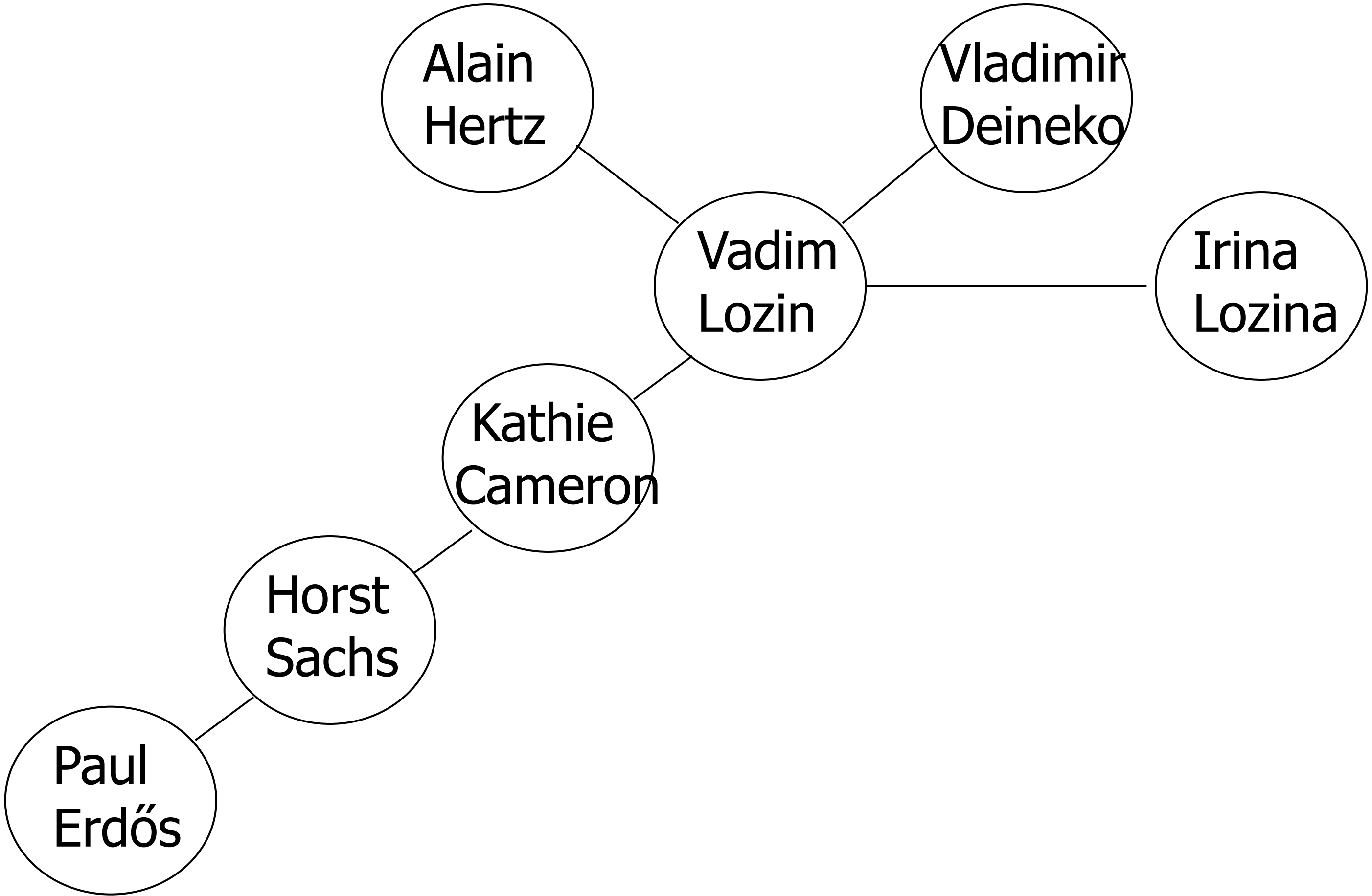
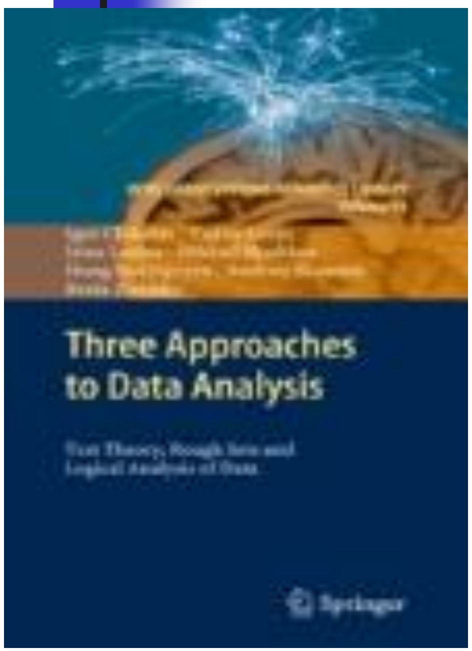
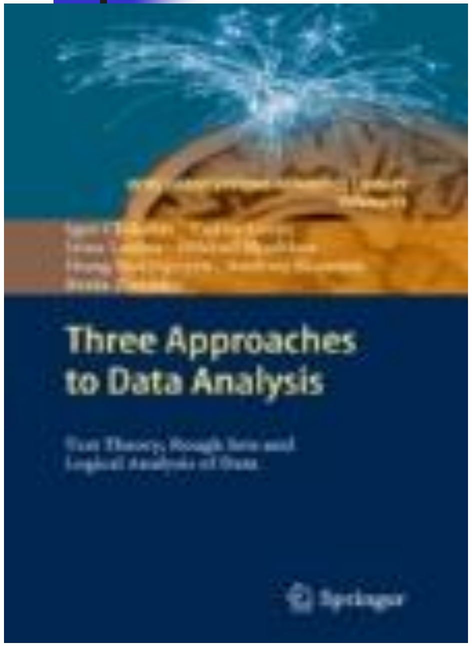



Three Approaches
to Data Analysis
Springer

Logical Analysis of Data



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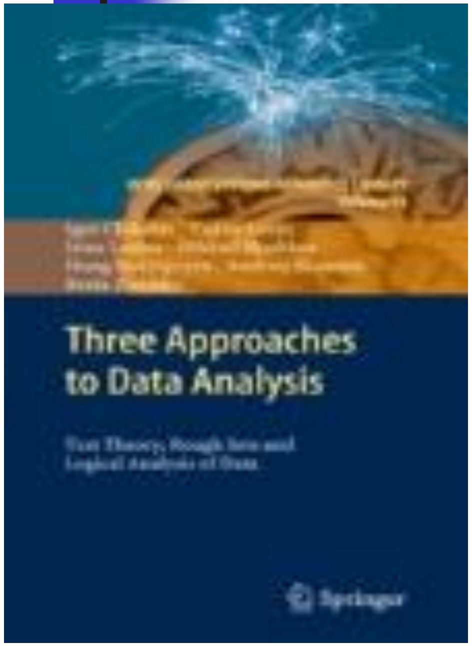

Peter Hammer

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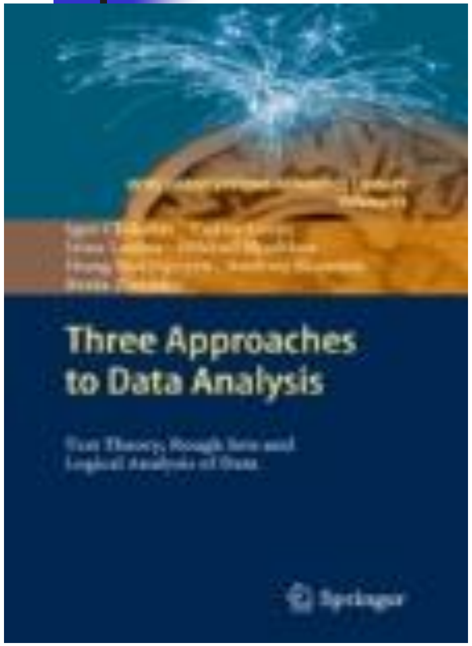

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Hammer founded the Rutgers University Center for Operations Research, and created and edited the journals *Discrete Mathematics*, *Discrete Applied Mathematics*, *Discrete Optimization*, *Annals of Discrete Mathematics*, *Annals of Operations Research*, and *SIAM Monographs on Discrete Mathematics and Applications*

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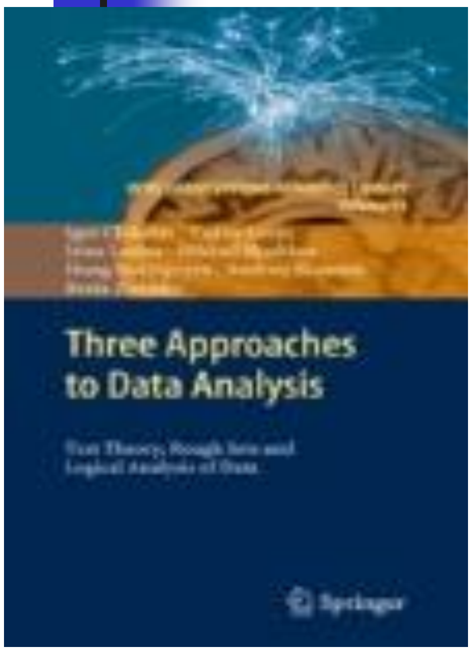

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He contributed to the fields of operations research and applied discrete mathematics through the study of pseudo-Boolean functions and their connections to graph theory and data mining.

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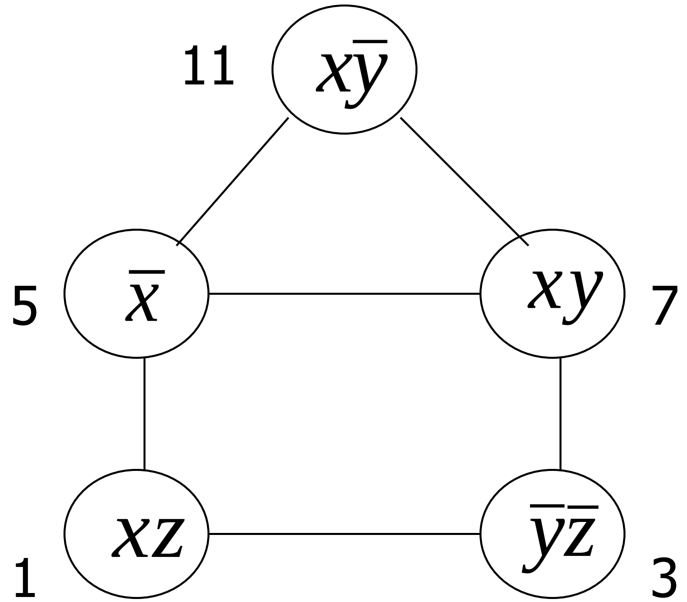
$$xz + 5\bar{x} + 11x\bar{y} + 7xy + 3\bar{y}\bar{z}$$ posiform

Conflict Graph



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Definition. In a graph, an independent set is a subset of vertices no two of which are adjacent.

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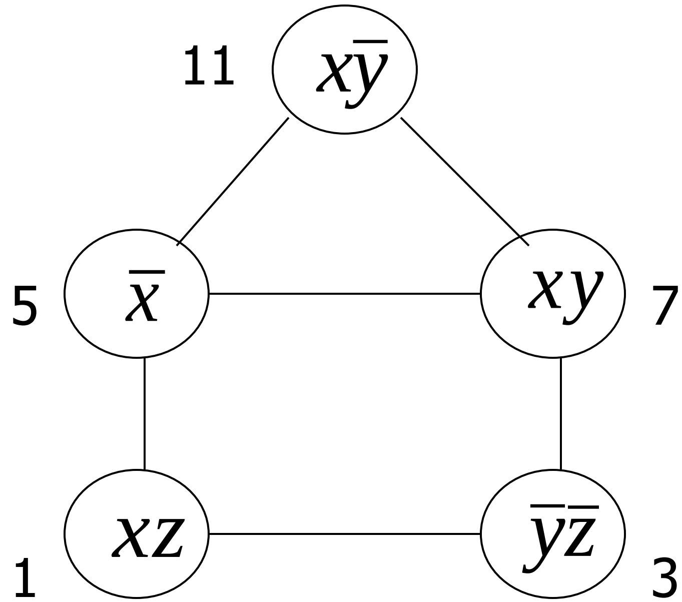

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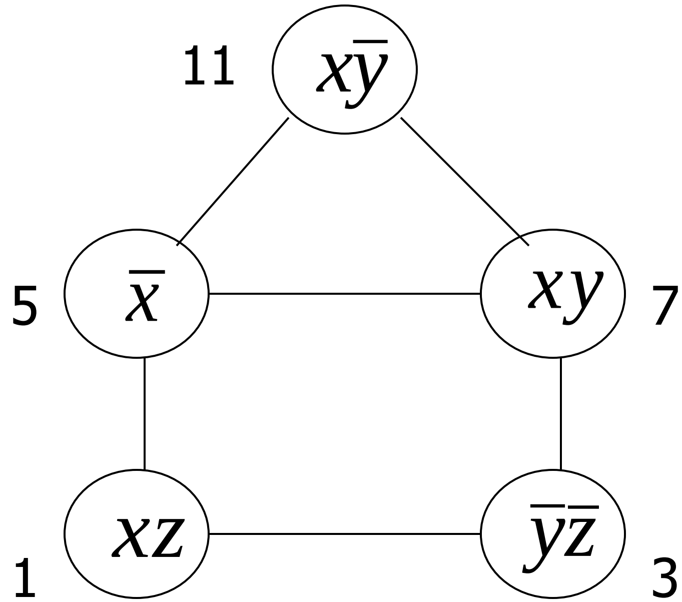

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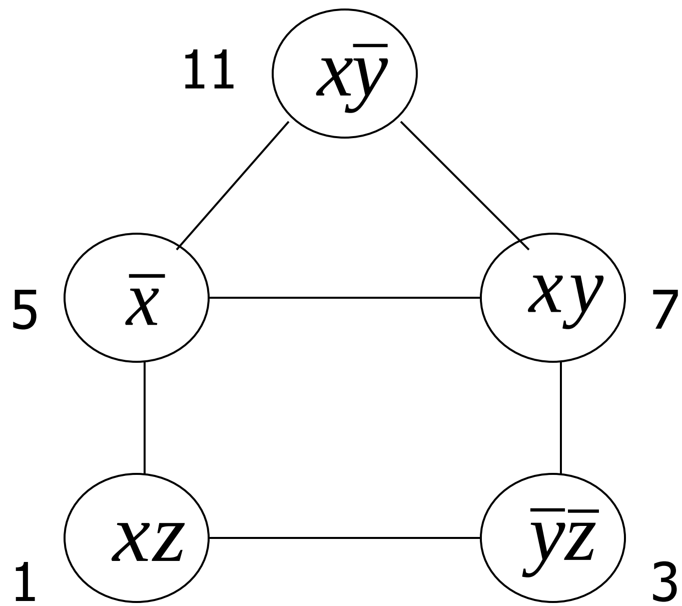

X=1 y=0
z=0

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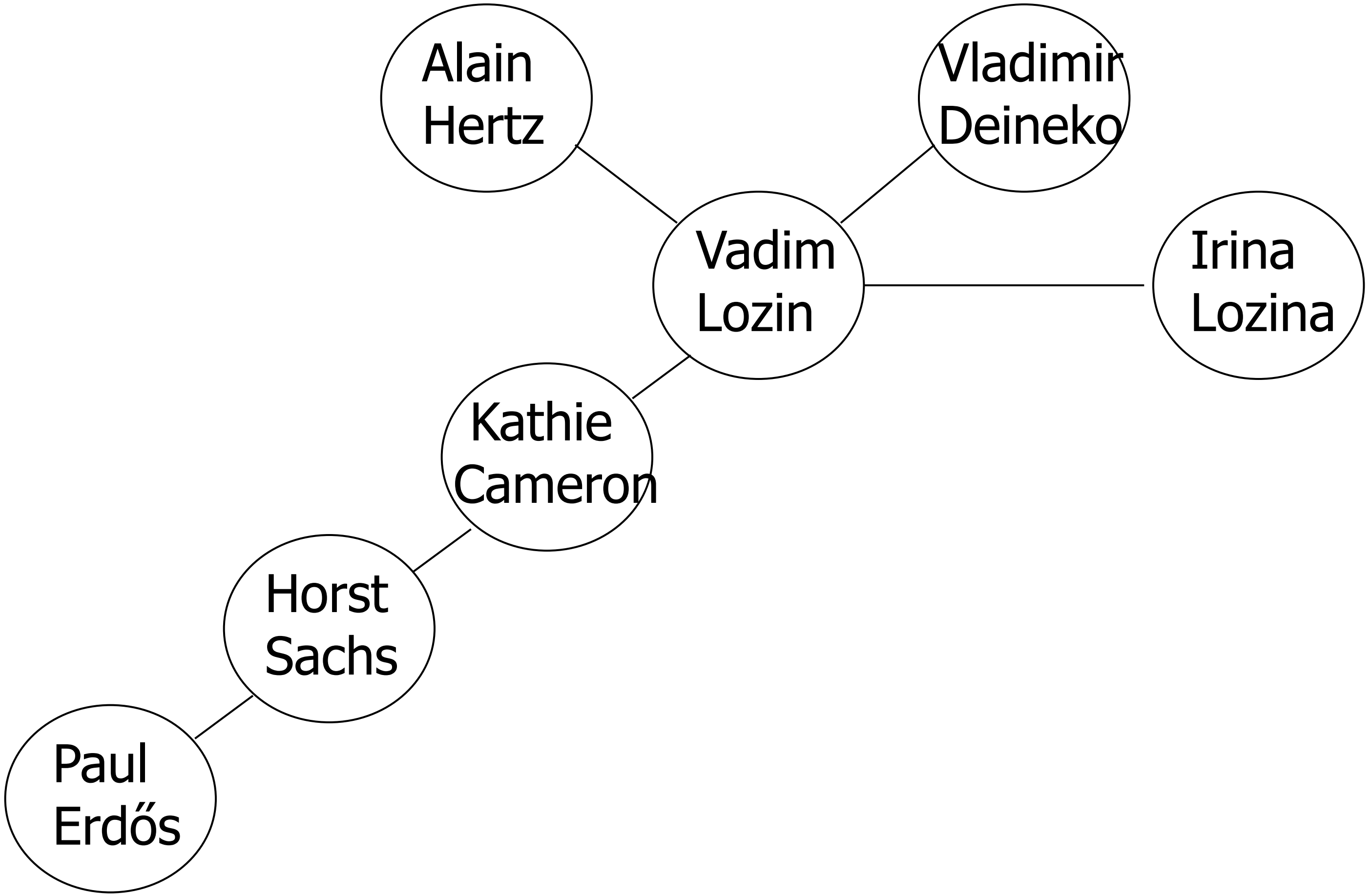
My Small World



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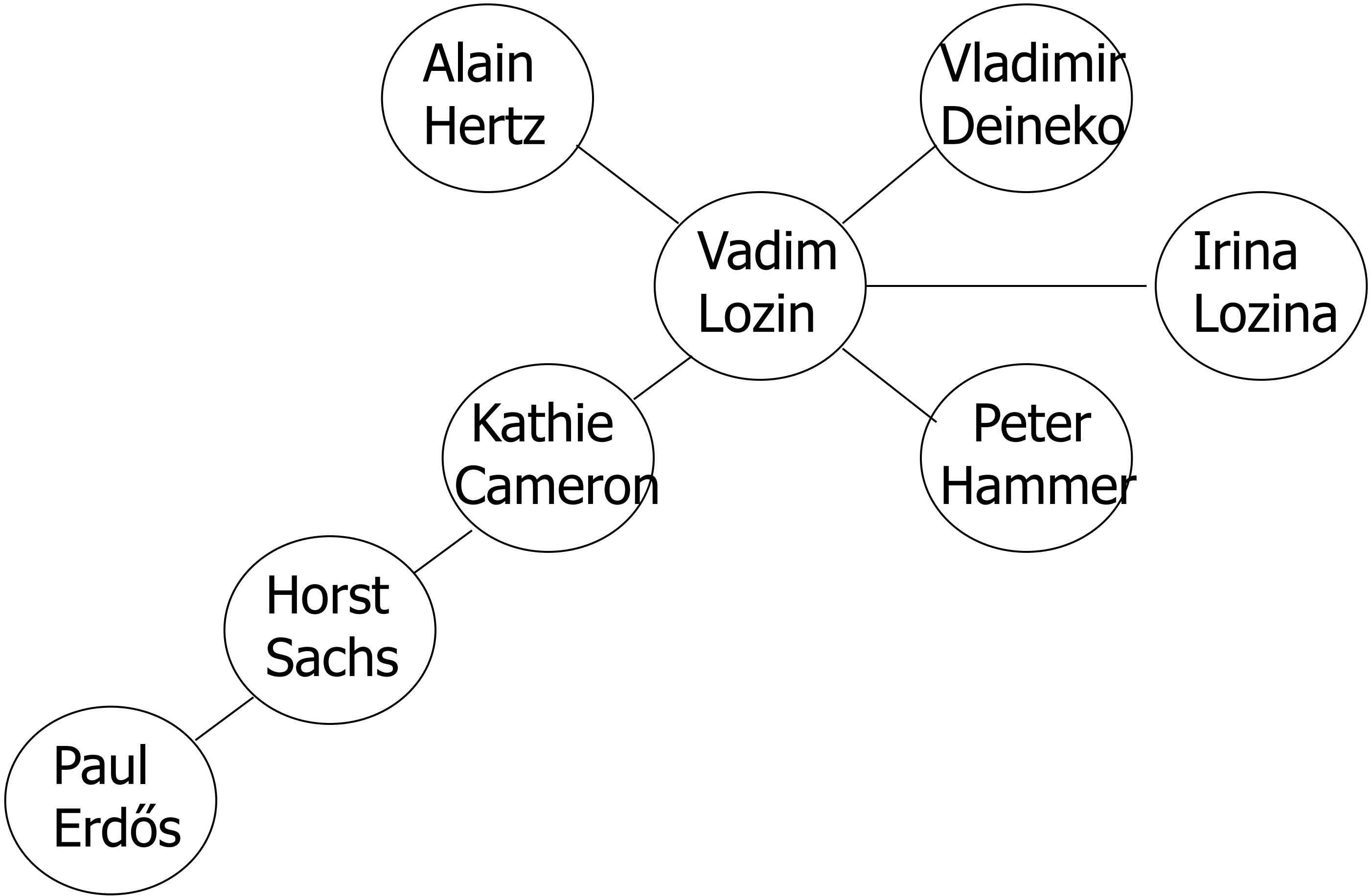

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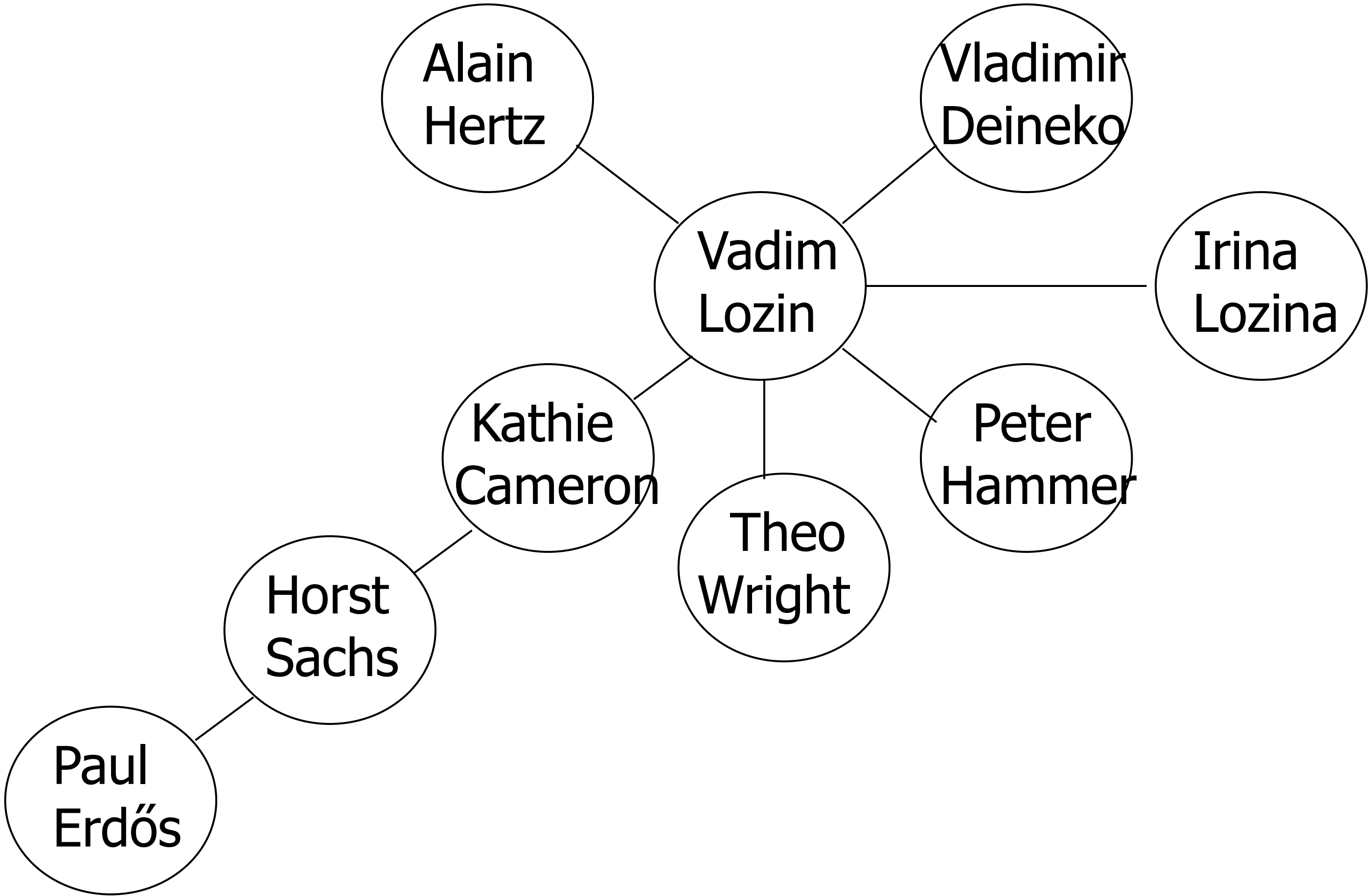


Textile project “Permutations”

Re: Maths/textile project - Internet Explore

https://outlook.office365.com/owa/#viewmodel=ReadMessageItem&ItemID=AAMkADhkZjlIM2Q5LTNjM2MtNDY3MS1iNzYyLTVkMjc1NWFiOTNINQBGAAAAAAZbsd%2BjudaRLjzNY4humpMBwDpLn8

Re: Maths/textile project

Theo Wright <weavingman@yahoo.com>
Sat 04/10/2014 07:56

To: ☐ Lozin, Vadim;

You forwarded this message on 18/11/2014 18:56

Theo Wright permutatio...
4 MB

1 attachment (4 MB)

Action Items

Vadim -

Thank you for agreeing to talk to me about my textile project Permutations.

I wonder if I could arrange a brief meeting with you at the University to discuss the project.
Let me know what would be convenient for you - I am based in Coventry and available on various dates in October/November (see below) and entirely free from the start of December.

17, 20, 21, 22, 27, 28, 29, 31 October
3, 5, 6 ,7 10, 11 November

I attach a short document that formed part of my bid to Arts Council England that helped me get the funding for the project.
I'm also keeping a blog on my website which will give an idea of the progress of the project http://www.theowright.co.uk/permutations-blog

Regards
Theo Wright

www.theowright.co.uk
07837 702317

Textile project "Permutations"

Theo Wright
Handwoven

Theo Wright is a weaver and textile designer based in Coventry, UK.

Originally trained as a computer scientist, he worked for many years in software development and technology research.

He subsequently retrained as a weaver, graduating from University College Falmouth in 2011, and now designs geometric textiles that explore pattern, contrast and colour.

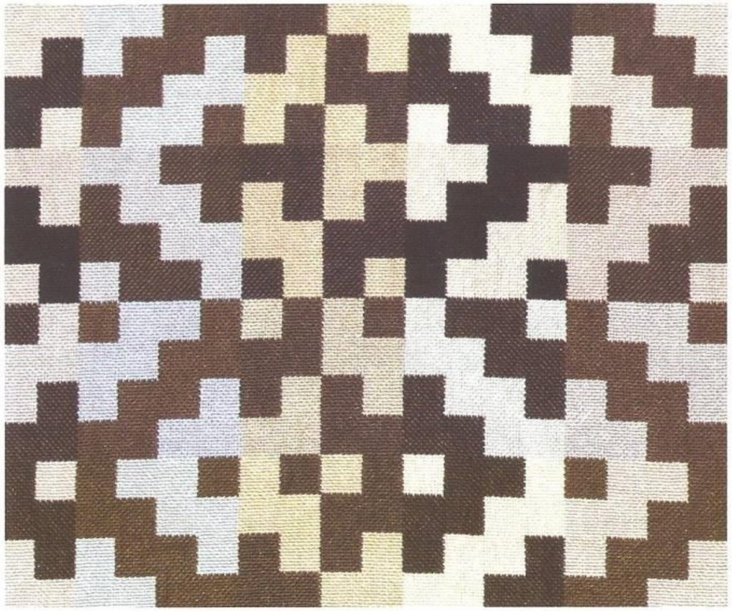
Since 2011 he has designed and woven scarves, selling online and at major craft fairs.

In 2014, Theo was a participant in the Crafts Council's Hothouse maker development programme and started to make larger non-functional textile works.

www.theowright.co.uk

LOTTERY FUNDED
Supported using public funding by
ARTS COUNCIL ENGLAND

Permutations



Theo Wright

2, 1, 4, 3 is a permutation of the set {1,2,3,4}

Textile project “Permutations”

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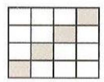
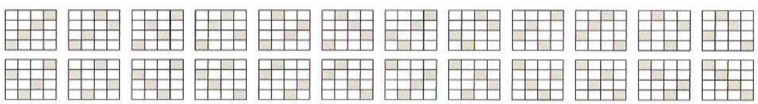
Textile project “Permutations”

Design

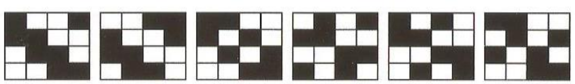
On a 4-shaft loom, each warp end is threaded onto one of the four shafts. The most common approach is straight threading.



The Permutations project was inspired by thinking about how many different ways a small section of warp could be threaded. For a group of four warp ends, each on a different shaft there are 4! (four factorial) ways: 4! = 4.3.2.1 = 24.

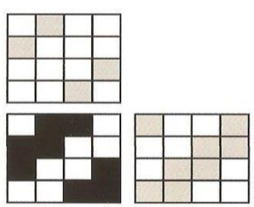


Using these warp threadings, and lifting two shafts on each pick of the weft yarn, I have created a set of 4 x 4 square patterns where each row and column is different and there are two dark squares in each row and column. There are 72 different patterns, e.g.



All of the Permutations textiles are made up of these 4 x 4 patterns.

I have selected and ordered the threadings and lifts differently for each work to create a unique large-scale design from these simple elements.



Although the original inspiration came from a group of just four warp ends and a simple weave structure, in order to scale up these designs I have used a 16-shaft mechanical dobby loom and woven the designs as a double cloth with one dark and one light layer, threaded in four blocks.

I use a computer in the design process, but I do the weaving itself entirely by hand.

Permutations and Combinations

I have used ideas of permutations and combinations in the textiles for both the pattern and the use of colour.

The maths behind the project is as follows. Suppose I choose k items from a set of n items (as a simple example, I might select two letters from the set of three letters: A, B, C. Here k=2, n=3)

To work out how many options there are I need to answer two questions:

> Is ordering significant (e.g. is AB considered to be different from BA)? If ordering is significant the options are called permutations; if it isn’t they are called combinations.
>
> Are repeat selections allowed (e.g. can I choose the letter A twice to give me AA) or do the selections have to be different?

All four types are useful in different circumstances. Here are the general formulae and my example of two letters selected from three.

Permutations (repeats allowed) n^k

e.g. $3^2 = 9$ (AA, AB, BA, AC, CA, BB, BC, CB, CC)

Permutations (repeats not allowed) $\dfrac{n!}{(n-k)!}$

e.g. 3!/(3-2)! = 6/1 = 6 (AB, BA, AC, CA, BC, CB)

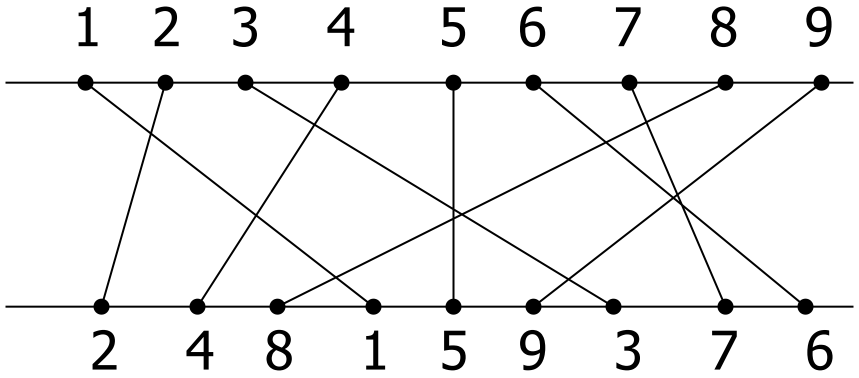
Combinations (repeats allowed) $\dfrac{(n+k-1)!}{k!\,(n-1)!}$

e.g. (3+2-1)!/2!(3-1)! = 4!/2!2! = 24/4 = 6 (AA AB AC BB BC CC)

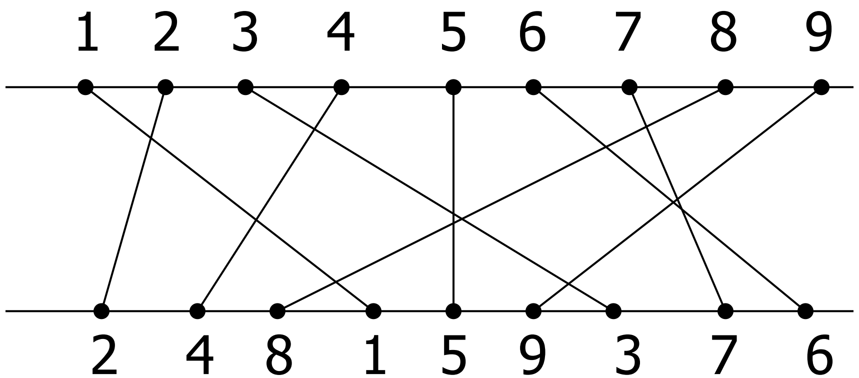
Combinations (repeats not allowed) $\dfrac{n!}{k!\,(n-k)!}$

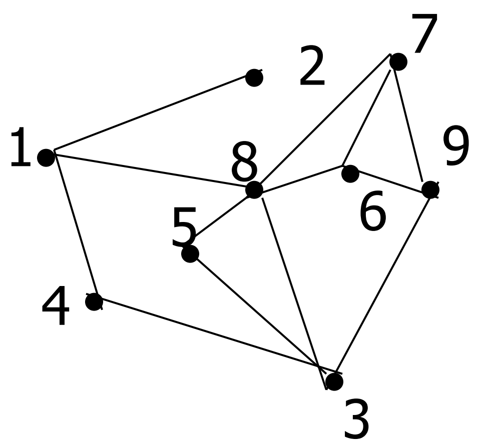
e.g. 3!/2!(3-2)! = 6/2!1! = 3 (AB, AC, BC)

From permutations to graphs



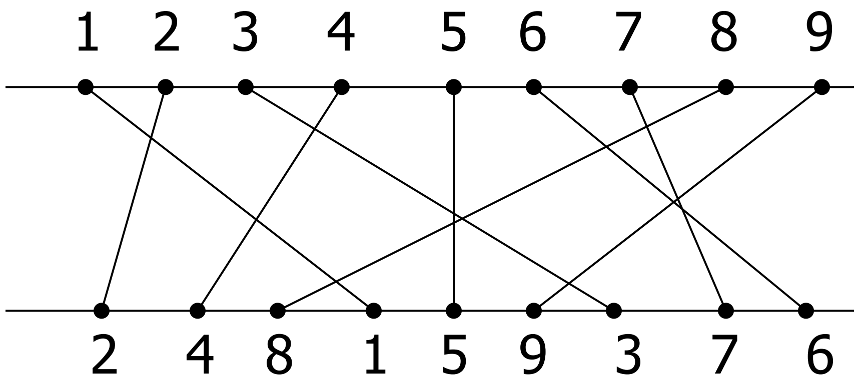
From permutations to graphs



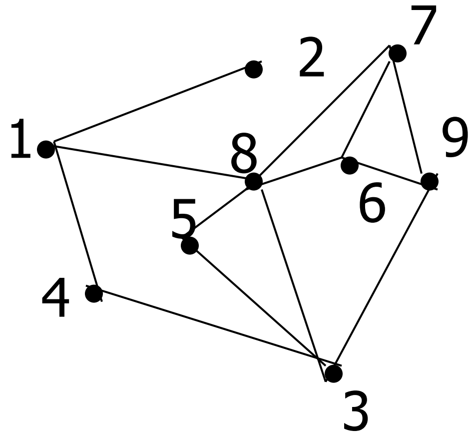


Permutation graph

From permutations to graphs



Longest increasing subsequence



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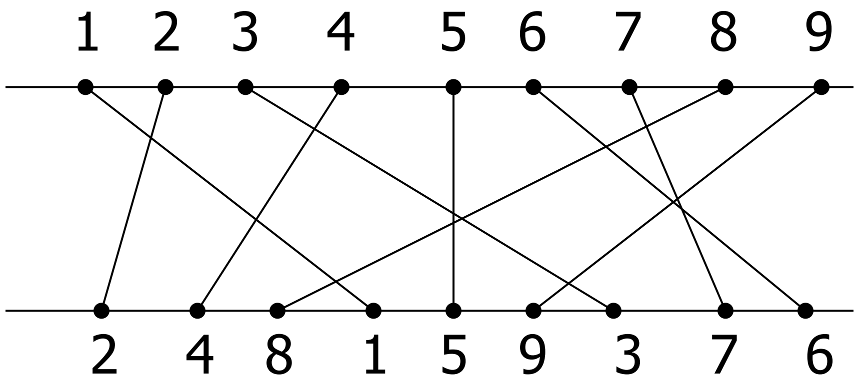
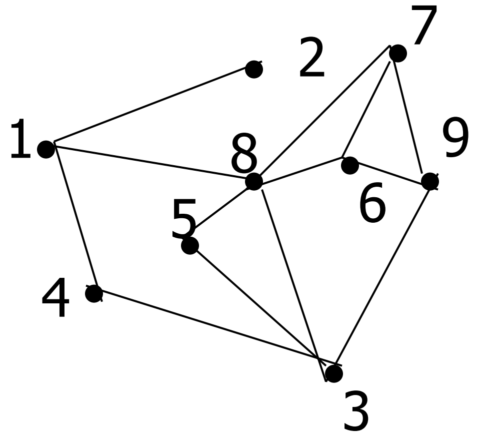
From permutations to graphs



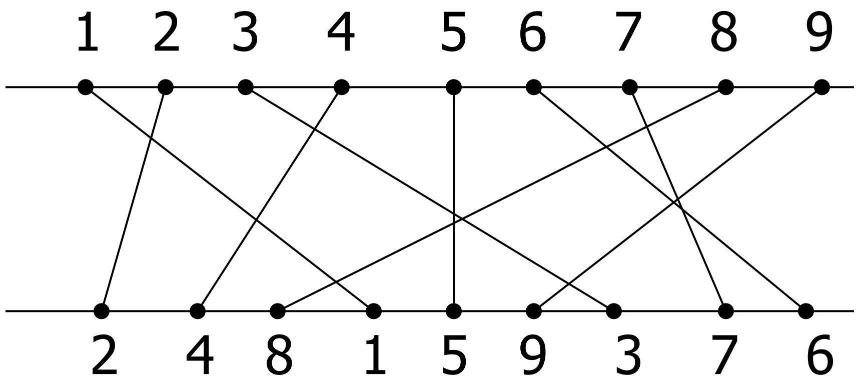
Longest increasing subsequence



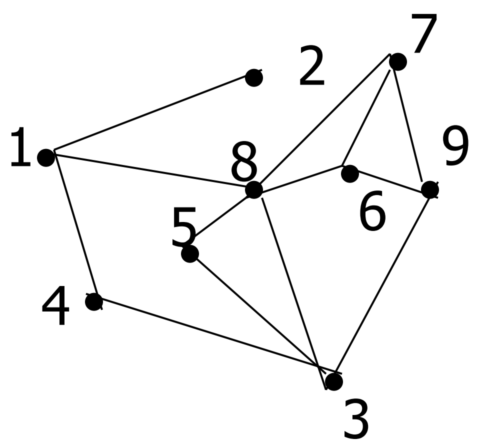
Permutation graph

Maximum independent set

From permutations to graphs



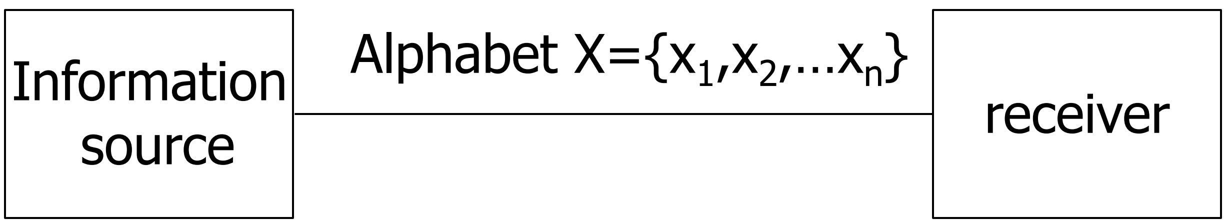
Longest decreasing subsequence



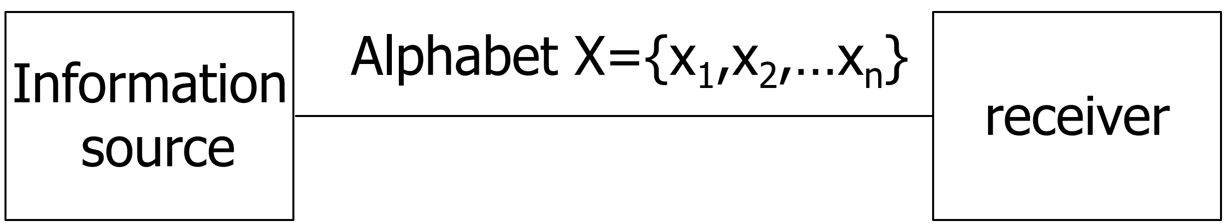
Permutation graph

Maximum clique, maximum subset of pairwise adjacent vertices

Maximum Independent set and Coding Theory

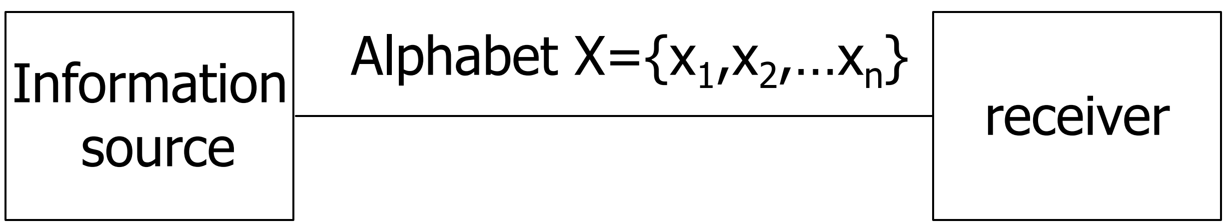


Maximum Independent set and Coding Theory



$G=(X,E)$ where $x_i x_j$ are adjacent if x_i and x_j can be interchanged

Maximum Independent set and Coding Theory



$G=(X,E)$ where $x_i x_j$ are adjacent if x_i and x_j can be interchanged

A largest noise-resistant code corresponds to a maximum independent set in G

Maximum cliques in Computer Vision and Pattern Recognition

Maximum cliques in Computer Vision and Pattern Recognition

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THIRTY YEARS OF GRAPH MATCHING IN PATTERN RECOGNITION

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A recent paper posed the question: "Graph Matching: What are we really talking about?". Far from providing a definite answer to that question, in this paper we will try to characterize the role that graphs play within the Pattern Recognition field. To this aim two

Maximum cliques in Computer Vision and Pattern Recognition

Matching of relational structures $\longrightarrow$ Maximum common subgraph

Maximum cliques in Computer Vision and Pattern Recognition

Matching of relational structures $\longrightarrow$ Maximum common subgraph

$G_1=(V_1,E_1)$ $\quad$ $G_2=(V_2,E_2)$

$$V = V_1 \times V_2$$

$G=(V,E)$

Association graph

$(i,j) \in V$ and $(k,l) \in V$ are adjacent in G if and only if $i \neq k$, $j \neq l$, and $ik \in E_1$ and $jl \in E_2$

Maximum cliques in Computer Vision and Pattern Recognition

Matching of relational structures $\longrightarrow$ Maximum common subgraph

$G_1=(V_1,E_1)$ $\quad$ $G_2=(V_2,E_2)$

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Association graph

$V = V_1 \times V_2$

$(i,j) \in V$ and $(k,l) \in V$ are adjacent in G if and only if $i \neq k$, $j \neq l$, and $ik \in E_1$ and $jl \in E_2$

A maximum common subgraph of G_1 and G_2 corresponds to a maximum clique in G

Maximum cliques in Computer Vision and Pattern Recognition

Marcello Pelillo

Follow

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Pattern Recognition, Computer Vision, Machine Learning
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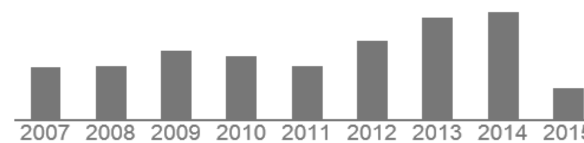


Regularity Lemma

The Szemeredi Regularity Lemma is one of the most fundamental and ingenious results in graph theory and discrete mathematics. It was originally advanced by Endre Szemeredi as an auxiliary result to prove a long standing conjecture of Erdős and Turán from 1936 (on the Ramsey properties of arithmetic progressions). Now the regularity lemma by itself is considered as one of the most important tools in graph theory.

A **very rough statement** of the regularity lemma could be made as follows:

Every graph can be approximated by random graphs. This is in the sense that every graph can be partitioned into a bounded number of equal parts such that:
1. Most edges run between different parts
2. And that these edges behave as if generated at random.

Regularity Lemma and Machine Learning

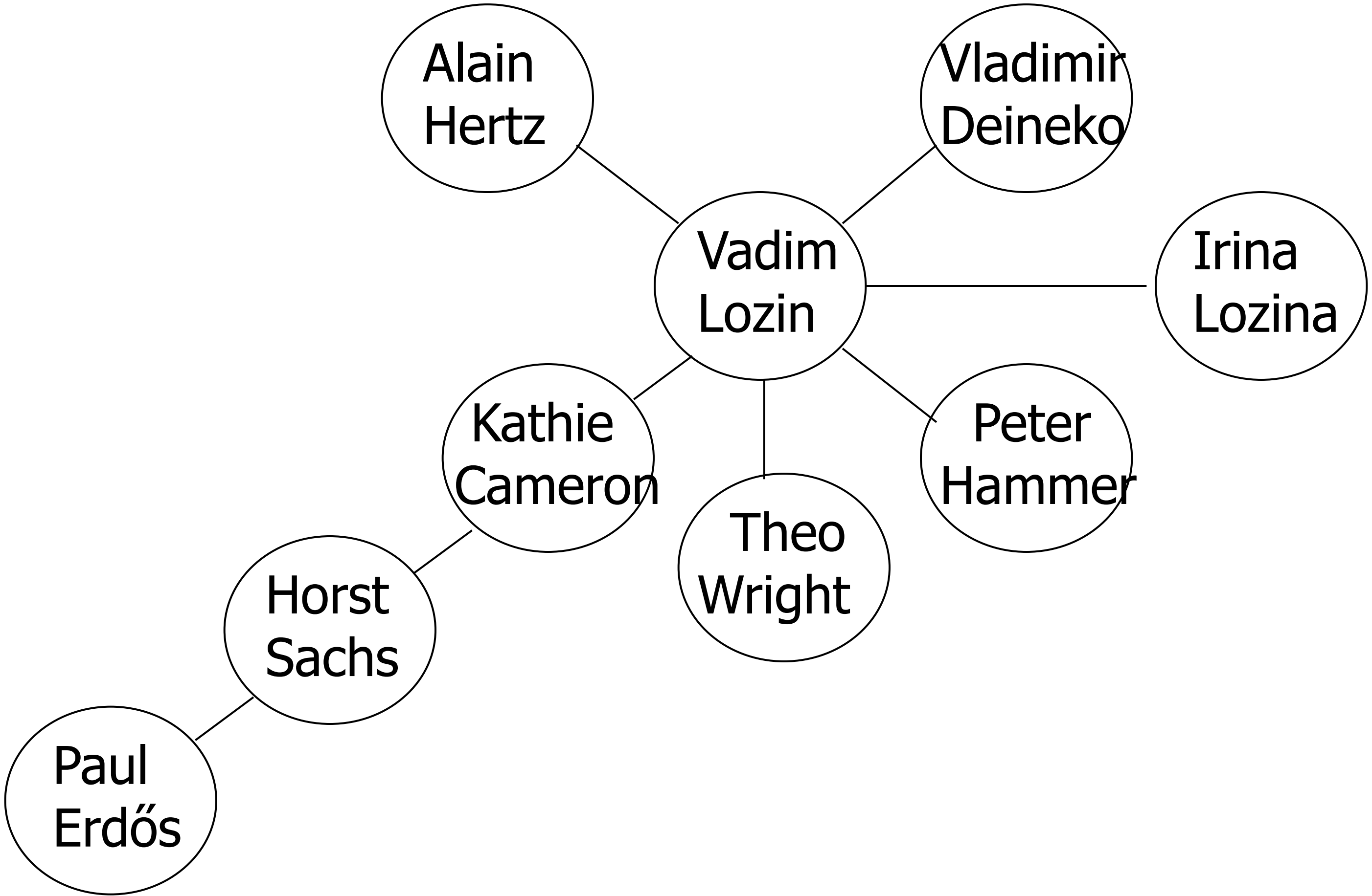
Importing the Szemerédi Regularity Lemma into Machine Learning

January 7, 2012 by Shubhendu Trivedi

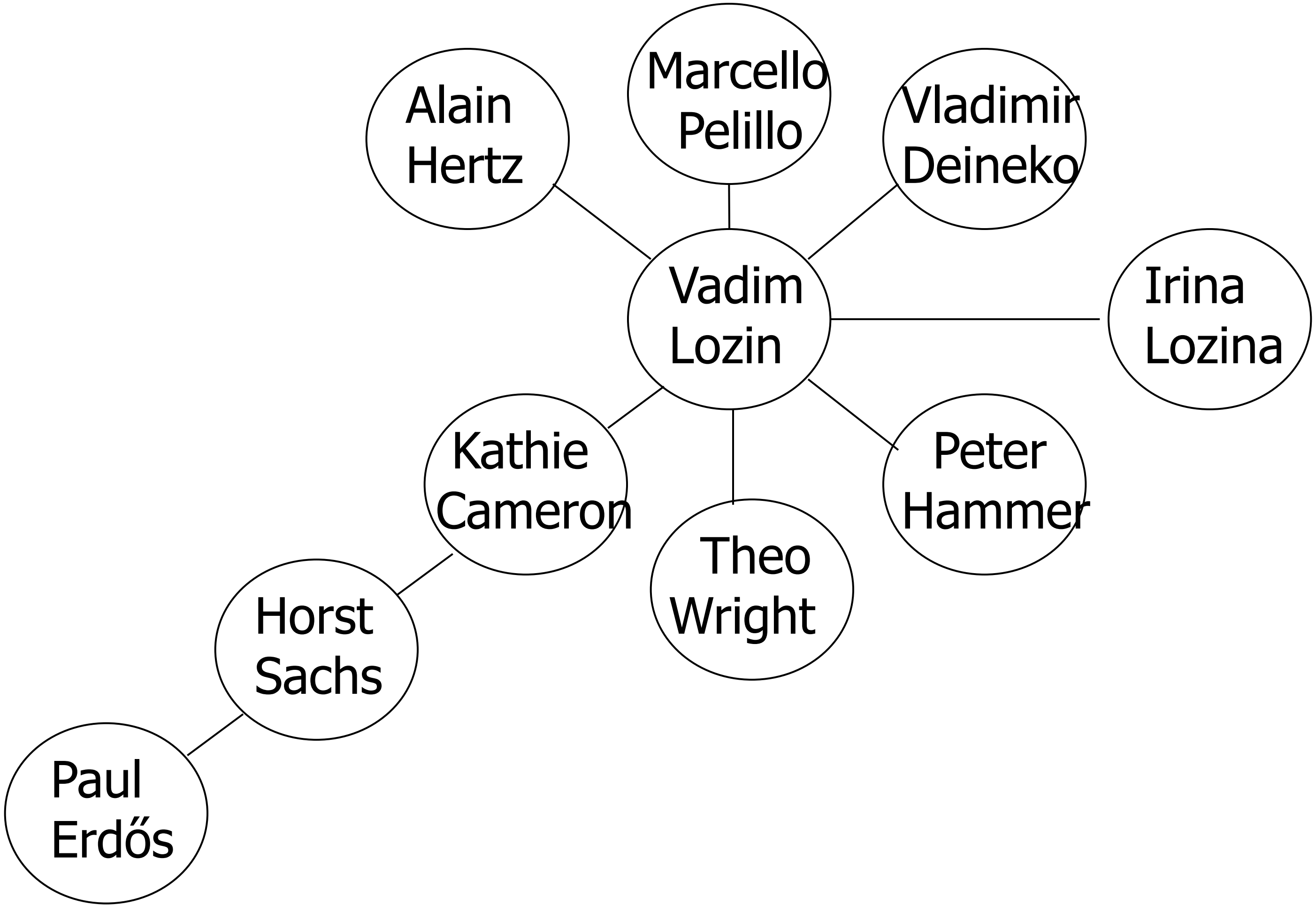
Synopsis of a recent direction of work with Gábor Sárközy, Endre Szemerédi and Fei Song — "The Regularity Lemma is a deep result from extremal graph theory having great utility as a fundamental tool to prove theoretical results, but can it be employed in more "practical" settings?"

More specifically we are interested in the problem of harnessing the power of the regularity lemma to do clustering. This blog post is organized as follows: We first sketch the regularity lemma, we then see that it is an existential predicate and state an algorithmic version, we then look at how this constructive version may be used for clustering/segmentation. It must be noted that the idea seems to have originated from an earlier interesting work by Sperotto and Pellilio.

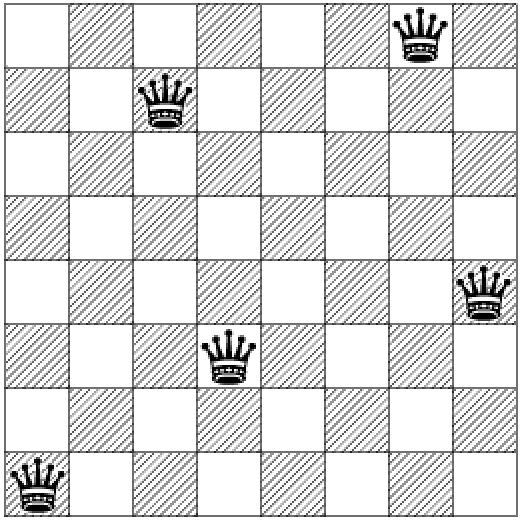
My Small World



My Small World



What is the minimum number of queens needed to occupy or attack all squares of an 8x8 board?



Minimum Dominating Set problem

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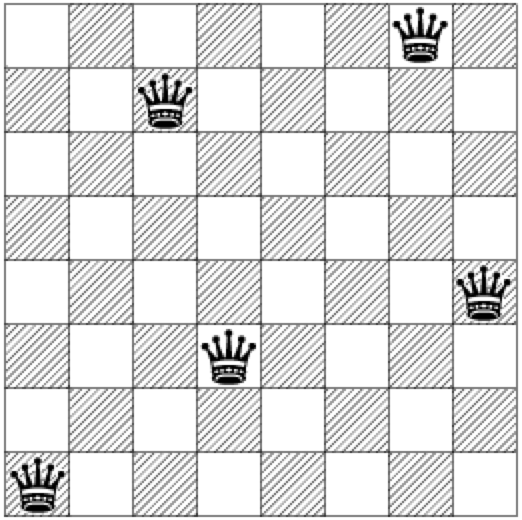

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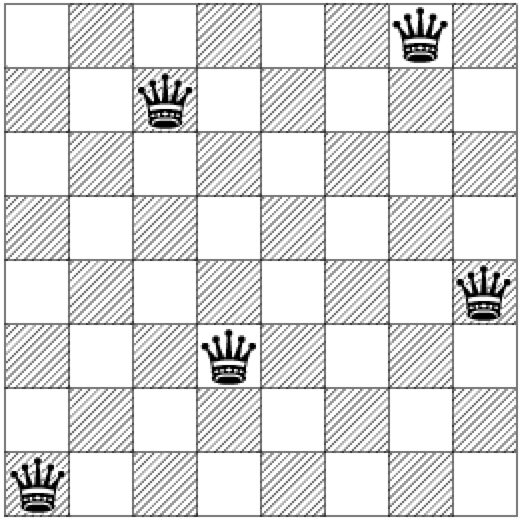

Let G be the graph in which every vertex corresponds to a square and two vertices are adjacent if and only if they belong to the same horizontal, vertical or diagonal line.

Minimum Dominating Set problem

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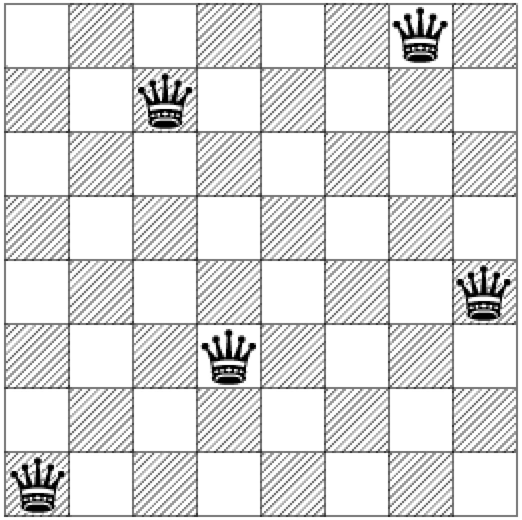
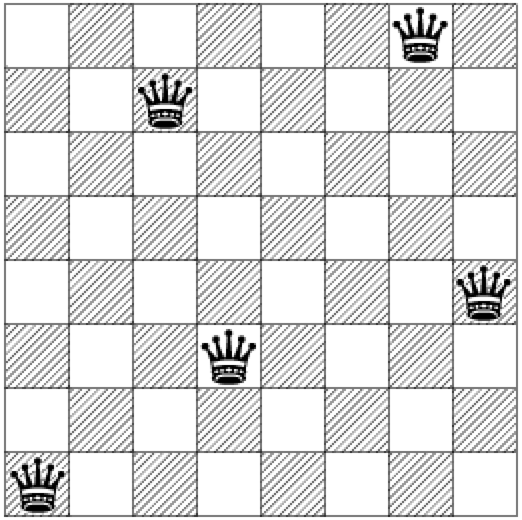
to occupy or attack all squares of an 8x8 board?



Let G be the graph in which every vertex corresponds to a square and two vertices are adjacent if and only if they belong to the same horizontal, vertical or diagonal line.

Definition. A set of vertices in a graph is dominating if every vertex outside of the set has a neighbour in the set.

Minimum Dominating Set problem



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Definition. A set of vertices in a graph is dominating if every vertex outside of the set has a neighbour in the set.

Problem. Find a dominating set of minimum size.

Minimum Dominating Set problem

Applications

Domination arises in facility location problems, where the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced.

Minimum Dominating Set problem

Applications

Domination arises in facility location problems, where the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced.

Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

Multi-Document Summarization via the Minimum Dominating Set

Chao Shen and **Tao Li**
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Florida Internation University
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Abstract

Multi-document summarization has been an important problem in information retrieval. It aims to distill the most important information from a set of documents to generate a compressed summary. Given a sentence graph generated from a set of documents where vertices represent sentences and edges indicate that the corresponding vertices are similar, the extracted summary can be described using the idea of graph domination. In this paper, we propose a new principled and versatile framework for multi-document summarization using the minimum dominating set. We show that four well-known

entire input set (Jurafsky and Martin, 2008). The generated summary can be generic where it simply gives the important information contained in the input documents without any particular information needs or query/topic-focused where it is produced in response to a user query or related to a topic or concern the development of an event (Jurafsky and Martin, 2008; Mani, 2001).

Recently, new summarization tasks such as update summarization (Dang and Owczarzak, 2008) and comparative summarization (Wang et al., 2009a) have also been proposed. Update summarization aims to generate short summaries of recent documents to capture new information different from earlier documents and comparative summarization aims to summarize the differences between comparable document groups.

Variations of Domination

Connected Domination
Independent Domination
Roman Domination
Fractional Domination
Total Domination
Paired Domination

SATISFIABILITY

SATISFIABILITY

Determine if a CNF formula is satisfiable

$$(x \vee y \vee \bar{z})(\bar{x} \vee y \vee z)(x \vee \bar{y} \vee z)$$

SATISFIABILITY in terms of graphs

Determine if a CNF formula is satisfiable

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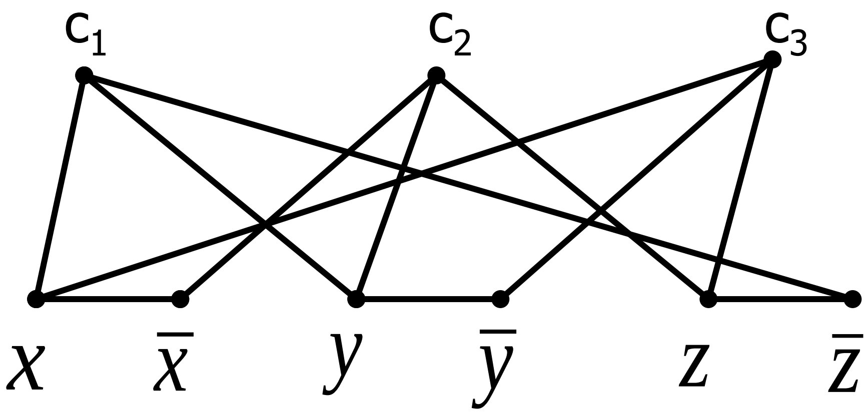

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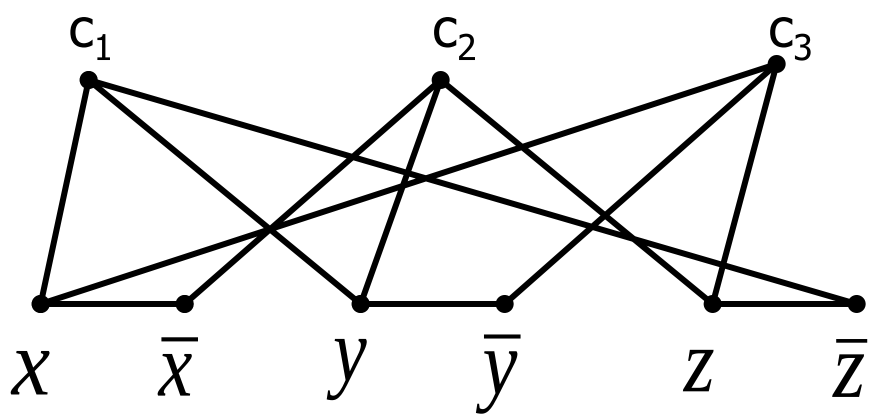

SAT: is there an independent set in the bottom part of the graph which dominates the upper part?

SATISFIABILITY in terms of graphs

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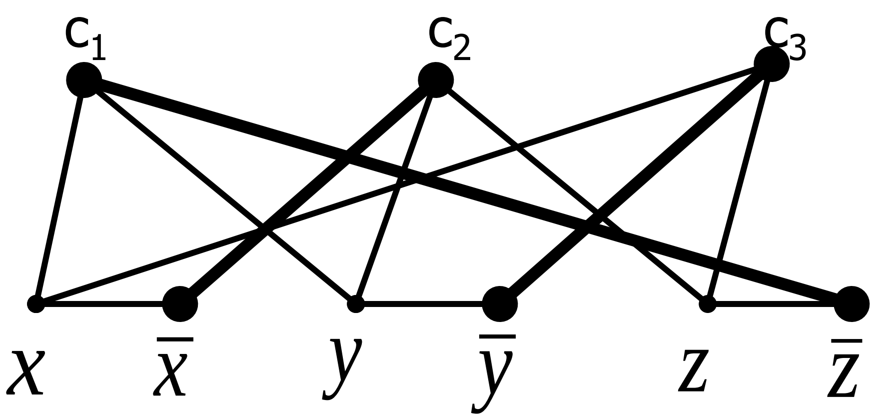

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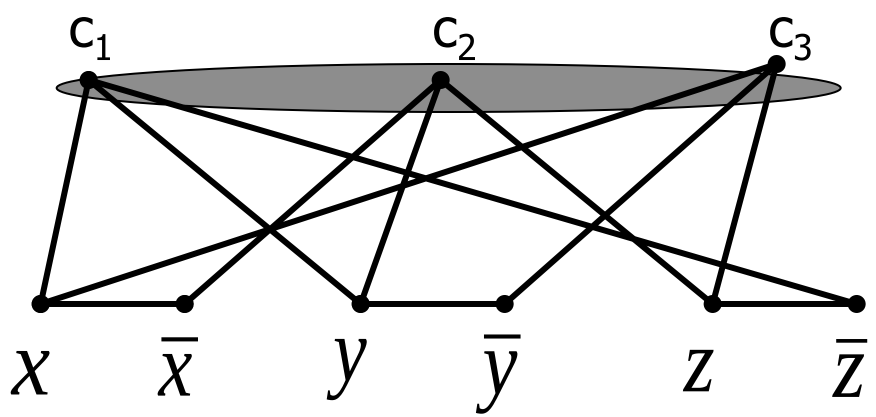

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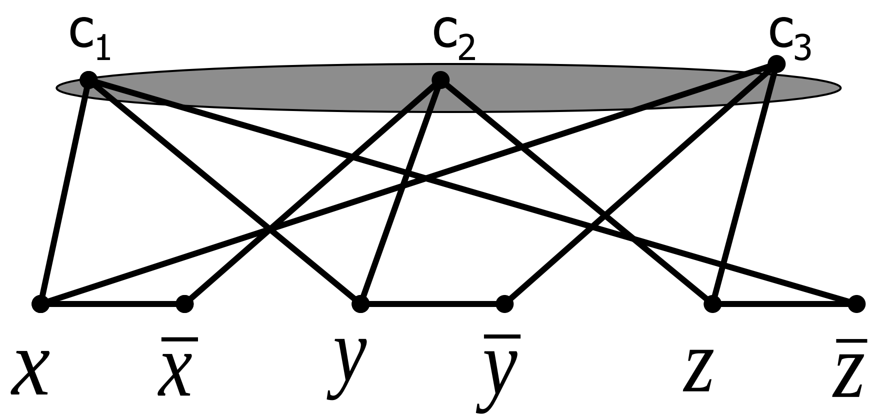

Independent Domination

||

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SATISFIABILITY in terms of graphs

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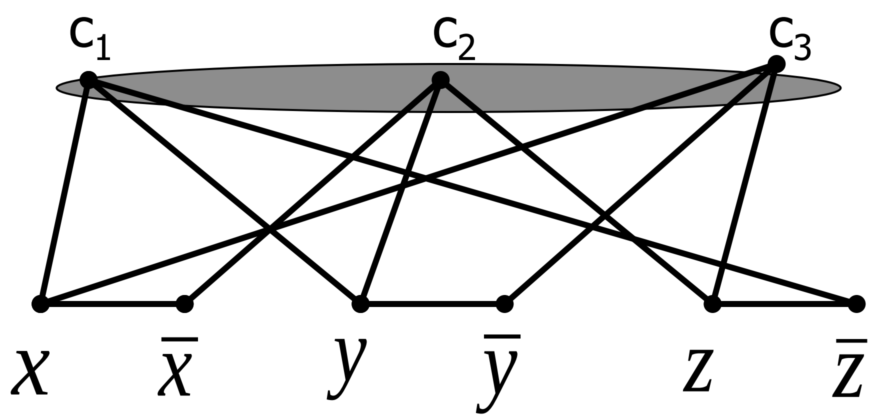

Independent Domination

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EDWARD BERTRAM AND PETER HORÁK

Some Applications of Graph Theory to Other Parts of Mathematics

Many mathematicians are now generally aware of the significance of graph theory as it is applied to other areas of science and even to societal problems. These areas include organic chemistry, solid state physics and statistical mechanics, electrical engineering (communications networks and coding theory), computer science (algorithms and computation), optimization theory, and operations research. The wide scope of these and other applications has been well documented (e.g., [4, 11]).

However, not everyone realizes that the powerful combinatorial methods found in graph theory have also been used to prove significant and well-known results in a variety of areas of *pure* mathematics. Perhaps the best known of these methods are related to a part of graph theory called *matching theory*. For example, results from this area can be used to prove Dilworth's chain decomposition theorem for finite partially ordered sets. A well-known application of matching in group theory shows that there is a common set of left and right coset representatives of a subgroup in a finite group. Also, the existence of matchings in certain infinite bipartite graphs played an important role in Laczkovich's affirmative answer to Tarski's 1925 problem of whether a circle is piecewise congruent to a square. Other applications of graph theory to pure mathematics may be found scattered throughout the literature.

Recently, a collection of examples [10] showing the application of a variety of combinatorial ideas to other areas has appeared. There, for example, matching theory is applied to give a very simple constructive proof of the existence of Haar measure on compact topological groups, but the other combinatorial applications do not focus on graph theory. The graph-theoretic applications presented here do not overlap with those in [10], and no attempt has been made at a survey. Rather, we present five examples, from

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Fermat's (Little) Theorem

There are many proofs of Fermat's Little Theorem, even short algebraic or number-theoretic proofs. The first known proof of the theorem was given by Euler, in his letter of 6 March 1742 to Goldbach. The idea of the graph-theoretic one presented below can be found in [5] where this method, together with some number–theoretic results, was used to prove Euler's generalization to nonprime modulus.

Theorem *(Fermat): Let p be a prime such that a is not divisible by p. Then, $a^p - a$ is divisible by p.*

Proof. Consider the graph $G = (V, E)$, where V is the set of all sequences $(a_1, a_2, \ldots, a_p)$ of natural numbers between 1 and a (inclusive), with $a_i \neq a_j$ for some $i \neq j$. Clearly, V has $a^p - a$ elements. For any $u \in V$, $u = (u_1, \ldots, u_{p-1}, u_p)$, let us say that $uv \in E$ just in case $v = (u_p, u_1, \ldots, u_{p-1})$. Clearly, each vertex of G is of degree 2, so each component of G is a cycle, of length p. But then, the number of components must be $(a^p - a)/p$, so $p | a^p - a$. □

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Methodology

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A graph-theory method for pattern identification in geographical epidemiology – a preliminary application to deprivation and mortality

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A graph-theory method for pattern identification in geographical epidemiology

Abstract

Background: Graph theoretical methods are extensively used in the field of computational chemistry to search datasets of compounds to see if they contain particular molecular sub-structures or patterns. We describe a preliminary application of a graph theoretical method, developed in computational chemistry, to geographical epidemiology in relation to testing a prior hypothesis. We tested the methodology on the hypothesis that if a socioeconomically deprived neighbourhood is situated in a wider deprived area, then that neighbourhood would experience greater adverse effects on mortality compared with a similarly deprived neighbourhood which is situated in a wider area with generally less deprivation.

A graph-theory method for pattern identification in geographical epidemiology

Data:

We used the Trent Region Health Authority area for this study. It had a population of approximately 5 million people. We used census enumeration districts (CED) as a proxy for neighbourhood areas, of which there were 10,665 in the Trent Region. CEDs were the lowest level of 1991 census geography at which detailed population information was available in England and Wales.

A graph-theory method for pattern identification in geographical epidemiology

Graph: the nodes represented CEDs and the edges were determined by whether or not CEDs were neighbours (i.e. they shared a common boundary).

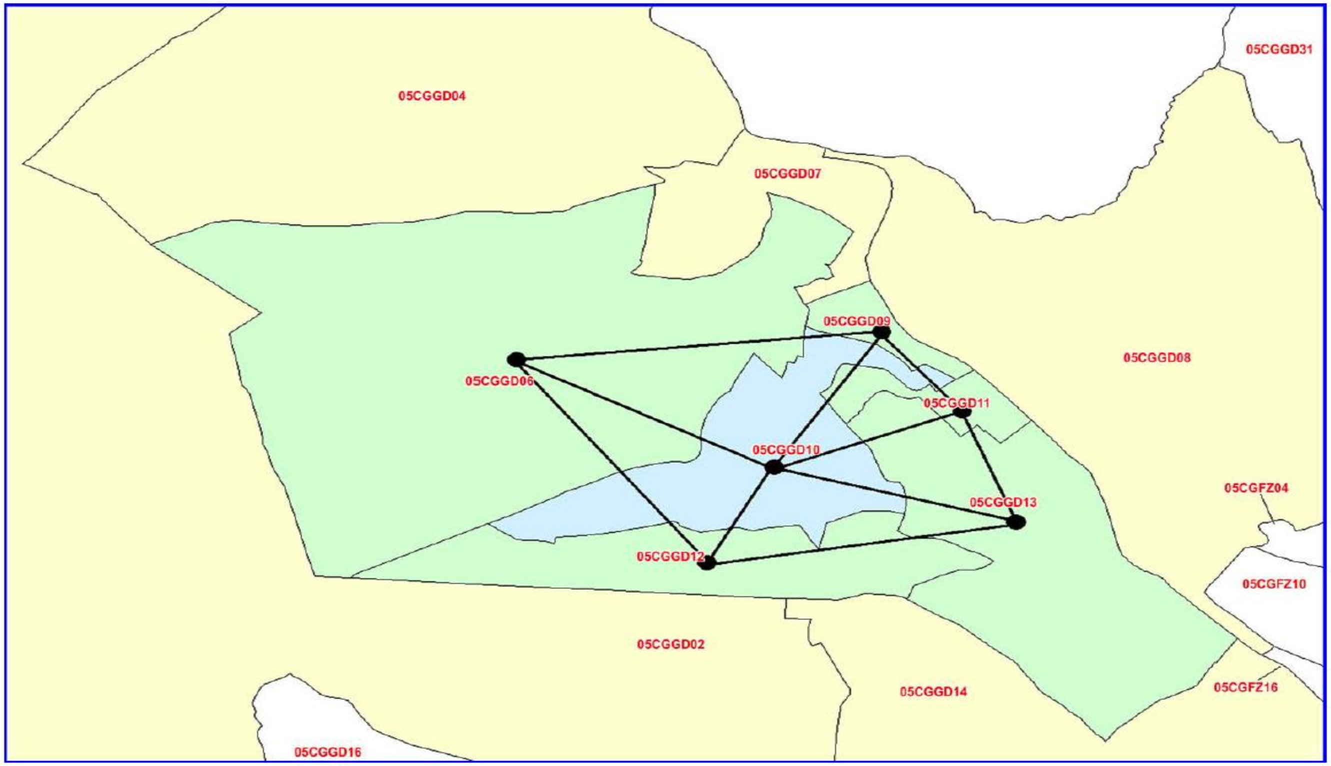


Figure 2. Map of part of the Trent region with the graph for enumeration district (ED) 05CGGD10 superimposed. The nodes (*black dots*) represent EDs and the edges (*black lines*) represent EDs that are adjacent to ED 05CGGD10.

A graph-theory method for pattern identification in geographical epidemiology

Graph: the nodes represented CEDs and the edges were determined by whether or not CEDs were neighbours (i.e. they shared a common boundary).

Each node was assigned the deprivation quintile (level) which is a number from 1 to 5

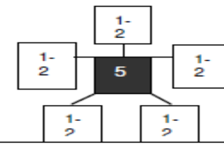
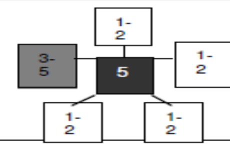
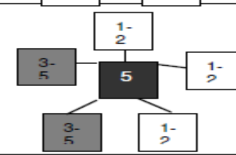
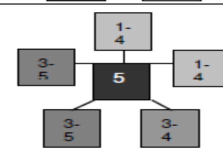
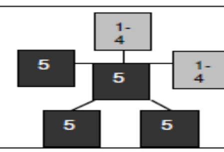
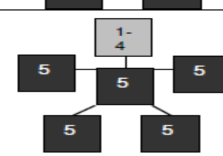
1 – affluent
2 – affluent
3 – neither affluent nor deprived
4 – neither affluent nor deprived
5 – deprived (2094 nodes)

A graph-theory method for pattern identification in geographical epidemiology

Table 1: Distribution of the number of neighbouring CEDs for 2094 deprived CEDs.

Number of adjacent CEDS	Frequency	Percent
2	27	1.3
3	160	7.6
4	347	16.6
5	506	24.2
6	433	20.7
7	303	14.5
8	125	6.0
9	78	3.7
10	49	2.3
11	31	1.5
12	12	0.6
13	9	0.4
14	6	0.3
15	5	0.2
16	1	0.05
17	1	0.05
19	1	0.05

A graph-theory method for pattern identification in geographical epidemiology

Group	Neighbours	Diagrams with deprivation quintile values
Group 1	5 affluent neighbours (deprivation quintiles 1 or 2)	
Group 2	4 affluent neighbours (deprivation quintiles 1 or 2) and 1 non-affluent neighbour (deprivation quintiles 3, 4 or 5).	
Group 3	3 affluent neighbours (deprivation quintiles 1 or 2) and 2 non-affluent neighbours (deprivation quintiles 3, 4 or 5).	
Group 4	2 non-affluent neighbours (deprivation quintiles 3, 4 or 5), 2 non-deprived neighbours (deprivation quintiles 1 to 4) and 1 non-affluent and non-deprived neighbour (deprivation quintiles 3 or 4).	
Group 5	3 deprived neighbours (deprivation quintile 5) and 2 non-deprived neighbours (deprivation quintiles 1 to 4).	
Group 6	4 deprived neighbours (deprivation quintile 5) and 1 non-deprived neighbour (deprivation quintiles 1 to 4).	
Group 7	5 deprived neighbours (deprivation quintile 5).	

A graph-theory method for pattern identification in geographical epidemiology

Table 3: Deaths, population counts and age and sex adjusted mortality rate ratios for deprived CEDs with five neighbours categorised by deprivation levels in the neighbouring CEDs.

Group*	Number of deprived CEDs	Deaths 1988–1998	Population count	Adjusted rate ratio (95% CI)
1	0	-	-	-
2	4	359	1189	1.08 (0.93 – 1.26)
3	17	1089	8089	0.86 (0.78 – 0.93)
4	214	16315	105424	1.00 (1.00 – 1.00)
5	95	6404	48388	0.98 (0.94 – 1.02)
6	81	4999	39001	1.02 (0.98 – 1.07)
7	85	4947	41519	1.02 (0.97 – 1.07)

* Group 1 – all five neighbouring CEDs were affluent; Group 7 – all five neighbouring CEDs were deprived.

Discussion

We found that the basic graph theory method we used to identify neighbourhoods which were surrounded by varying levels of deprivation showed that there was some evidence of a trend towards higher mortality in neighbourhoods surrounded by deprived areas.

Graph Mining

Graph Mining for Business Processes

Investigation of Graph Mining for Business Processes

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***Abstract*—Business process management and business intelligence are fields which gain a lot of attention in recent years. These techniques try to improve not only efficiency of processes but also save considerable cost. Graph based representation of concepts (objects, data) are also used in business domain to support aforementioned techniques. Graph mining methods are successful in many fields for discovery of new relations, knowledge, and visualization. In this paper, we briefly discuss the fields in which graph mining is successfully applied. We also discuss challenges of applying graph mining in business processes and what are the benefits.**

***Keywords*-Graph mining, business processes, graph mining applications, business intelligence, business process analysis**

frequently used to accomplish defined tasks? What are the common characteristics and relationships between activities, business objects, and their flow or relation (business process executions)? What common features can we discover between successful and unsuccessful scenarios? Prediction on a certain business flow whether it will lead to a desired state or failure. What commonalities can we find between the executions of business applications by users? How should an organization be structured to get maximum benefits from the employees (dealing with social network analysis)? What would be the next information request from the user of a system during business process execution?

Graph Mining for Business Processes

An often used example of graph mining in media industry uses IMDB[1] (Internet Movie Database) website as a resource website. This website contains information about movies and television programs. It provides movies and programs detail information to users through online queries freely. Graph mining is used on IMDB movie database in which movies attributes (actors, director, producer, etc.) are represented as graph nodes, and then graph mining methods are applied to discover new relations and knowledge. This knowledge is used to make a predictions like how much business will a certain movie do during a time-span? Will it be nominated for an award or not? The results of applying graph mining on IMDB and to some other fields are discussed in [4].

Applications of graph theory to an English rhyming corpus

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COMPUTER SPEECH AND LANGUAGE

www.elsevier.com/locate/csl

Applications of graph theory to an English rhyming corpus

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Available online 21 May 2010

Applications of graph theory to an English rhyming corpus

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Applications of graph theory to an English rhyming corpus

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This paper explores the connection between pronunciation and network structure in sets of rhymes. It discusses how rhyme graphs could be used for historical pronunciation reconstruction.

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This paper explores the connection between pronunciation and network structure in sets of rhymes. It discusses how rhyme graphs could be used for historical pronunciation reconstruction.

In particular, the author builds classifiers to separate half from full groups of rhymes, based on the groups' *rhyme graphs.*

Applications of graph theory to an English rhyming corpus

Data:

Although the long-term goal of this project is to infer historical pronunciation, this paper uses recent poetry, where the pronunciation is known, to develop and evaluate methods.

Our corpus consists of rhymes from poetry written by English authors around 1900. The contents of the corpus, itemized by author, are summarized in Table 1

Table 1
Summary of authors of rhymes used in the corpus. "Georgian Poets" are contributors to the *Georgian Poetry* anthologies (Marsh, 1916–1922).

Poet	# Rhymes (10^3)	Sources
A.E. Housman (1859–1936)	1.52	Housman (1896, 1922, 1936, 1939)
Rudyard Kipling (1865–1936)	2.60	Kipling (1889–1896, 1892, 1886)
T.W.H. Crosland (1865–1924)	0.60	Crosland (1917)
Walter de la Mare (1873–1956)	1.74	de la Mare (1901–1918)
G.K. Chesterton (1874–1936)	1.29	Chesterton (1911)
Edward Thomas (1878–1917)	0.52	Thomas (1917)
Rupert Brooke (1887–1915)	1.05	Brooke (1915)
Georgian Poets (c. 1890)	3.07	Georgian Poetry (1911–1919)

Applications of graph theory to an English rhyming corpus

Data:

Poems were first hand-annotated by rhyme scheme, then parsed using Perl scripts to extract rhyming pairs.

Some definitions

A *rhyme* is a pair of two words, w_1 and w_2, observed in rhyming position in a text.

A word's *short rhyme stem* is the nucleus and coda of its final syllable, and its *long rhyme stem* is all segments from the primary stressed nucleus on.

The rhyme is *full* if the rhyme stems of w_1 and w_2 are the same, and *half* otherwise.

Applications of graph theory to an English rhyming corpus

The rhyme graph:

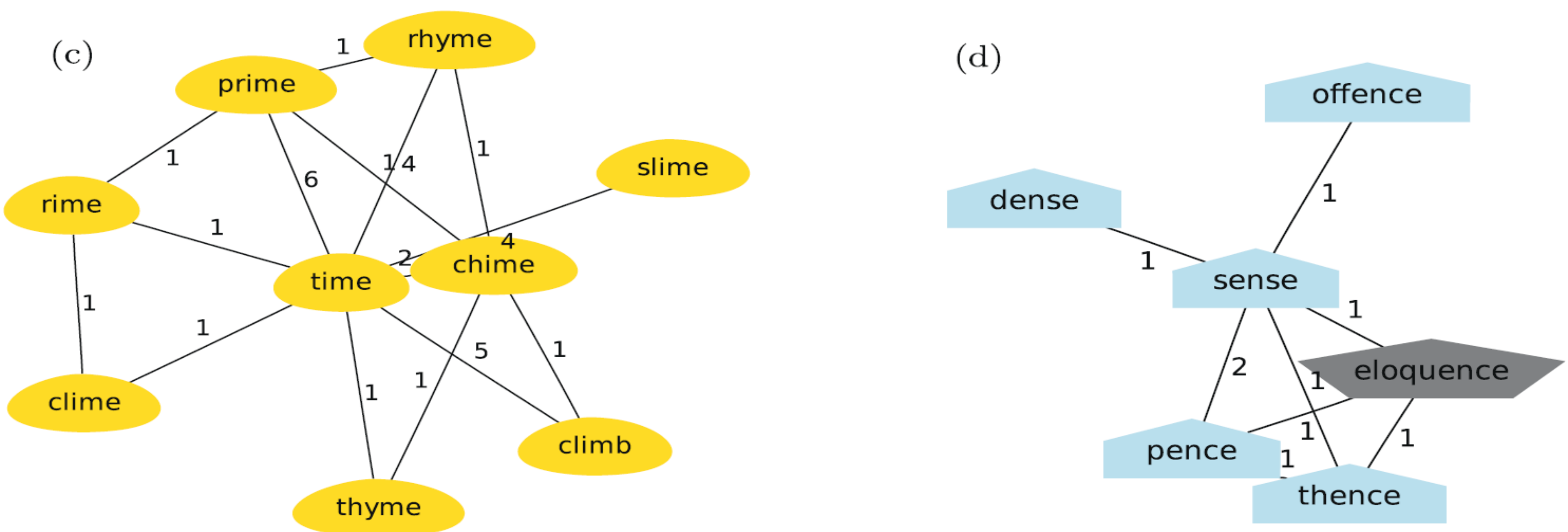
The number of nodes (words) – 4464
The number of edges (rhymes) – 6350
The weight of an edge – the number of times the rhyme was observed

The graph has 70 connected components.

Applications of graph theory to an English rhyming corpus

The rhyme graph: common components

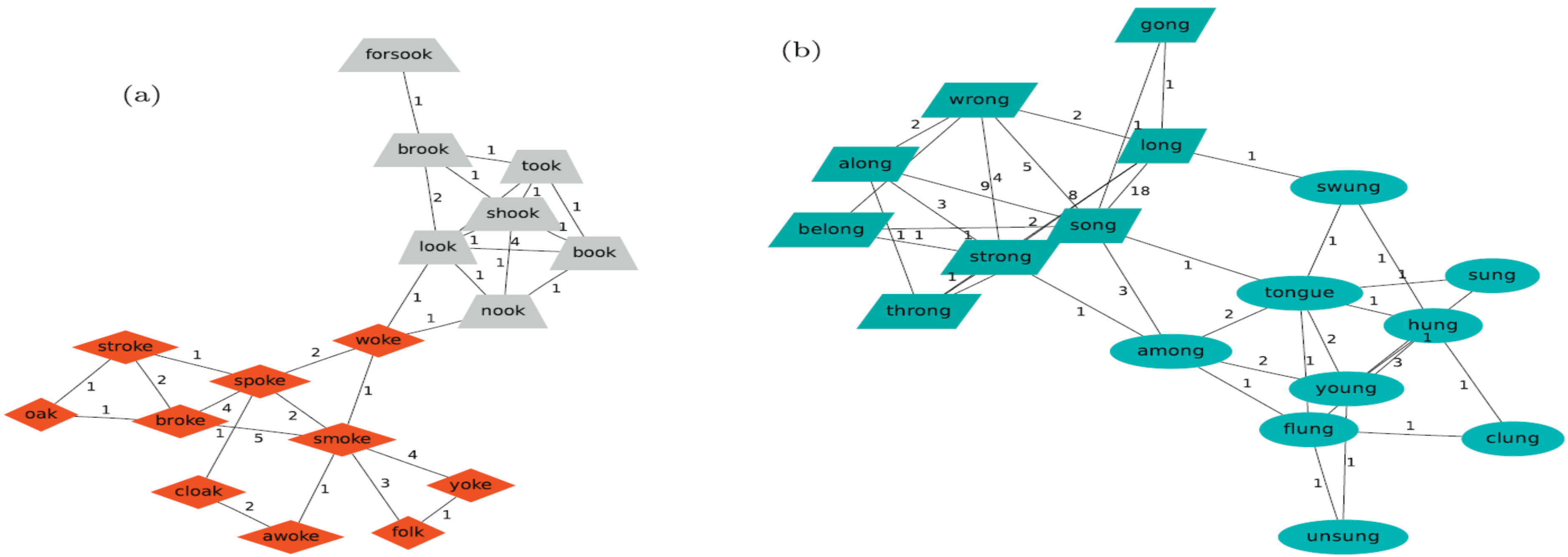
- all or nearly all words have the same stem



Applications of graph theory to an English rhyming corpus

The rhyme graph: common components

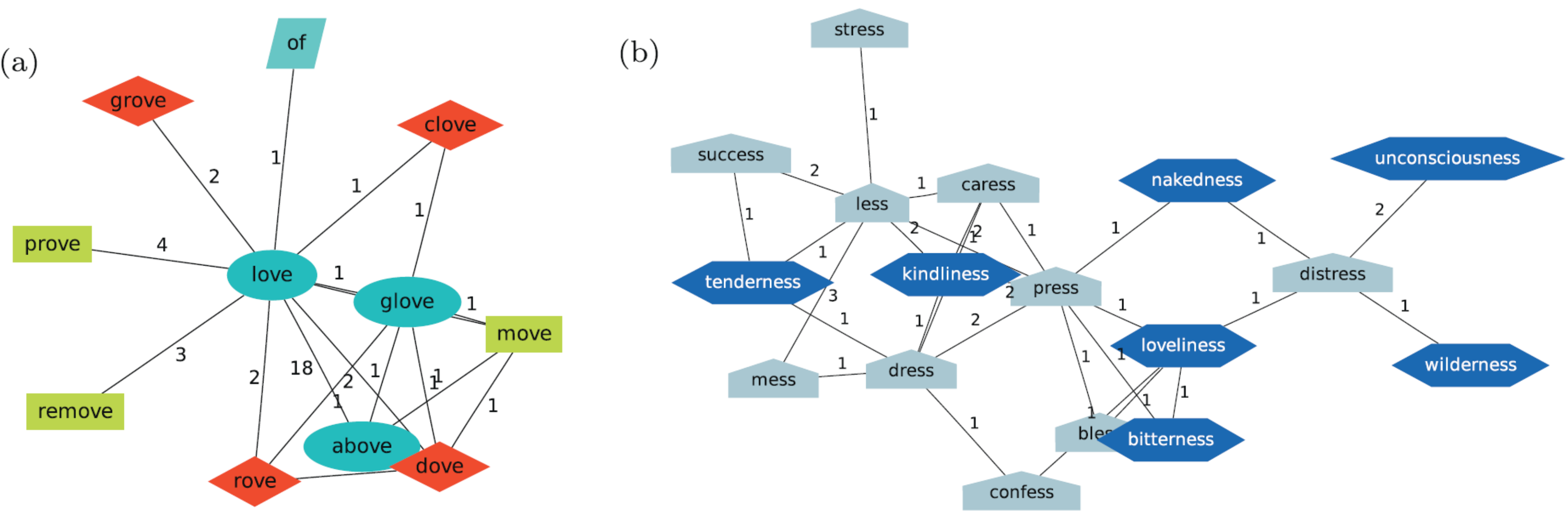
- two or more dense clusters corresponding to different stems with relatively few edges between the clusters.



Applications of graph theory to an English rhyming corpus

The rhyme graph: less common components

- contain many edges corresponding to half rhymes between words with similar spellings (*spelling rhymes*) and *poetic pronunciation conventions*



Applications of graph theory to an English rhyming corpus

Classification problem: predict which group a given component falls into, using features derived from its graph structure.

Applications of graph theory to an English rhyming corpus

Classification problem: predict which group a given component falls into, using features derived from its graph structure.

Feature set:

10 non-spectral features

- mean/max degree
- edge ratio
- max clique size
- max vertex betweenness centrality
- diameter
- mean shortest path
- radius
- mean clustering coefficient
- log size

7 spectral features

- cut lower bound 1: $\frac{\lambda_{00}}{2}$
- cut upper bound 1: $\sqrt{1-(1-\lambda_{00})^2}$
- cut lower bound 2: $\frac{\lambda_{10}}{2}$
- cut upper bound 2: $\sqrt{1-\left(1-\frac{\lambda_{10}}{\delta}\right)^2}$
- cut lower bound 3: $\frac{\lambda_{11}}{2}$
- cut upper bound 3: $2\sqrt{\lambda_{11}}$
- subset perim/area bound: $\frac{1-(1-\mu_{00})^2}{1+(1-\mu_{00})^2}$

Applications of graph theory to an English rhyming corpus

Classification problem: predict which group a given component falls into, using features derived from its graph structure.

Binary classification task

For both short and long rhyme stem data, we wish to classify components of the rhyme graph as "positive" (consisting primarily of true rhymes) or "negative" (otherwise). As a measure of component goodness, we use the percentage of vertices corresponding to the most common rhyme stem.

Classifiers

There are 33 positive/37 negative components for long rhyme stems, and 39 positive/31 negative components for short rhyme stems.
We use three non-trivial classifiers: *k*-nearest neighbors, classification and regression trees and support vector machines

Applications of graph theory to an English rhyming corpus

Some conclusions:

We have found that spectral features are more predictive of component goodness than non-spectral features; and that classifiers using a single spectral feature have 85–90% accuracy.

Graph structure for the most part transparently reflects actual pronunciation. It is (in principle) possible to “read off” pronunciation from structure.

Considering linguistic data as graphs (or networks) gives new insights into how language is structured and used. Specifically, we found a strong and striking association between graph spectra and linguistic properties.

Use of Graph Theory to Evaluate Brain Networks: A Clinical Tool for a Small World?[1]

Jeffrey R. Petrella, MD

This issue of *Radiology* features an article by Whitlow et al (1) in which graph theory methods are applied to neuroimaging data to extract information on how the brain is organized. Whitlow et al used resting-state functional magnetic resonance (MR) imaging to show that it is possible to accurately obtain graph theory metrics of large-scale brain network connectivity in as little as 2 minutes.

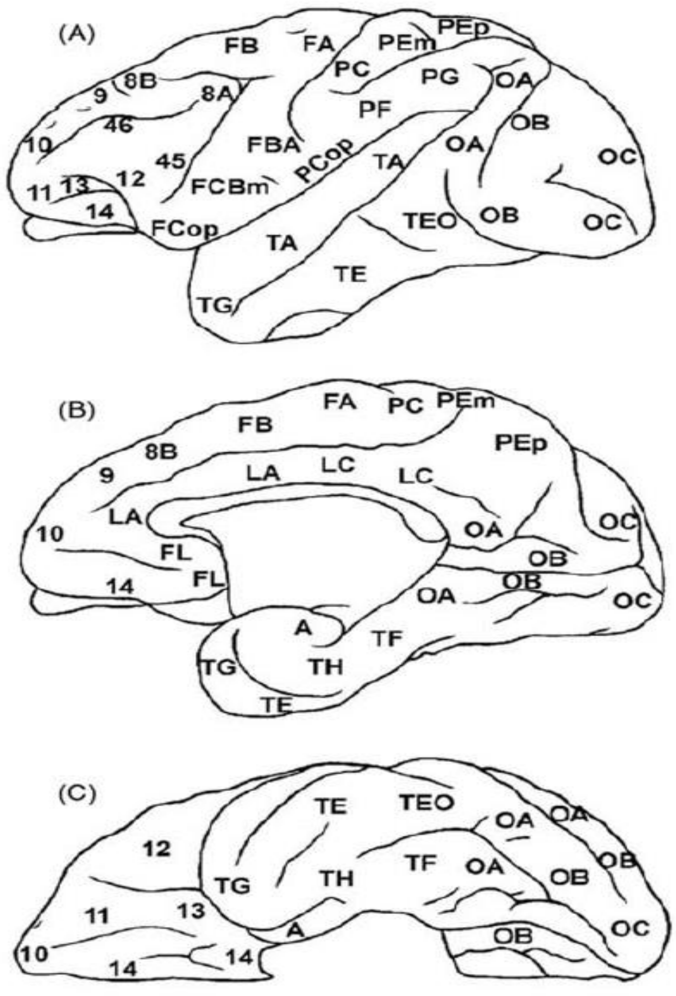
Graph theory is a branch of mathematics developed in the 18th century that deals with global and local characteristics of networks, systems modeled as a collection of elements, or nodes linked a powerful tool with which to model relations and process dynamics in many physical, biologic, and social systems. As recently as 1998, it was recognized that certain common properties were inherent in diverse and efficient networks in nature, such as the neural network of the *Caenorhabditis elegans* worm, the power grid of the western United States, and the social network of the Screen Actors Guild (3). These networks were labeled small-world networks, a term that came from the small-world phenomenon, more popularly known as six degrees of separation. Until this time, networks had been considered

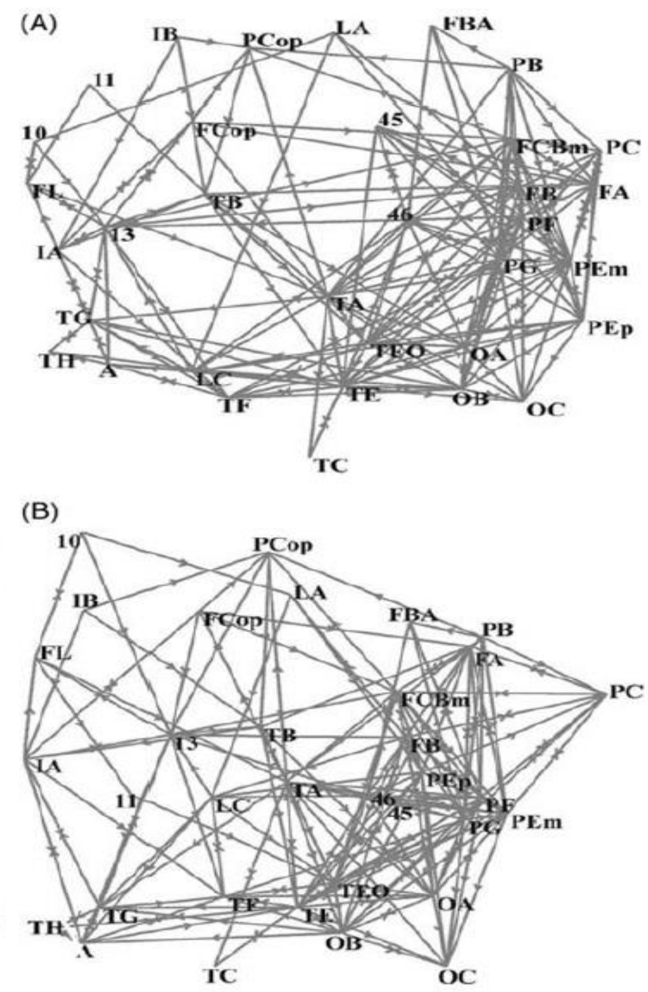
Use of Graph Theory to Evaluate Brain Networks

The brain can be considered a network on multiple scales.

- At the most elementary level, there are synaptic connections between neurons;
- at a higher level, there are corticocortical or cortico-deep gray connections between different cell types;
- at a yet higher level, there are large-scale connections between brain regions in the form of white matter bundles or fascicles.

Use of Graph Theory to Evaluate Brain Networks





Use of Graph Theory to Evaluate Brain Networks

Graph theory can help us understand the biologic underpinnings of behavioral function and dysfunction.

A number of psychiatric and neurocognitive disorders can be classified as disconnection syndromes, in which there is damage to white matter connections.

The emergence of particular symptoms can be theoretically related to particular types of damage to large-scale brain networks.

A number of studies have shown abnormalities in intrinsic brain networks in patients with different abnormal conditions, including Alzheimer disease (AD).

Use of Graph Theory to Evaluate Brain Networks

For example, a significant decrease in the clustering coefficient and small-world properties was found in patients with AD compared with control subjects.

Also, a group of researchers examined the effect of random deletions of nodes and links versus targeted deletions of highly interconnected nodes and long-distance links in healthy subjects and those with AD.

In healthy subjects, the network was resistant to both types of attack; however, in patients with AD, the network was approximately as robust to random failures but was particularly vulnerable to targeted attacks, presumably as a result of altered network organization (disrupted small world architecture).

Use of Graph Theory to Evaluate Brain Networks

In addition to helping us understand the biologic underpinnings of a number of brain disorders, brain network measures may have applications in patient care, such as early diagnosis.

Evidence is starting to accumulate in patients with disorders such as schizophrenia, depression, and attention deficit hyperactivity disorder that suggests a possible role for graph theory network measures in early diagnosis of these conditions.

CONCEPTS & SYNTHESIS
EMPHASIZING NEW IDEAS TO STIMULATE RESEARCH IN ECOLOGY

Ecology, 82(5), 2001, pp. 1205–1218

LANDSCAPE CONNECTIVITY: A GRAPH-THEORETIC PERSPECTIVE

DEAN URBAN[1,3] AND TIMOTHY KEITT[2,4]

[1]*Nicholas School of the Environment, Duke University, Durham, North Carolina 27708 USA*
[2]*National Center for Ecological Analysis and Synthesis, Santa Barbara, California 93101 USA*

Abstract. Ecologists are familiar with two data structures commonly used to represent landscapes. Vector-based maps delineate land cover types as polygons, while raster lattices represent the landscape as a grid. Here we adopt a third lattice data structure, the graph. A graph represents a landscape as a set of *nodes* (e.g., habitat patches) connected to some degree by *edges* that join pairs of nodes functionally (e.g., via dispersal). Graph theory is well developed in other fields, including geography (transportation networks, routing applications, siting problems) and computer science (circuitry and network optimization). We present an overview of basic elements of graph theory as it might be applied to issues of connectivity in heterogeneous landscapes, focusing especially on applications of metapopulation theory in conservation biology. We develop a general set of analyses using a hypothetical landscape mosaic of habitat patches in a nonhabitat matrix. Our results suggest that a simple graph construct, the minimum spanning tree, can serve as a powerful guide to decisions about the relative importance of individual patches to overall landscape connectivity. We then apply this approach to an actual conservation scenario involving the threatened Mexican Spotted Owl (*Strix occidentalis lucida*). Simulations with an incidence-function metapopulation model suggest that population persistence can be maintained despite substantial losses of habitat area, so long as the minimum spanning tree is protected. We believe that graph theory has considerable promise for applications concerned with connectivity and ecological flows in general. Because the theory is already well developed in other disciplines, it might be brought to bear immediately on pressing ecological applications in conservation biology and landscape ecology.

Yan *et al. Proteome Science* 2011, **9**(Suppl 1):S17
http://www.proteomesci.com/content/9/S1/S17

PROTEOME SCIENCE

PROCEEDINGS Open Access

Applications of graph theory in protein structure identification

Yan Yan[1,2], Shenggui Zhang[1], Fang-Xiang Wu[2*]

From International Workshop on Computational Proteomics
Hong Kong, China. 18-21 December 2010

Abstract

There is a growing interest in the identification of proteins on the proteome wide scale. Among different kinds of protein structure identification methods, graph-theoretic methods are very sharp ones. Due to their lower costs, higher effectiveness and many other advantages, they have drawn more and more researchers' attention nowadays. Specifically, graph-theoretic methods have been widely used in homology identification, side-chain cluster identification, peptide sequencing and so on. This paper reviews several methods in solving protein structure identification problems using graph theory. We mainly introduce classical methods and mathematical models including homology modeling based on clique finding, identification of side-chain clusters in protein structures upon graph spectrum, and *de novo* peptide sequencing via tandem mass spectrometry using the spectrum graph model. In addition, concluding remarks and future priorities of each method are given.

Mackaness, W. A., and Beard, M. K. (1993), Use of Graph Theory to Support Map Generalization.
Cartography and Geographic Information Systems, **20**, 210 - 221.

USE OF GRAPH THEORY TO SUPPORT GENERALISATION

William Mackaness and Kate Beard
National Center for Geographic Information and Analysis,
348 Boardman Hall,
University of Maine,
ME 04469, USA

Abstract
This paper discusses the utilization of graph theory to aid in the subtle application of specific generalization techniques during map design. The visualization of space encapsulates the notion of context, the representation of the interdependence of salient variables and even that of aesthetics; many of the subtleties of the cartographic hand rely on a rich understanding of those relationships. It is argued that any equivalent automated system needs to have the same rich knowledge explicitly or implicitly stored with each feature. The consequences for database design from this perspective are very different from ones that require efficiency or cater to spatial query.

J. Chem. Inf. Comput. Sci. **1985**, *25*, 334–343

Applications of Graph Theory in Chemistry

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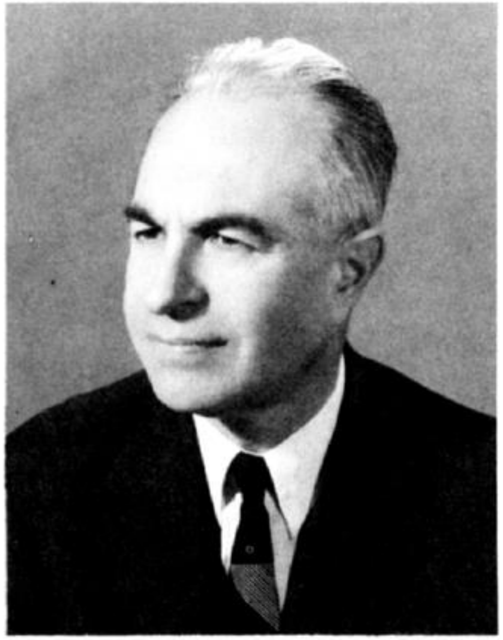
Received March 12, 1985

Graph theoretical (GT) applications in chemistry underwent a dramatic revival lately. Constitutional (molecular) graphs have points (vertices) representing atoms and lines (edges) symbolizing covalent bonds. This review deals with definition, enumeration, and systematic coding or nomenclature of constitutional or steric isomers, valence isomers (especially of annulenes), and condensed polycyclic aromatic hydrocarbons. A few key applications of graph theory in theoretical chemistry are pointed out. The complete set of all possible monocyclic aromatic and heteroaromatic compounds may be explored by a combination of Pauli's principle, Pôlya's theorem, and electronegativities. Topological indices and some of their applications are reviewed. Reaction graphs and synthon graphs differ from constitutional graphs in their meaning of vertices and edges and find other kinds of chemical applications. This paper ends with a review of the use of GT applications for chemical nomenclature (nodal nomenclature and related areas), coding, and information processing/storage/retrieval.

INTRODUCTION

All structural formulas of covalently bonded compounds are graphs: they are therefore called molecular graphs or, better, constitutional graphs. From the chemical compounds described and indexed so far, more than 90% are organic or contain organic ligands in whose constitutional formulas the lines (edges of the graph) symbolize covalent two-electron bonds and the points (vertices of the graph) symbolize atoms or, more exactly, atomic cores excluding the valence electrons. Constitutional graphs represent only one type of graphs that are of interest to chemists. Other kinds of graphs (synthon graphs, reaction graphs, etc.) will be mentioned later. This review will try to highlight applications of such graphs in chemistry: graph theory provides the basis for definition, enumeration, systematization, codification, nomenclature, correlation, and computer programming.[1–7]

Chemistry is privileged to be, both potentially and actually, the best documented branch of science. This is mainly due to the facts that most of the chemical information is associated



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Web link(s)
Web Page

RESEARCH

Research interests

- Combinatorial Optimization
- Graph Theory
- Algorithmics
- Heuristics and metaheuristics
- Decision aid systems
- Scheduling problems
- Vehicle routing problems

Professor Alain Hertz

Who Am I ?

Research activities

- Combinatorial Optimization
- Algorithmics in Graph Theory
- Evolutionary algorithms
- Scheduling and Timetabling
- Distribution and Transportation Problems

List of students

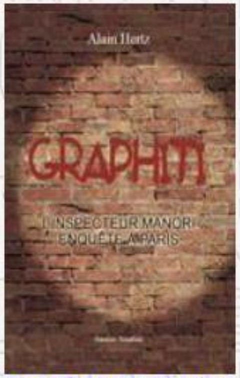
Publications

Lecture Notes on Graphs and Networks

Lecture Notes on Combinatorial Optimization

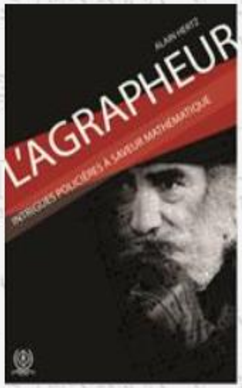
Books

GRAPHITI
L'inspecteur Manori enquête à Paris



Beth'El Vallée FM 98.4

L'AGRAPHEUR
Intrigues policières à saveur mathématique



Radio Canada interview
CJN article
Reb(((o)))nds
RTBF OPositif (7.03.13)

Der Graf der Graphen
Kriminalistische Verwicklungen mit mathmatischer Pointe

Quick on the draw
Crime-busting with a mathematical twist

Jewish Tribune

Jewish prof pens unique crime-busting novel

Mike Cohen
Quebec Bureau Chief

MONTREAL – Alain Hertz, a Jewish professor in the department of mathematical and industrial engineering of the École Polytechnique of the Université de Montréal, has used the expertise gained in his profession to write a fairly unique novel. It is a mystery based on mathematics.

The book has already been published in French and German, with the English version set to hit the shelves as early as this week.

Quick on the Draw: Crime-Busting with a Mathematical Twist deals with a theft and a hold-up, an impostor trying to collect an inheritance, the disappearance of a lab mouse worth several hundred thousand dollars, and a number of other cases.

These are the investigations led by Hertz's lead character, Maurice Manori, a police inspector known for being quick on the draw. He owes his reputation to his highly effective – but very unconventional – methods. His secret weapon? Graph theory. In search of the truth, Inspector Manori draws graphs that will introduce readers to the ins and outs of a mathematical discipline with countless handy applications.


PROFESSOR ALAIN HERTZ

Hertz said that the novel provides the layperson with an excellent breakdown of a science that's not very well known, using it to model a wide range of everyday situations.

"Thanks to its fun approach, it's great for both Sudoku and logic puzzle lovers and for math and science students and teachers," explained Hertz, a Swiss Jew who has called Montreal home for 11 years. "Learning while having fun. Isn't that what we'd all like to do?"

Quick on the Draw, Hertz said, speaks to having a fun approach to learning graph theory – a mathematical discipline with tons of everyday applications – using crime stories.

graph theory. Teachers can use the companion lecture notes, which will help them integrate the book's cases into their course material with fun exercises. But anyone who loves logic games and Sudoku will enjoy solving the cases set out in the story."

Graph theory makes it possible to model a wide range of everyday situations. Inspector Manori uses this science to identify the criminals in the cases he leads. Because he's quick to draw his graphs, he's managed to nail a number of suspects, which has earned him the reputation of being quick on

To the memory of my mother, Irène-Reiz bat Yéhouda (zal).
Your shining joy was contagious.
Your star still sparkles in the sky and in my heart.

Yéhouda ben Yossef

To the four other women who light up my life,
Muryel, Anaëlle, Sarah and Céline.

Alain

August 13, 2012

For my very good friend

VADIM

I hope you will enjoy.

Amitiés,

TABLE OF CONTENTS

The case of the missing files

Important files were stolen from an archive on the night of 10 April.

The case of the missing files

Important files were stolen from an archive on the night of 10 April.

The list of suspects includes 7 people who visited the archive 9 and 10 April.

The Case of the Missing Files 41

Suspects	Thursday, April 9, 2009	Friday, April 10, 2009
Tait	x	x
Bonneau		x
Épiney	x	x
Sporov	x	x
Lippo	x	x
Melkain	x	
Guerel	x	x

The case of the missing files

Important files were stolen from an archive on the night of 10 April.

The list of suspects includes 7 people who visited the archive 9 and 10 April.

The suspects were interviewed and the information of who saw whom is reported in the two tables on the right.

The Case of the Missing Files 41

Suspects	Thursday, April 9, 2009	Friday, April 10, 2009
Tait	x	x
Bonneau		x
Épiney	x	x
Sporov	x	x
Lippo	x	x
Melkain	x	
Guerel	x	x

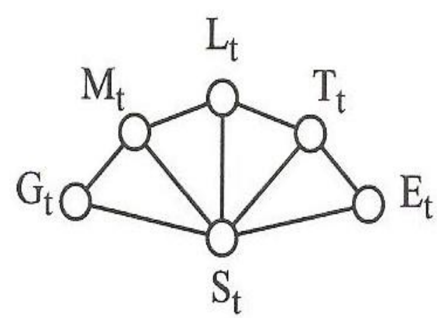
Thursday, April 9, 2009

	T	B	E	S	L	G	M
T			x	x	x		
B							
E	x			x			
S	x		x		x	x	x
L	x			x			x
G				x			x
M				x	x	x	

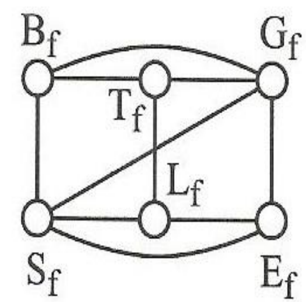
Friday, April 10, 2009

	T	B	E	S	L	G	M
T		x			x	x	
B	x			x		x	
E				x	x	x	
S		x	x		x	x	
L	x		x	x			
G	x	x	x	x			
M							

The case of the missing files



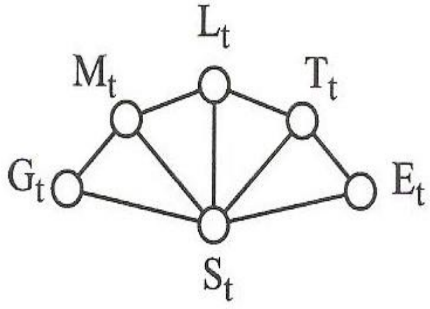
Thursday graph



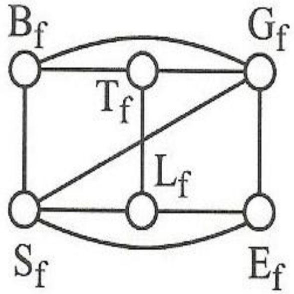
Friday graph

The case of the missing files

Every suspect claims that on each day of the visit, (s)he entered and left the archive exactly once.



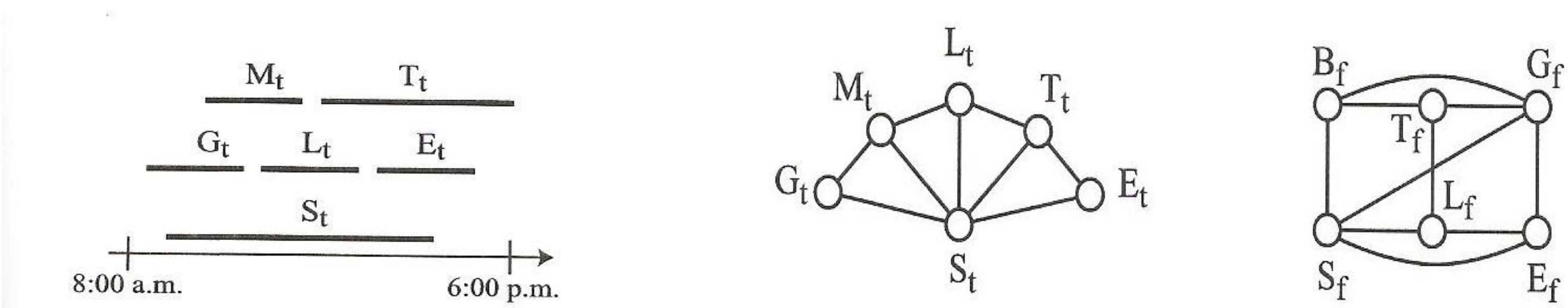
Thursday graph



Friday graph

The case of the missing files

Every suspect claims that on each day of the visit, (s)he entered and left the archive exactly once.

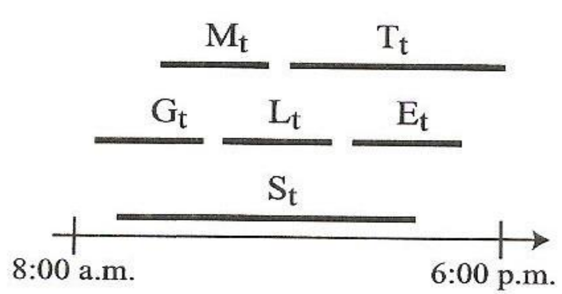


Thursday   Thursday graph   Friday graph

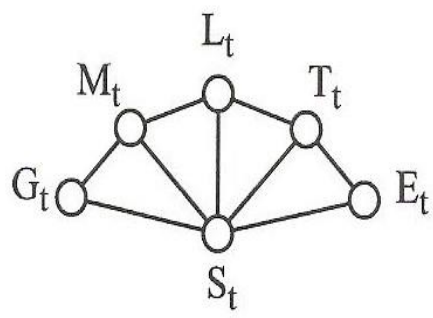
The case of the missing files

Every suspect claims that on each day of the visit, (s)he entered and left the archive exactly once.

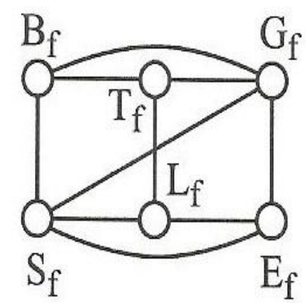
Preliminary investigation showed that the thief is the person who was the last in the archive on Thursday.



Thursday



Thursday graph



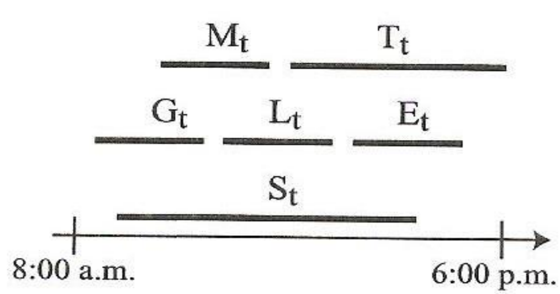
Friday graph

The case of the missing files

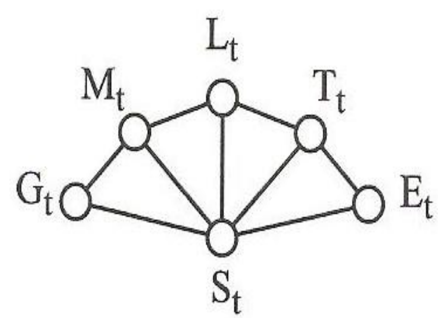
Every suspect claims that on each day of the visit, (s)he entered and left the archive exactly once.

Preliminary investigation showed that the thief is the person who was the last in the archive on Thursday.

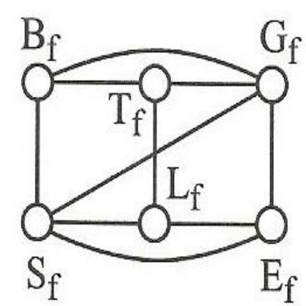
Therefore, the thief is T.



Thursday



Thursday graph



Friday graph

Thank you